

Solutions Manual

*to accompany*

# Communication Systems

An Introduction to Signals and Noise in  
Electrical Communication

Fourth Edition

A. Bruce Carlson  
*Rensselaer Polytechnic Institute*

Paul B. Crilly  
*University of Tennessee*

Janet C. Rutledge  
*University of Maryland at Baltimore*



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COMMUNICATION SYSTEMS: AN INTRODUCTION TO SIGNALS AND NOISE IN ELECTRICAL COMMUNICATION,  
FOURTH EDITION

A. BRUCE CARLSON, PAUL B. CRILLY, AND JANET C. RUTLEDGE

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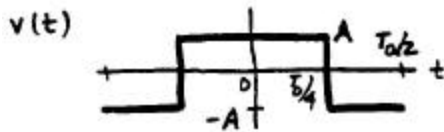
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## Chapter 2

2.1-1

$$c_n = \frac{Ae^{jf}}{T_0} \int_{-T_0/2}^{T_0/2} e^{j2p(m-n)f\delta} dt = Ae^{jf} \text{sinc}(m-n) = \begin{cases} Ae^{jf} & n = m \\ 0 & \text{otherwise} \end{cases}$$

2.1-2

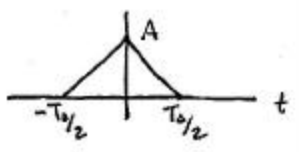


$$c_0 \langle v(t) \rangle = 0$$

$$c_n = \frac{2}{T_0} \int_0^{T_0/4} A \cos \frac{2pnt}{T_0} dt + \int_{T_0/4}^{T_0/2} (-A) \cos \frac{2pnt}{T_0} dt = \frac{2A}{pn} \sin \frac{pn}{2}$$

$n$	0	1	2	3	4	5	6	7
$ c_n $	0	$2A/p$	0	$2A/3p$	0	$2A/5p$	0	$2A/7p$
$\arg c_n$		0		$\pm 180^\circ$		0		$\pm 180^\circ$

2.1-3

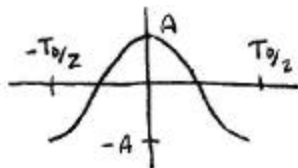


$$c_0 = \langle v(t) \rangle = A/2$$

$$c_n = \frac{2}{T_0} \int_0^{T_0/2} \left( A - \frac{2At}{T_0} \right) \cos \frac{2pnt}{T_0} dt = \frac{A}{pn} \sin pn - \frac{A}{(pn)^2} (\cos pn - 1)$$

$n$	0	1	2	3	4	5	6
$ c_n $	$0.5A$	$0.2A$	0	$0.02A$	0	$0.01A$	0
$\arg c_n$	0	0		0		0	

2.1-4



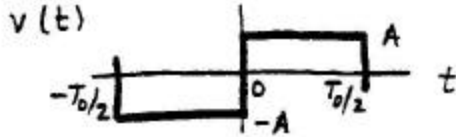
$$c_0 = \frac{2}{T_0} \int_0^{T_0/2} A \cos \frac{2pt}{T_0} dt = 0$$

(cont.)

$$c_n = \frac{2}{T_0} \int_0^{T_0/2} A \cos \frac{2pt}{T_0} \cos \frac{2pnt}{T_0} dt = \frac{2A}{T_0} \left[ \frac{\sin(\mathbf{p} - \mathbf{p}n) 2t / T_0}{4(\mathbf{p} - \mathbf{p}n) / T_0} + \frac{\sin(\mathbf{p} + \mathbf{p}n) 2t / T_0}{4(\mathbf{p} + \mathbf{p}n) / T_0} \right]_0^{T_0/2}$$

$$= \frac{A}{2} [\text{sinc}(1-n) + \text{sinc}(1+n)] = \begin{cases} A/2 & n = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

2.1-5

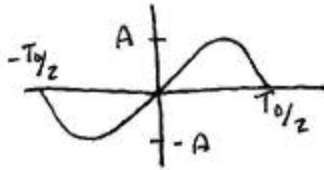


$$c_0 = \langle v(t) \rangle = 0$$

$$c_n = -j \frac{2}{T_0} \int_0^{T_0/2} A \sin \frac{2pnt}{T_0} dt = -j \frac{A}{pn} (1 - \cos pn)$$

$n$	1	2	3	4	5
$ c_n $	$2A/p$	0	$2A/3p$		$2A/5p$
$\arg c_n$	$-90^\circ$		$-90^\circ$		$-90^\circ$

2.1-6



$$c_0 = \langle v(t) \rangle = 0$$

$$c_n = -j \frac{2}{T_0} \int_0^{T_0/2} A \sin \frac{2pt}{T_0} \sin \frac{2pnt}{T_0} dt = -j \frac{2A}{T_0} \left[ \frac{\sin(\mathbf{p} - \mathbf{p}n) 2t / T_0}{4(\mathbf{p} - \mathbf{p}n) / T_0} - \frac{\sin(\mathbf{p} + \mathbf{p}n) 2t / T_0}{4(\mathbf{p} + \mathbf{p}n) / T_0} \right]_0^{T_0/2}$$

$$= -j \frac{A}{2} [\text{sinc}(1-n) - \text{sinc}(1+n)] = \begin{cases} \mp jA/2 & n = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

2.1-7

$$c_n = \frac{1}{T_0} \left[ \int_0^{T_0/2} v(t) e^{-jn\omega_0 t} dt + \int_{T_0/2}^{T_0} v(t) e^{-jn\omega_0 t} dt \right]$$

$$\text{where } \int_{T_0/2}^{T_0} v(t) e^{-jn\omega_0 t} dt = \int_0^{T_0/2} v(l + T_0/2) e^{-jn\omega_0 l} e^{-jn\omega_0 T_0/2} dl$$

$$= -e^{jnp} \int_0^{T_0/2} v(t) e^{-jn\omega_0 t} dt$$

since  $e^{jnp} = 1$  for even  $n$ ,  $c_n = 0$  for even  $n$

2.1-8

$$P = |c_0|^2 + 2 \sum_{n=1}^{\infty} |c_n|^2 = |Af_0 t|^2 + 2|Af_0 t \operatorname{sinc} f_0 t|^2 + 2|Af_0 t \operatorname{sinc} 2f_0 t|^2 + 2|Af_0 t \operatorname{sinc} 3f_0 t|^2 + \dots$$

where  $\frac{1}{t} = 4f_0$

$$|f| > \frac{1}{t} \quad P = \frac{A^2}{16} \left[ 1 + 2\operatorname{sinc}^2 \frac{1}{4} + 2\operatorname{sinc}^2 \frac{1}{2} + 2\operatorname{sinc}^2 \frac{3}{4} \right] = 0.23A^2$$

$$|f| > \frac{2}{t} \quad P = \frac{A^2}{16} \left[ 1 + 2\operatorname{sinc}^2 \frac{1}{4} + 2\operatorname{sinc}^2 \frac{1}{2} + 2\operatorname{sinc}^2 \frac{3}{4} + 2\operatorname{sinc}^2 \frac{5}{4} + 2\operatorname{sinc}^2 \frac{3}{2} + 2\operatorname{sinc}^2 \frac{7}{4} \right] = 0.24A^2$$

$$|f| > \frac{1}{2t} \quad P = \frac{A^2}{16} \left[ 1 + 2\operatorname{sinc}^2 \frac{1}{4} + 2\operatorname{sinc}^2 \frac{1}{2} \right] = 0.21A^2$$

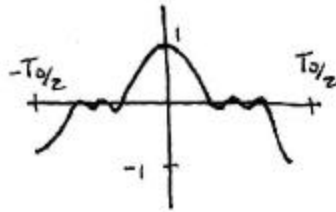
2.1-9

$$c_n = \begin{cases} 0 & n \text{ even} \\ \left( \frac{2}{pn} \right)^2 & n \text{ odd} \end{cases}$$

$$a) \quad P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \left( 1 - \frac{4|t|}{T_0} \right)^2 dt = \frac{2}{T_0} \int_0^{T_0/2} \left( 1 - \frac{4t}{T_0} \right)^2 dt = \frac{1}{3}$$

$$P' = 2 \left( \frac{4}{p^2} \right)^2 + 2 \left( \frac{4}{9p^2} \right)^2 + 2 \left( \frac{4}{25p^2} \right)^2 = 0.332 \quad \text{so } P'/P = 99.6\%$$

$$b) \quad v'(t) = \frac{8}{p^2} \cos w_0 t + \frac{8}{9p^2} \cos 3w_0 t + \frac{8}{25p^2} \cos 5w_0 t$$



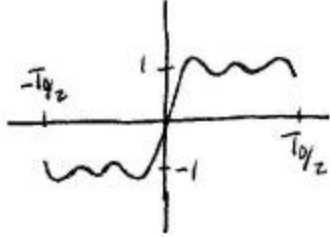
2.1-10

$$c_n = \begin{cases} 0 & n \text{ even} \\ \frac{-j2}{pn} & n \text{ odd} \end{cases}$$

$$a) \quad P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1)^2 dt = 1 \quad P' = 2 \left[ \left( \frac{2}{p} \right)^2 + \left( \frac{2}{3p} \right)^2 + \left( \frac{2}{5p} \right)^2 \right] = 0.933 \quad \text{so } P'/P = 93.3\%$$

(cont.)

$$\begin{aligned} \text{b) } v'(t) &= \frac{4}{\mathbf{p}} \cos(\mathbf{w}_0 t - 90^\circ) + \frac{4}{3\mathbf{p}} \cos(3\mathbf{w}_0 t - 90^\circ) + \frac{4}{5\mathbf{p}} \cos(5\mathbf{w}_0 t - 90^\circ) \\ &= \frac{4}{\mathbf{p}} \sin(\mathbf{w}_0 t) + \frac{4}{3\mathbf{p}} \sin(3\mathbf{w}_0 t) + \frac{4}{5\mathbf{p}} \sin(5\mathbf{w}_0 t) \end{aligned}$$



2.1-11

$$P = \frac{1}{T_0} \int_0^{T_0} \left( \frac{t}{T_0} \right)^2 dt = \frac{1}{3} \quad |c_n| = \begin{cases} 1/2 & n=0 \\ 1/2\mathbf{p}n & n \neq 0 \end{cases}$$

$$P = 2 \sum_{n \text{ odd}} \left( \frac{2}{\mathbf{p}n} \right)^4 = 2 \left( \frac{2}{\mathbf{p}} \right)^4 \left( \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \right) = \frac{1}{3}$$

$$\text{Thus, } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{4\mathbf{p}^2}{2} \left( \frac{1}{3} - \frac{1}{4} \right) = \frac{\mathbf{p}^2}{6}$$

2.1-12

$$P = \frac{2}{T_0} \int_0^{T_0/2} \left( 1 - \frac{4t}{T_0} \right)^2 dt = \frac{1}{3} \quad |c_n| = \begin{cases} 0 & n \text{ even} \\ (2/\mathbf{p}n)^2 & n \text{ odd} \end{cases}$$

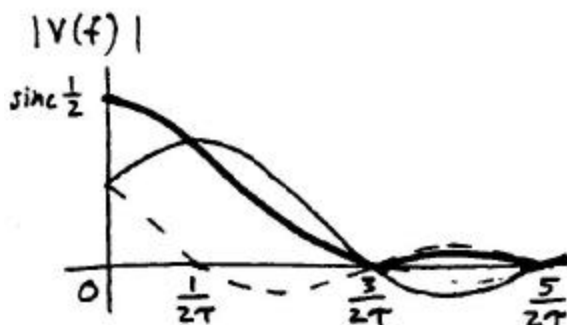
$$P = \left( \frac{1}{2} \right)^2 + 2 \sum_{n=1}^{\infty} \left( \frac{1}{2\mathbf{p}n} \right)^2 = \frac{1}{4} + \frac{2}{4\mathbf{p}^2} \left( \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right) = \frac{1}{3}$$

$$\text{Thus, } \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots = \frac{\mathbf{p}^4}{2 \cdot 2^4} \frac{1}{3} = \frac{\mathbf{p}^4}{96}$$

2.2-1

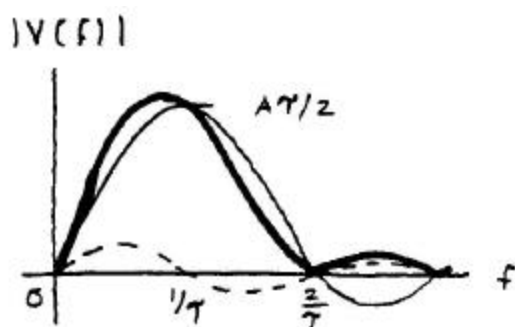
$$\begin{aligned} V(f) &= 2 \int_0^{t/2} A \cos \frac{\mathbf{p}t}{t} \cos 2\mathbf{p}ft dt \\ &= 2A \left[ \frac{\sin \left( \frac{\mathbf{p}}{t} - 2\mathbf{p}f \right) \frac{t}{2}}{2 \left( \frac{\mathbf{p}}{t} - 2\mathbf{p}f \right)} + \frac{\sin \left( \frac{\mathbf{p}}{t} + 2\mathbf{p}f \right) \frac{t}{2}}{2 \left( \frac{\mathbf{p}}{t} + 2\mathbf{p}f \right)} \right] = \frac{At}{2} [\text{sinc}(f\mathbf{t} - 1/2) + \text{sinc}(f\mathbf{t} + 1/2)] \end{aligned}$$

(cont.)



2.2-2

$$\begin{aligned}
 V(f) &= -j2 \int_0^{t/2} A \sin \frac{2pt}{t} \cos 2pf t dt \\
 &= -j2A \left[ \frac{\sin \left( \frac{2p}{t} - 2pf \right) \frac{t}{2}}{2 \left( \frac{2p}{t} - 2pf \right)} - \frac{\sin \left( \frac{2p}{t} + 2pf \right) \frac{t}{2}}{2 \left( \frac{2p}{t} + 2pf \right)} \right] = -j \frac{At}{2} [\text{sinc}(ft - 1) - \text{sinc}(ft + 1)]
 \end{aligned}$$



2.2-3

$$V(f) = 2 \int_0^t \left( A - A \frac{t}{t} \right) \cos \omega t dt = \frac{2At}{(\omega t)^2} \left[ 2 \sin^2 \left( \frac{\omega t}{2} \right) - 1 + 1 \right] = At \text{sinc}^2 ft$$

2.2-4

$$\begin{aligned}
 V(f) &= -j2 \int_0^t A \frac{t}{t} \sin \omega t dt = -j \frac{2At}{(\omega t)^2} (\sin \omega t - \omega t \cos \omega t) \\
 &= -j \frac{A}{pf} (\text{sinc} 2ft - \cos 2pf t)
 \end{aligned}$$

2.2-5

$$\begin{aligned}
 v(t) &= \text{sinc} 2Wt \leftrightarrow \frac{1}{2W} \Pi \left( \frac{f}{2W} \right) \\
 \int_{-\infty}^{\infty} |\text{sinc} 2Wt|^2 dt &= \int_{-\infty}^{\infty} \left| \frac{1}{2W} \Pi \left( \frac{f}{2W} \right) \right|^2 df = \int_{-\infty}^{\infty} \frac{1}{4W^2} df = \frac{1}{2W}
 \end{aligned}$$

2.2-6

$$E = \int_0^\infty (Ae^{-bt})^2 dt = \frac{A^2}{2b} \quad E' = 2 \int_0^W \frac{A^2}{b^2 + (2pf)^2} df = \frac{A^2}{pb} \arctan \frac{2pW}{b}$$

$$\frac{E'}{E} = \frac{2}{p} \arctan \frac{2pW}{b} = \begin{cases} 50\% & W = b/2p \\ 84\% & W = 2b/p \end{cases}$$

2.2-7

$$\begin{aligned} \int_{-\infty}^\infty v(t)w(t)dt &= \int_{-\infty}^\infty v(t) \left[ \int_{-\infty}^\infty W(f)e^{j\omega t} df \right] dt \\ &= \int_{-\infty}^\infty W(f) \left[ \int_{-\infty}^\infty v(t)e^{-j(-\omega)t} dt \right] df = \int_{-\infty}^\infty W(f)V(-f)df \\ V(-f) &= V^*(f) \text{ when } v(t) \text{ is real, so } \int_{-\infty}^\infty v^2(t)dt = \int_{-\infty}^\infty V(f)V^*(f)df = \int_{-\infty}^\infty |V(f)|^2 df \end{aligned}$$

2.2-8

$$\int_{-\infty}^\infty w^*(t)e^{-j2pft} dt = \left[ \int_{-\infty}^\infty w(t)e^{j2pft} dt \right]^* = \left[ \int_{-\infty}^\infty w(t)e^{-j2p(-f)t} dt \right]^* = W^*(f)$$

Let  $z(t) = w^*(t)$  so  $Z(f) = W^*(-f)$  and  $W^*(f) = Z(-f)$

$$\text{Hence } \int_{-\infty}^\infty v(t)z(t)dt = \int_{-\infty}^\infty V(f)Z(-f)df$$

2.2-9

$$\Pi\left(\frac{t}{A}\right) \leftrightarrow A \operatorname{sinc} Af \quad \text{so} \quad \operatorname{sinc} At \leftrightarrow \frac{1}{A} \Pi\left(\frac{f}{A}\right)$$

$$v(t) = \operatorname{sinc} \frac{2t}{t} \leftrightarrow V(f) = \frac{t}{2} \Pi\left(\frac{ft}{2}\right) \text{ for } A = \frac{2}{t}$$

2.2-10

$$B \cos \frac{pt}{t} \Pi\left(\frac{t}{t}\right) \leftrightarrow \frac{Bt}{2} [\operatorname{sinc}(ft - 1/2) + \operatorname{sinc}(ft + 1/2)]$$

$$\text{so } \frac{Bt}{2} [\operatorname{sinc}(ft - 1/2) + \operatorname{sinc}(ft + 1/2)] \leftrightarrow B \cos \frac{p(-f)}{t} \Pi\left(\frac{-f}{t}\right) = B \cos \frac{pf}{t} \Pi\left(\frac{f}{t}\right)$$

$$\text{Let } B = A \text{ and } t = 2W \Rightarrow z(t) = AW [\operatorname{sinc}(2Wt - 1/2) + \operatorname{sinc}(2Wt + 1/2)]$$

2.2-11

$$B \sin \frac{2pt}{t} \Pi\left(\frac{t}{t}\right) \leftrightarrow -j \frac{Bt}{2} [\operatorname{sinc}(ft - 1) + \operatorname{sinc}(ft + 1)]$$

$$\text{so } -j \frac{Bt}{2} [\operatorname{sinc}(ft - 1) + \operatorname{sinc}(ft + 1)] \leftrightarrow B \sin \frac{2p(-f)}{t} \Pi\left(\frac{-f}{t}\right) = -B \sin \frac{2pf}{t} \Pi\left(\frac{f}{t}\right)$$

$$\text{Let } B = -jA \text{ and } t = 2W \Rightarrow z(t) = AW [\operatorname{sinc}(2Wt - 1) + \operatorname{sinc}(2Wt + 1)]$$



2.2-12

$$e^{-b|f|} \leftrightarrow \frac{2b}{b^2 + (2pf)^2} \Rightarrow e^{-2pa|f|} \leftrightarrow \frac{4pa}{(2pa)^2 + (2pf)^2} = \frac{a/p}{a^2 + f^2}$$

$$\int_{-\infty}^{\infty} (e^{-2pa|f|})^2 df = \frac{1}{2pa} = \int_{-\infty}^{\infty} \left| \frac{a/p}{a^2 + f^2} \right| df = 2 \left( \frac{a}{p} \right)^2 \int_0^{\infty} \frac{df}{(a^2 + f^2)^2}$$

$$\text{Thus, } \int_0^{\infty} \frac{dx}{(a^2 + x^2)^2} = \frac{1}{2} \left( \frac{p}{a} \right)^2 \frac{1}{2pa} = \frac{p}{4a^3}$$

2.3-1

$$z(t) = v(t-T) + v(t+T) \text{ where } v(t) = A\Pi(t/T) \leftrightarrow At \operatorname{sinc} ft$$

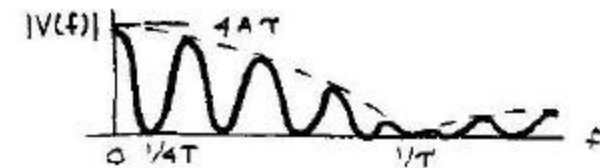
$$\text{so } Z(f) = V(f)e^{-j\omega T} + V(f)e^{j\omega T} = 2At \operatorname{sinc} ft \cos 2pfT$$



2.3-2

$$z(t) = v(t-2T) + 2v(t) + v(t+2T) \text{ where } v(t) = a\Pi(t/T) \leftrightarrow At \operatorname{sinc} ft$$

$$Z(f) = V(f)e^{-j2\omega T} + V(f) + V(f)e^{j2\omega T} = 2At(\operatorname{sinc} ft)(1 + \cos 4pfT)$$



2.3-3

$$z(t) = v(t-2T) - 2v(t) + v(t+2T) \text{ where } v(t) = a\Pi(t/T) \leftrightarrow At \operatorname{sinc} ft$$

$$Z(f) = V(f)e^{-j2\omega T} - 2V(f) + V(f)e^{j2\omega T} = 2At(\operatorname{sinc} ft)(\cos 4pfT - 1)$$



2.3-4

$$v(t) = A\Pi\left(\frac{t-T}{2T}\right) + (B-A)\Pi\left(\frac{t-T/2}{T}\right)$$

$$V(f) = 2AT \operatorname{sinc} 2fTe^{-j\omega T} + (B-A)T \operatorname{sinc} fTe^{-j\omega T/2}$$

2.3-5

$$v(t) = A\Pi\left(\frac{t-2T}{4T}\right) + (B-A)\Pi\left(\frac{t-2T}{2T}\right)$$

$$V(f) = 4AT \operatorname{sinc} 4fT e^{-j2\omega T} + 2(B-A)T \operatorname{sinc} 2fT e^{-j2\omega T}$$

2.3-6

$$\text{Let } w(t) = v(at) \leftrightarrow W(f) = \frac{1}{|a|} V(f/a)$$

$$\text{Then } z(t) = v[a(t - t_d/a)] = w(t - t_d/a) \text{ so } Z(f) = W(f) e^{-j\omega t_d/a} = \frac{1}{|a|} V(f/a) e^{-j\omega t_d/a}$$

2.3-7

$$\mathbf{F} \left[ v(t) e^{j\omega_c t} \right] = \int_{-\infty}^{\infty} v(t) e^{j\omega_c t} e^{-j\omega t} dt = \int_{-\infty}^{\infty} v(t) e^{-j2p(f-f_c)t} dt = V(f-f_c)$$

2.3-8

$$v(t) = A\Pi(t/t) \cos \mathbf{w}_c t \text{ with } \mathbf{w}_c = 2p f_c = p/t$$

$$V(f) = \frac{At}{2} \operatorname{sinc}(f-f_c)t + \frac{At}{2} \operatorname{sinc}(f+f_c)t = \frac{At}{2} [\operatorname{sinc}(ft-1/2) + \operatorname{sinc}(ft+1/2)]$$

2.3-9

$$v(t) = A\Pi(t/t) \cos(\mathbf{w}_c t - p/2) \text{ with } \mathbf{w}_c = 2p f_c = 2p/t$$

$$\begin{aligned} V(f) &= \frac{e^{-jp/2}}{2} At \operatorname{sinc}(f-f_c)t + \frac{e^{jp/2}}{2} At \operatorname{sinc}(f+f_c)t \\ &= -j \frac{At}{2} [\operatorname{sinc}(ft-1) - \operatorname{sinc}(ft+1)] \end{aligned}$$

2.3-10

$$v(t) = v(t) \cos \mathbf{w}_c t \quad v(t) = Ae^{-|t|} \leftrightarrow \frac{2A}{1+(2pf)^2}$$

$$Z(f) = \frac{1}{2} V(f-f_c) + \frac{1}{2} V(f+f_c) = \frac{A}{1+4p^2(f-f_c)^2} + \frac{A}{1+4p^2(f+f_c)^2}$$

2.3-11

$$v(t) = v(t) \cos(\mathbf{w}_c t - p/2) \quad v(t) = Ae^{-t} \text{ for } t \geq 0 \leftrightarrow \frac{A}{1+j2pf}$$

$$\begin{aligned} Z(f) &= \frac{e^{-jp/2}}{2} V(f-f_c) + \frac{e^{jp/2}}{2} V(f+f_c) = \frac{-jA/2}{1+j2p(f-f_c)} + \frac{jA/2}{1+j2p(f+f_c)} \\ &= \frac{A/2}{j-2p(f-f_c)} - \frac{A/2}{j-2p(f+f_c)} \end{aligned}$$

2.3-12

$$v(t) = t \quad z(t) = \frac{A}{t} \Pi\left(\frac{t}{t}\right) \leftrightarrow 2A \operatorname{sinc} 2ft$$

$$\frac{d}{df} Z(f) = 2A \frac{d}{df} \left[ \frac{\sin 2pft}{2pft} \right] = \frac{2A}{(2pft)^2} \left[ (2pt)^2 f \cos 2pft - 2pt \sin 2pft \right]$$

$$V(f) = \frac{1}{-j2p} \frac{d}{df} Z(f) = \frac{-jA}{pf} (\operatorname{sinc} 2ft - \cos 2pft)$$

2.3-13

$$z(t) = tv(t) \quad v(t) = Ae^{-bt} \leftrightarrow \frac{2Ab}{b^2 + (2pf)^2}$$

$$Z(f) = \frac{1}{-j2p} \frac{d}{df} \left[ \frac{2Ab}{b^2 + (2pf)^2} \right] = \frac{j2Abf}{[b^2 + (2pf)^2]^2}$$

2.3-14

$$z(t) = t^2 v(t) \quad v(t) = Ae^{-t} \text{ for } t \geq 0 \leftrightarrow \frac{A}{b + j2pf}$$

$$Z(f) = \frac{1}{(-j2pf)^2} \frac{d}{df} \left[ \frac{A}{b + j2pf} \right] = \frac{2A}{[b + j2pf]^3}$$

2.3-15

$$v(t) = e^{-p(bt)^2} \leftrightarrow V(f) = \frac{1}{b} e^{-p(f/b)^2}$$

$$(a) \frac{d}{dt} v(t) = -2pb^2 t e^{-p(bt)^2} \leftrightarrow \frac{j2pf}{b} e^{-p(f/b)^2}$$

$$(b) t e^{-p(bt)^2} \leftrightarrow \frac{1}{-j2p} \frac{d}{df} V(f) = \frac{f}{jb} e^{-p(f/b)^2}$$

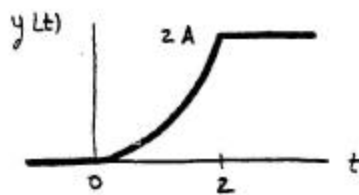
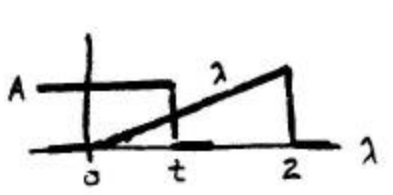
Both results are equivalent to  $bte^{-p(bt)^2} \leftrightarrow -jf e^{-p(f/b)^2}$

2.4-1

$$y(t) = 0 \quad t < 0$$

$$= \int_0^t A l \, dl = \frac{At^2}{2} \quad 0 < t < 2$$

$$= \int_0^2 A l \, dl = 2A \quad t > 2$$



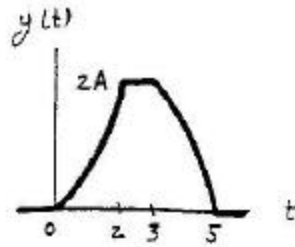
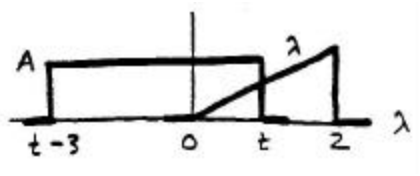
2.4-2

$$y(t) = 0 \quad t < 0, t > 5$$

$$= \int_0^t A l \, dl = \frac{At^2}{2} \quad 0 < t < 2$$

$$= \int_0^2 A l \, dl = 2A \quad 2 < t < 3$$

$$= \int_{t-3}^2 A l \, dl = \frac{A}{2} [4 - (t-3)^2] \quad 3 < t < 5$$



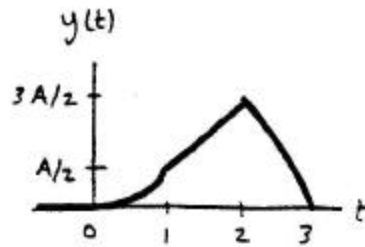
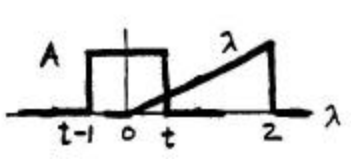
2.4-3

$$y(t) = 0 \quad t < 0, t > 3$$

$$= \int_0^t A l \, dl = \frac{At^2}{2} \quad 0 < t < 1$$

$$= \int_{t-1}^t A l \, dl = \frac{A}{2} (2t-1) \quad 1 < t < 2$$

$$= \int_{t-1}^2 A l \, dl = \frac{A}{2} [4 - (t-1)^2] \quad 2 < t < 3$$

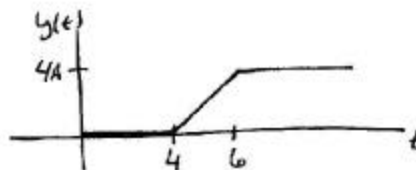
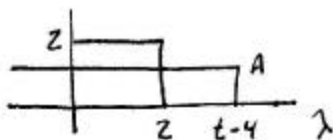


2.4-4

$$y(t) = 0 \quad t < 4$$

$$= \int_4^t 2A \, dl = 2At - 8A \quad 4 \leq t \leq 6$$

$$= \int_6^8 2A \, dl = 4A \quad t > 6$$

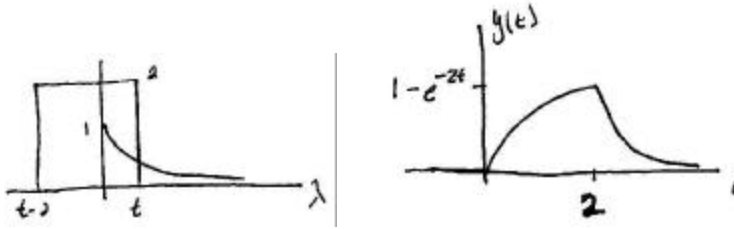


2.4-5

$$y(t) = 0 \quad t < 0$$

$$= \int_0^t 2e^{-2I} dI = 1 - e^{-2t} \quad 0 \leq t \leq 2$$

$$= \int_{t-2}^t 2e^{-2I} dI = e^{-2t} [e^4 - 1] \quad t > 2$$



2.4-6

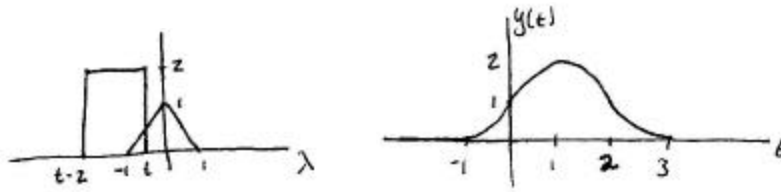
$$y(t) = 0 \quad t < -1, t \geq 3$$

$$= \int_{-1}^t 2(1+I) dI = t^2 + 2t + 1 \quad -1 \leq t < 0$$

$$= \int_{-1}^0 2(1+I) dI + \int_0^t 2(1-I) dI = -t^2 + 2t + 1 \quad 0 \leq t < 1$$

$$= \int_{t-2}^0 2(1+I) dI + \int_0^1 2(1-I) dI = -t^2 + 2t + 1 \quad 1 \leq t < 2$$

$$= \int_{t-2}^1 2(1-I) dI = t^2 - 6t + 9 \quad 2 \leq t < 3$$



2.4-7

$$y(t) = 0 \quad t \leq 0$$

$$= \int_0^t A e^{-aI} B e^{-b(t-I)} dI = [AB/(a-b)][e^{-bt} - e^{-at}] \quad t > 0$$

2.4-8

$$v(t) = A e^{-at} \quad w(t) = \sin pt = \frac{j}{2} e^{jpt} - \frac{j}{2} e^{-jpt} = B_1 e^{-b_1 t} + B_2 e^{-b_2 t}$$

$$y(t) = v * w_1(t) + v * w_2(t) = [AB_1/(a-b_1)][e^{-b_1 t} - e^{-at}] + [AB_2/(a-b_2)][e^{-b_2 t} - e^{-at}]$$

Let  $B_1 = j/2$ ,  $b_1 = -jp$ ,  $B_2 = -j/2$ ,  $b_2 = jp$  and simplify

2.4-9

$$v * w(t) = \int_{-\infty}^{\infty} v(I) w(t-I) dI \quad \text{let } m = t - I$$

$$= - \int_{-\infty}^{\infty} v(t-m) w(m) dm = \int_{-\infty}^{\infty} w(m) v(t-m) dm = w * v(t)$$

2.4-10

Let  $y(t) = \int_{-\infty}^{\infty} v(I)w(t-I)dI$  where  $v(-t) = v(t)$ ,  $w(-t) = w(t)$

$$\begin{aligned} y(-t) &= \int_{-\infty}^{\infty} v(I)w(-t-I)dI = \int_{-\infty}^{\infty} v(I)w(t+I)dI \\ &= -\int_{-\infty}^{\infty} v(-\mathbf{m})w(t-\mathbf{m})d\mathbf{m} = \int_{-\infty}^{\infty} v(\mathbf{m})w(t-\mathbf{m})d\mathbf{m} = y(t) \end{aligned}$$

2.4-11

Let  $y(t) = \int_{-\infty}^{\infty} v(I)w(t-I)dI$  where  $v(-t) = -v(t)$ ,  $w(-t) = -w(t)$

$$\begin{aligned} y(-t) &= \int_{-\infty}^{\infty} v(I)w(-t-I)dI = -\int_{-\infty}^{\infty} v(I)w(t+I)dI \\ &= \int_{-\infty}^{\infty} v(-\mathbf{m})w(t-\mathbf{m})d\mathbf{m} = \int_{-\infty}^{\infty} v(\mathbf{m})w(t-\mathbf{m})d\mathbf{m} = y(t) \end{aligned}$$

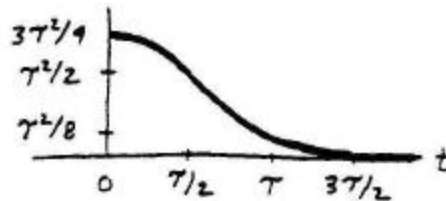
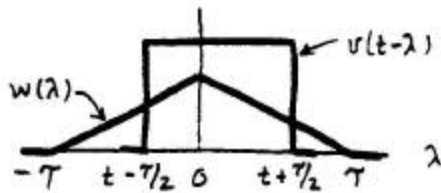
2.4-12

Let  $w(t) = v * v(t) = t\Lambda(t/t)$

$$v * w(t) = \int_{t-t/2}^0 (t+I)dI + \int_0^{t+t/2} (t-I)dI = \frac{3}{4}t^2 - t^2 \quad 0 \leq t < t/2$$

$$= \int_{t-t/2}^t (t-I)dI = \frac{1}{2} \left( t - \frac{3}{2}t \right)^2 \quad t/2 \leq t < 3t/2$$

$$\text{Thus } v * v * v(t) = \begin{cases} \frac{3}{4}t^2 - t^2 & |t| < t/2 \\ \frac{1}{2} \left( |t| - \frac{3}{2}t \right)^2 & t/2 \leq |t| < 3t/2 \\ 0 & |t| \geq 3t/2 \end{cases}$$



2.4-13

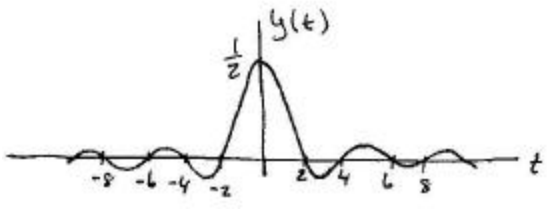
$$\mathbf{F} \{ v(t) * [w(t) * z(t)] \} = V(f)[W(f)Z(f)] = [V(f)W(f)]Z(f)$$

$$\text{so } v(t) * [w(t) * z(t)] = \mathbf{F}^{-1} \{ [V(f)W(f)]Z(f) \} = [v(t) * w(t)] * z(t)$$

2.4-14

$$V(f) = \frac{1}{4} \Pi\left(\frac{f}{4}\right) \quad W(f) = 4\Pi(2f)$$

$$Y(f) = V(f)W(f) = \Pi(2f) \leftrightarrow y(t) = (1/2)\text{sinc}(t/2)$$



2.5-1

$$z(t) = A\Pi\left(\frac{t}{t}\right) \cos \omega_c t \quad Z(f) = \frac{At}{2} \text{sinc}(f - f_c)t + \frac{At}{2} \text{sinc}(f + f_c)t$$

As  $t \rightarrow 0$  the cosine pulse  $z(t)$  gets narrower and narrower while maintaining height A.

This is not the same as an impulse since the area under the curve is also getting smaller.

As  $t \rightarrow 0$  the main lobe and side lobes of the spectrum  $Z(f)$  get wider and wider, however the height gets smaller and smaller. Eventually the spectrum will cover all frequencies with almost zero energy at each frequency. Again this is different from what happens in the case of an impulse.

2.5-2

$$\begin{aligned} W(f) &= v(f)e^{-j2\pi f t_d} = \left[ \sum_n c_v(nf_0) \mathbf{d}(f - nf_0) \right] e^{-j2\pi f t_d} \\ &= \sum_n \left[ c_v(nf_0) e^{-j2\pi n f_0 t_d} \right] \mathbf{d}(f - nf_0) \Rightarrow c_w(nf_0) = c_v(nf_0) e^{-j2\pi n f_0 t_d} \end{aligned}$$

2.5-3

$$\begin{aligned} W(f) &= j2\pi f V(f) = j2\pi f \left[ \sum_n c_v(nf_0) \mathbf{d}(f - nf_0) \right] = \sum_n [j2\pi n f_0 c_v(nf_0)] \mathbf{d}(f - nf_0) \\ &\Rightarrow c_w(nf_0) = j2\pi n f_0 c_v(nf_0) \end{aligned}$$

2.5-4

$$\begin{aligned} W(f) &= \frac{1}{2} [V(f - mf_0) + V(f + mf_0)] = \frac{1}{2} \left[ \sum_n c_v(nf_0) \mathbf{d}(f - kf_0 - mf_0) + \sum_n c_v(nf_0) \mathbf{d}(f - kf_0 + mf_0) \right] \\ &= \frac{1}{2} \left[ \sum_k c_v[(k - m)f_0] \mathbf{d}(f - kf_0) + \sum_k c_v[(k + m)f_0] \mathbf{d}(f - kf_0) \right] \\ &= \sum_n \frac{1}{2} \{ c_v[(n - m)f_0] + c_v[(n + m)f_0] \} \mathbf{d}(f - nf_0) \\ \text{so } c_w(nf_0) &= \frac{1}{2} \{ c_v[(n - m)f_0] + c_v[(n + m)f_0] \} \end{aligned}$$

2.5-5

$$v(t) = Au(t) - Au(t - 2t)$$

$$V(f) = A \left\{ \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f) - \left[ \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f) \right] e^{-j4pft} \right\}$$

But  $\mathbf{d}(f)e^{-j4pft} = e^{-j0} \mathbf{d}(f)$ , so

$$V(f) = \frac{A}{j2pf} (1 - e^{-j4pft}) = 2At \operatorname{sinc} 2ft e^{-j2pft}$$

$$\text{Agrees with } v(t) = \Pi\left(\frac{t-t}{2t}\right) \leftrightarrow 2At \operatorname{sinc} 2ft e^{-j2pft}$$

2.5-6

$$v(t) = A - Au(t + t) + Au(t - t)$$

$$V(f) = A \left\{ \mathbf{d}(f) - \left[ \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f) \right] e^{j2pft} - \left[ \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f) \right] e^{-j2pft} \right\}$$

But  $\mathbf{d}(f)e^{j2pft} = \mathbf{d}(f)e^{-j2pft} = e^{j0} \mathbf{d}(f)$ , so

$$V(f) = A \left[ \mathbf{d}(f) - \frac{1}{j2pf} (e^{j2pft} - e^{-j2pft}) \right] = A \mathbf{d}(f) - 2At \operatorname{sinc} 2ft$$

$$\text{Agrees with } v(t) = A - A\Pi(t/2t) \leftrightarrow A \mathbf{d}(f) - 2At \operatorname{sinc} 2ft$$

2.5-7

$$v(t) = A - Au(t + T) - Au(t - T)$$

$$V(f) = A \left\{ \mathbf{d}(f) - \left[ \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f) \right] e^{j2p f T} - \left[ \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f) \right] e^{-j2p f T} \right\}$$

But  $\mathbf{d}(f)e^{j2p f T} = \mathbf{d}(f)e^{-j2p f T} = e^{j0} \mathbf{d}(f) = \mathbf{d}(f)$ , so

$$V(f) = \frac{-A}{j2pf} (e^{j2p f T} + e^{-j2p f T}) = \frac{-A}{jpf} \cos 2p f T$$

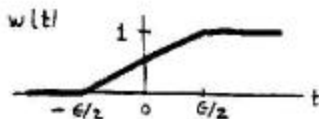
$$\text{If } T \rightarrow 0, \quad v(t) = -A \operatorname{sng} t \leftrightarrow V(f) = \frac{-A}{jpf}, \text{ which agrees with Eq. (17)}$$

2.5-8

$$V(f) = \operatorname{sinc} f \mathbf{e} \text{ and } V(0) = 1, \text{ so}$$

$$W(f) = \frac{\operatorname{sinc} f \mathbf{e}}{j2pf} + \frac{1}{2} \mathbf{d}(f)$$

$$\text{If } \mathbf{e} \rightarrow 0, \quad w(t) = u(t) \text{ and } W(f) = \frac{1}{j2pf} + \frac{1}{2} \mathbf{d}(f), \text{ which agrees with Eq. (18)}$$



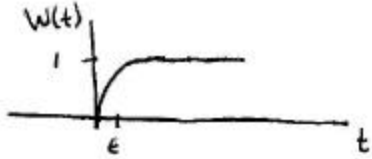


2.5-9

$$V(f) = \frac{1/e}{1/e + j2pf} \quad \text{and } V(0) = 1, \text{ so}$$

$$W(f) = \frac{1/e}{(j2pf)(1/e + j2pf)} + \frac{1}{2}d(f)$$

$$\text{If } e \rightarrow 0, \quad w(t) = u(t) \quad \text{and} \quad W(f) = \frac{1}{j2pf} + \frac{1}{2}d(f), \text{ which agrees with Eq. (18)}$$



2.5-10

$$z(t) = A\Pi(t/t) * [d(t-T) + d(t+T)]$$

$$\text{so } Z(f) = (At \operatorname{sinc} ft) (e^{-j\omega T} + e^{j\omega T}) = 2At \operatorname{sinc} ft \cos 2p fT$$

2.5-11

$$z(t) = A\Pi(t/t) * [d(t-2T) + 2d(t) + d(t+2T)]$$

$$\text{so } Z(f) = (At \operatorname{sinc} ft) (e^{-j\omega 2T} + 2 + e^{j\omega 2T}) = 2At \operatorname{sinc} ft (1 + \cos 4p fT)$$

2.5-12

$$z(t) = A\Pi(t/t) * [d(t-2T) - 2d(t) + d(t+2T)]$$

$$\text{so } Z(f) = (At \operatorname{sinc} ft) (e^{-j\omega 2T} - 2 + e^{j\omega 2T}) = 2At \operatorname{sinc} ft (\cos 4p fT - 1)$$

2.5-13

$n$	0	1	2	3	4	5	6	7	8
$\sin(p\hat{t})d(t-0.5n)$	0	1	0	1	0	1	0	1	0
$v(t)$	0	1	1	2	2	3	3	4	4

2.5-14

$n$	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
$\cos(2p\hat{t})d(t-0.1n)$	1	0.81	0.31	-0.31	-0.81	-1	-0.81	-0.31	0.31	0.81	1
$v(t)$	1	1.81	2.12	1.81	1	0	-0.81	-1.12	-0.81	0	1
$v(\hat{t}) \text{ for } n=1,10$	1.81	2.12	1.81	1	0	-0.81	-1.12	-0.81	0	1	

## Chapter 3

3.1-1

$$y(t) = h(t) * A[\mathbf{d}(t+t_d) - \mathbf{d}(t-t_d)] = A[h(t+t_d) - h(t-t_d)]$$

$$Y(f) = H(f)A(e^{j\omega t_d} - e^{-j\omega t_d}) = j2AH(f)\sin 2\pi f t_d$$

3.1-2

$$y(t) = h(t) * A[\mathbf{d}(t+t_d) + \mathbf{d}(t)] = A[h(t+t_d) + h(t)]$$

$$Y(f) = H(f)A(e^{j\omega t_d} + 1) = 2AH(f)\cos \pi f t_d e^{j\pi f t_d}$$

3.1-3

$$y(t) = h(t) * Ah(t-t_d) = Ah(t) * h(t-t_d)$$

$$Y(f) = H(f)AH(f)e^{-j\omega t_d} = AH^2(f)e^{-j\omega f t_d}$$

3.1-4

$$y(t) = h(t) * Au(t-t_d) = A \int_{-\infty}^{t-t_d} h(\mathbf{l}) d\mathbf{l}$$

$$Y(f) = H(f)A \left[ \frac{1}{j2\pi f} + \frac{1}{2} \mathbf{d}(f) \right] e^{-j\omega t_d} = \frac{A}{j2\pi f} H(f) e^{-j2\pi f t_d} + \frac{A}{2} H(0) \mathbf{d}(f)$$

3.1-5

$$\mathbf{F}[g(t)] = H(f) \left[ \frac{1}{j2\pi f} + \frac{1}{2} \mathbf{d}(f) \right] = \frac{1}{j2\pi f} H(f) + \frac{1}{2} H(0) \mathbf{d}(f)$$

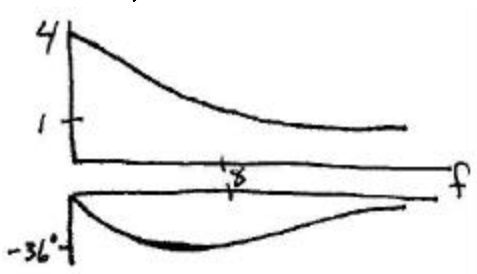
$$\mathbf{F} [dg(t) / dt] = j2\pi f \mathbf{F} [g(t)] = H(f) = \mathbf{F} [h(t)] \quad \text{Thus } h(t) = dg(t) / dt$$

3.1-6

$$j2\pi f Y(f) + 4\pi Y(f) = j2\pi f X(f) + 16\pi X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{j2\pi f + 2\pi 8}{j2\pi f + 2\pi 2} = \frac{8 + jf}{2 + jf}$$

$$|H(f)| = \sqrt{\frac{64 + f^2}{4 + f^2}} \quad \arg H(f) = \arctan \frac{f}{8} - \arctan \frac{f}{2}$$

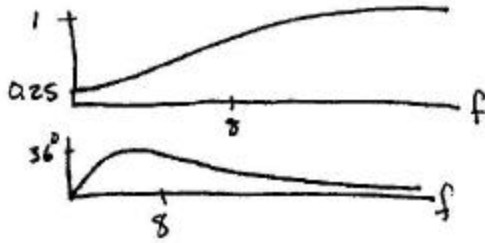


3.1-7

$$j2\pi f Y(f) + 16 Y(f) = j2\pi f X(f) + 4 X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{j2\pi f + 2 \cdot 2}{j2\pi f + 2 \cdot 8} = \frac{2 + jf}{8 + jf}$$

$$|H(f)| = \sqrt{\frac{4 + f^2}{64 + f^2}} \quad \arg H(f) = \arctan \frac{f}{2} - \arctan \frac{f}{8}$$

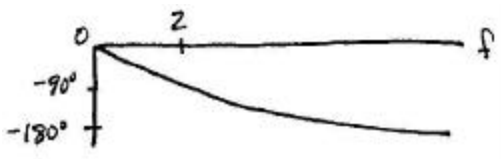


3.1-8

$$j2\pi f Y(f) - 4 Y(f) = -j2\pi f X(f) + 4 X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{-j2\pi f + 2 \cdot 2}{j2\pi f - 2 \cdot 2} = \frac{2 - jf}{2 + jf}$$

$$|H(f)| = \sqrt{\frac{2 + f^2}{2 + f^2}} = 1 \text{ for all } f \quad \arg H(f) = -2 \arctan \frac{f}{2}$$



3.1-9

$$H(f) \approx \frac{B}{jf} \quad \text{for } |f| \geq W \ll B$$

$$\text{Thus } Y(f) \approx \frac{B}{jf} X(f) = 2\pi B \frac{1}{j2\pi f} X(f) \quad \text{for } |f| \geq W$$

$$\text{and } y(t) \approx 2\pi B \int_{-\infty}^t x(\tau) d\tau \quad \text{since } X(0) \approx 0$$

3.1-10

$$H(f) \approx \frac{jf}{B} \quad \text{for } |f| \leq W \ll B$$

$$\text{Thus } Y(f) \approx \frac{jf}{B} X(f) = \frac{1}{2\pi B} j2\pi f X(f) \quad \text{for } |f| \leq W$$

$$\text{so } y(t) \approx \frac{1}{2\pi B} \frac{dx(t)}{dt}$$

3.1-11

$$x(t) = 2\text{sinc}4Wt \leftrightarrow X(f) = \frac{2}{4W} \Pi\left(\frac{f}{4W}\right) = \frac{1}{2W} \Pi\left(\frac{f}{4W}\right)$$

$$E_x = \int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-2W}^{2W} \frac{1}{4W^2} df = \frac{1}{W}$$

$$Y(f) = \frac{1}{1 + j(f/B)} \frac{1}{2W} \Pi\left(\frac{f}{4W}\right)$$

$$E_y = 2 \int_0^{2W} \frac{1/4W^2}{1 + (f/B)^2} df = \frac{B}{2W^2} \arctan \frac{2W}{B}$$

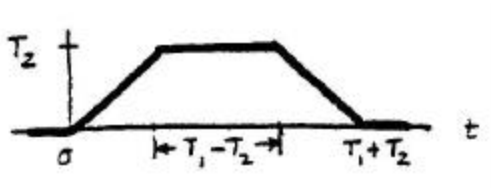
$$\frac{E_y}{E_x} = \frac{B}{2W} \arctan \frac{2W}{B}$$

3.1-12

$$h(t) = \mathbf{F}^{-1} [H_1(f)H_2(f)] = h_1(t) * h_2(t)$$

$$\text{where } h_1(t) = u(t) - u(t - T_1)$$

$$h_2(t) = u(t) - u(t - T_2)$$

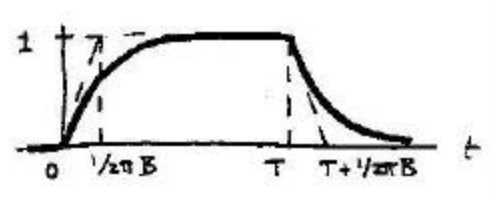


3.1-13

$$h(t) = \mathbf{F}^{-1} [H_1(f)H_2(f)] = h_1(t) * h_2(t)$$

$$\text{where } h_1(t) = 2p B e^{-2p B t} u(t)$$

$$h_2(t) = u(t) - u(t - T)$$



3.1-14

$$H(f) = \frac{j2pf}{1 + jK2pf} = \frac{1}{K} j2pf \frac{1}{1/K + j2pf}$$

$$h(t) = \frac{1}{K} \frac{d}{dt} [e^{-t/K} u(t)] = \frac{1}{K} \mathbf{d}(t) - \frac{1}{K^2} e^{-t/K} u(t)$$

$$g(t) = \mathbf{F}^{-1} \left\{ \frac{1/K}{1/K + j2pf} \right\} = \frac{1}{K} e^{-t/K} u(t)$$

3-1-15

$$H(f) = \frac{K}{1 + Kj2\pi f} = \frac{1}{\frac{1}{K} + j2\pi f}$$

so  $h(t) = e^{-t/K} u(t)$

$$g(t) = \int_{-\infty}^t h(l) dl = K(1 - e^{-t/K}) u(t)$$

3.1-16

Since  $h(t)$  is real,  $H_r(f) = H_e(f)$  and  $H_i(f) = H_o(f)$ , so

$$\begin{aligned} h(t) &= \int_{-\infty}^{\infty} [H_e(f) + jH_o(f)] e^{j\omega t} df = 2 \int_0^{\infty} H_r(f) \cos \omega t df + j2 \int_0^{\infty} jH_i(f) \sin \omega t df \\ &= 2 \left[ \int_0^{\infty} H_r(f) \cos \omega t df - \int_0^{\infty} H_i(f) \sin \omega t df \right] \end{aligned}$$

$$h(t) = 0 \text{ for } t < 0 \Rightarrow \int_0^{\infty} H_i(f) \sin \omega t df = \int_0^{\infty} H_r(f) \cos \omega t df$$

$$\text{Hence, for } t > 0, \quad - \int_0^{\infty} H_i(f) \sin \omega t df = \int_0^{\infty} H_r(f) \cos \omega t df$$

$$\text{so } h(t) = \int_0^{\infty} H_i(f) \sin \omega t df = \int_0^{\infty} H_r(f) \cos \omega t df$$

3.2-1

$$|H(f)| = \left[ 1 + (f/B)^2 \right]^{-1/2} = 1 - \frac{1}{2} (f/B)^2 + \dots \approx 1$$

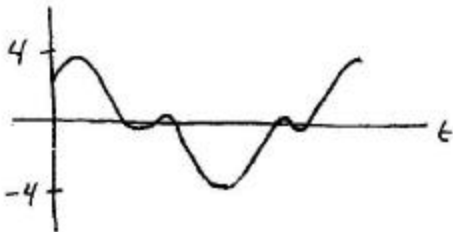
$$\arg H(f) = -\arctan \frac{f}{B} = -\frac{f}{B} + \frac{1}{3} \left( \frac{f}{B} \right)^3 + \dots \approx -\frac{f}{B}$$

for  $|f| \leq W \ll B$

3.2-2

$$|H(nf_0)| = \left[ 1 + (n/3)^2 \right]^{-1/2} \quad \arg H(nf_0) = -\arctan(n/3)$$

$$\begin{aligned} y(t) &= (0.95)(4) \cos(\omega_0 t - 18^\circ) + (0.71)(4/9) \cos(3\omega_0 t - 45^\circ) + (0.5)(4/25) \cos(5\omega_0 t - 59^\circ) \\ &= 3.79 \cos(\omega_0 t - 18^\circ) + 0.31 \cos(3\omega_0 t - 45^\circ) + 0.08 \cos(5\omega_0 t - 59^\circ) \end{aligned}$$

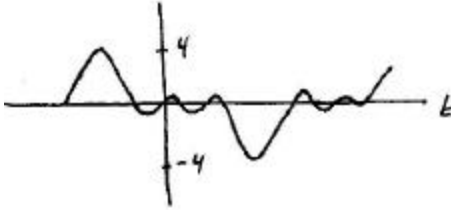


3.2-3

$$|H(nf_0)| = \frac{n/3}{\sqrt{1+(n/3)^2}} \quad \arg H(nf_0) = 90^\circ - \arctan(n/3)$$

$$y(t) = (0.32)(4)\cos(\omega_0 t - 72^\circ) + (0.71)(4/9)\cos(3\omega_0 t - 45^\circ) + (0.86)(4/25)\cos(5\omega_0 t - 31^\circ)$$

$$= 1.28\cos(\omega_0 t - 72^\circ) + 0.31\cos(3\omega_0 t - 45^\circ) + 0.14\cos(5\omega_0 t - 31^\circ)$$

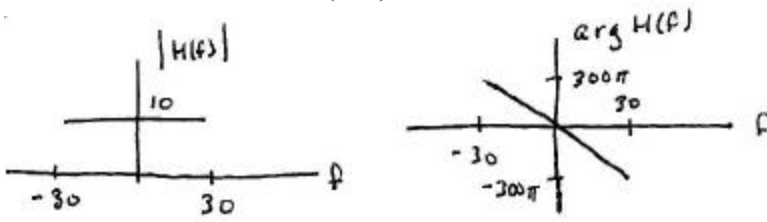


3.2-4

$$X(f) = \frac{2}{40} \Pi\left(\frac{f}{40}\right) = \frac{1}{20} \Pi\left(\frac{f}{40}\right)$$

$$Y(f) = \frac{20}{40} \Pi\left(\frac{f}{40}\right) e^{-j\omega 5}$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{\frac{1}{2} \Pi\left(\frac{f}{40}\right) e^{-j\omega 5}}{\frac{1}{20} \Pi\left(\frac{f}{40}\right)} = 10e^{-j\omega 5}$$



Note that  $300\pi = 2\pi$  and the phase actually wrapped around several times. Under normal plotting conventions we would go from  $-\pi$  to  $\pi$  and repeat this pattern 300 times.

3.2-5

$$t_d(f) = \frac{-\arctan(f/B)}{2\pi f} \quad B = 2\text{kHz} \quad \lim_{f \rightarrow 0} t_d(f) = \frac{1}{2\pi B}$$

$f$ kHz	$t_d(f)$ , ms
0	-0.08
0.5	-0.078
1	-0.074
2	-0.062

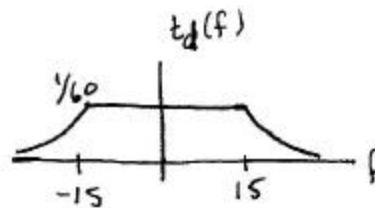
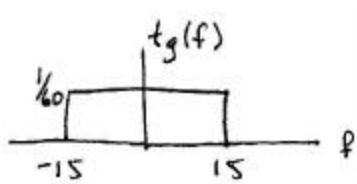
3.2-6

$$\arg H(f) = \begin{cases} -\frac{pf}{30} & |f| \leq 15 \\ -\frac{p}{2} & |f| > 15 \end{cases}$$

$$t_g(f) = -\frac{1}{2p} \frac{d}{df} \arg H(f) = \begin{cases} -\frac{1}{2p} \left( -\frac{p}{30} \right) = \frac{1}{60} & |f| \leq 15 \\ 0 & |f| > 15 \end{cases}$$

$$t_d(f) = \frac{-\arg H(f)}{2pf} = \begin{cases} \frac{pf/30}{2pf} = \frac{1}{60} & |f| \leq 15 \\ \frac{p/2}{2pf} = \frac{1}{4f} & |f| > 15 \end{cases}$$

so  $t_d(f) = t_g(f)$  for  $|f| \leq 15$

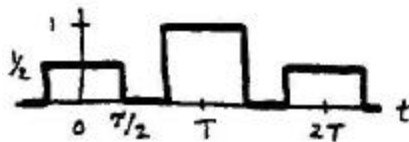


3.2-7

$$(a) H_c(f) = \left[ 1 + 2a \frac{1}{2} (e^{j\omega T} + e^{-j\omega T}) \right] e^{-j\omega T} = a + e^{-j\omega T} + a e^{-j\omega 2T}$$

Thus,  $y(t) = ax(t) + x(t-T) + ax(t-2T)$

$$(b) t = \frac{2T}{3}$$



$$t = \frac{4T}{3}$$



3.2-8

$$\exp[-j(\mathbf{w}T - \mathbf{a} \sin \mathbf{w}T)] = e^{-j\mathbf{w}T} e^{j\mathbf{a} \sin \mathbf{w}T} = e^{-j\mathbf{w}T} \left( 1 + j\mathbf{a} \sin \mathbf{w}T - \frac{\mathbf{a}^2}{2} \sin^2 \mathbf{w}T + \dots \right)$$

$$\begin{aligned} \text{If } |\mathbf{a}| \ll \mathbf{p}/2, \quad H_c(f) &\approx e^{-j\mathbf{w}T} + j\mathbf{a} \sin \mathbf{w}T e^{-j\mathbf{w}T} = e^{-j\mathbf{w}T} + \frac{\mathbf{a}}{2} (e^{j\mathbf{w}T} - e^{-j\mathbf{w}T}) e^{-j\mathbf{w}T} \\ &\approx \frac{\mathbf{a}}{2} + e^{-j\mathbf{w}T} - \frac{\mathbf{a}}{2} e^{-j2\mathbf{w}T} \end{aligned}$$

$$\text{Thus, } y(t) \approx \underbrace{\frac{\mathbf{a}}{2} x(t)}_{\text{leading echo}} + x(t-T) - \underbrace{\frac{\mathbf{a}}{2} x(t-2T)}_{\text{inverted trailing echo}}$$

3.2-9

$$H_{eq}(f) = K e^{-j\mathbf{w}(t_d - T)} e^{j0.4 \sin \mathbf{w}T}$$

$$e^{j0.4 \sin \mathbf{w}T} = 1 + j0.4 \sin \mathbf{w}T - 0.8 \sin^2 \mathbf{w}T + \dots \approx 1 + 0.2 (e^{j\mathbf{w}T} - e^{-j\mathbf{w}T})$$

$$\text{so } H_{eq}(f) \approx K e^{-j\mathbf{w}(t_d - T)} (1 + 0.2 e^{j\mathbf{w}T} - 0.2 e^{-j\mathbf{w}T})$$

$$\text{Take } K=1 \text{ and } t_d = 2T, \text{ so } H_{eq}(f) \approx (0.2 e^{j\mathbf{w}T} + 1 - 0.2 e^{-j\mathbf{w}T}) e^{-j\mathbf{w}T}$$

$$\text{Hence, } \Delta = T, \quad M = 1, \quad c_{-1} = 0.02, \quad c_0 = 1, \quad c_1 = -0.2$$

3.2-10

$$H_{eq}(f) = K e^{-j\mathbf{w}(t_d - T)} (1 + 0.8 \cos \mathbf{w}T)^{-1}$$

Expanding using the first 3 terms

$$(1 + 0.8 \cos \mathbf{w}T)^{-1} = 1 - 0.8 \cos \mathbf{w}T + 0.64 \cos^2 \mathbf{w}T - 0.51 \cos^3 \mathbf{w}T$$

$$\text{where } \cos \mathbf{w}T = \frac{1}{2} (e^{j\mathbf{w}T} + e^{-j\mathbf{w}T}), \quad \cos^2 \mathbf{w}T = \frac{1}{2} + \frac{1}{2} \cos 2\mathbf{w}T = \frac{1}{2} + \frac{1}{4} (e^{j2\mathbf{w}T} + e^{-j2\mathbf{w}T})$$

$$\cos^3 \mathbf{w}T = \frac{1}{4} (3 \cos \mathbf{w}T + \cos 3\mathbf{w}T) = \frac{3}{8} (e^{j\mathbf{w}T} + e^{-j\mathbf{w}T}) + \frac{1}{8} (e^{j3\mathbf{w}T} + e^{-j3\mathbf{w}T})$$

Take  $K=1$  and  $t_d = 4T$ , so

$$\begin{aligned} H_{eq}(f) = & \left[ -\frac{0.13}{2} e^{j3\mathbf{w}T} + \frac{0.64}{4} e^{j2\mathbf{w}T} - \left( \frac{0.8}{2} + \frac{0.38}{2} \right) e^{j\mathbf{w}T} + 1 + \frac{0.64}{2} - \left( \frac{0.8}{2} + \frac{0.38}{2} \right) e^{-j\mathbf{w}T} \right. \\ & \left. + \frac{0.64}{4} e^{-j2\mathbf{w}T} - \frac{0.13}{2} e^{-j3\mathbf{w}T} \right] e^{-j3\mathbf{w}T} \end{aligned}$$

$$\text{Hence, } \Delta = T, \quad M = 3, \quad c_{-3} = c_3 = -0.065, \quad c_{-2} = c_2 = 0.16, \quad c_{-1} = c_1 = -0.59, \quad c_0 = 1.32$$



3.2-11

$$y(t) = 2A \cos \omega_0 t - 3A^3 \cos^3 \omega_0 t \quad 3A^3 \cos^3 \omega_0 t = \frac{9A^3}{4} \cos \omega_0 t + \frac{3A^3}{4} \cos 3\omega_0 t$$

$$\text{so } y(t) = \left( 2A - \frac{9A^3}{4} \right) \cos \omega_0 t - \frac{3A^3}{4} \cos 3\omega_0 t$$

2<sup>nd</sup> harmonic distortion = 0

$$3^{\text{rd}} \text{ harmonic distortion} = \left| \frac{\frac{3A^3}{4}}{2A - \frac{9A^3}{4}} \right| \times 100 = \begin{cases} 300\% & A = 1 \\ 42\% & A = 2 \end{cases}$$

3.2-12

$$y(t) = 5A \cos \omega_0 t - 2A^2 \cos^2 \omega_0 t + 4A^3 \cos^3 \omega_0 t$$

$$2A^2 \cos^2 \omega_0 t = A^2 + A^2 \cos 2\omega_0 t \quad 4A^3 \cos^3 \omega_0 t = 3A^3 \cos \omega_0 t + A^3 \cos 3\omega_0 t$$

$$\text{so } y(t) = -A^2 + (5A + 3A^3) \cos \omega_0 t - A^2 \cos 2\omega_0 t + A^3 \cos 3\omega_0 t$$

$$2^{\text{nd}} \text{ harmonic distortion} = \left| \frac{A^2}{5A + 3A^3} \right| \times 100 = \begin{cases} 12.5\% & A = 1 \\ 11.8\% & A = 2 \end{cases}$$

$$3^{\text{rd}} \text{ harmonic distortion} = \left| \frac{A^3}{5A + 3A^3} \right| \times 100 = \begin{cases} 12.5\% & A = 1 \\ 23.5\% & A = 2 \end{cases}$$

3.3-1

$$P_{in} = 0.5W = 27\text{dBm} \quad \ell = 50\text{km} \quad a = 2\text{dB/km}$$

$$P_{out} = 50\text{mW} = 17\text{dBm} \quad 20\mu\text{W} = -17\text{dBm}$$

$$27\text{dBm} - 2\ell_1 = -17\text{dBm} \Rightarrow \ell_1 = 22\text{km} \Rightarrow \ell_3 = 50 - 22 = 28\text{km}$$

$$-17\text{dBm} + g_2 - 2 \times 28 = -17\text{dBm} \Rightarrow g_2 = 56\text{dB}$$

$$-17\text{dBm} + g_4 = 17\text{dBm} \Rightarrow g_4 = 34\text{dB}$$

3.3-2

$$P_{in} = 100\text{mW} = 20\text{dBm} \quad \ell = 50\text{km} \quad a = 2\text{dB/km}$$

$$P_{out} = 0.1W = 20\text{dBm} \quad 20\mu\text{W} = -17\text{dBm}$$

$$20\text{dBm} - 2\ell_1 = -17\text{dBm} \Rightarrow \ell_1 = 18.5\text{km} \Rightarrow \ell_3 = 40 - 18.5 = 21.5\text{km}$$

$$-17\text{dBm} + g_2 - 2 \times 21.5 = -17\text{dBm} \Rightarrow g_2 = 43\text{dB}$$

$$-17\text{dBm} + g_4 = 20\text{dBm} \Rightarrow g_4 = 37\text{dB}$$

3.3-3

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{50 \times 10^{-3}}{2} = -16 \text{ dB}, \quad mL_i = 0.4 \times 400 = 160 \text{ dB}, \quad g_i \leq 30 \text{ dB}$$

$$m \times 30 \text{ dB} - 160 \geq -16 \Rightarrow m \geq 4.8 \quad \text{so } m = 5$$

$$g = (160 - 16)/5 = 28.8 \text{ dB}$$

3.3-4

$$L_i = 0.5 \times 3000 / m = 1500 / m \text{ dB} \quad P_{\text{in}} = 5 \text{ mW} = 7 \text{ dBm}$$

$$\frac{P_{\text{in}}}{L_1} \geq 67 \mu\text{W} = -11.75 \text{ dBm}$$

$$7 \text{ dBm} - \frac{1500}{m} \geq -11.75 \Rightarrow m \geq 80$$

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{mg_i}{mL_i} = 1 \Rightarrow g_i = L_i = \frac{1500}{80} = 18.75 \text{ dB}$$

3.3-5

$$L_i = 2.5 \times 3000 / m = 7500 / m \text{ dB} \quad P_{\text{in}} = 5 \text{ mW} = 7 \text{ dBm}$$

$$\frac{P_{\text{in}}}{L_1} \geq 67 \mu\text{W} = -11.75 \text{ dBm}$$

$$7 \text{ dBm} - \frac{7500}{m} \geq -11.75 \Rightarrow m \geq 400$$

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{mg_i}{mL_i} = 1 \Rightarrow g_i = L_i = \frac{7500}{400} = 18.75 \text{ dB}$$

3.3-6

$$\frac{P_{\text{out}}}{P_{\text{in}}} = 2 \times 10^{-6} / 5 = -64 \text{ dB}, \quad L = 92.4 - 6 + 26 = 112.4 \text{ dB}$$

$$\frac{g^2}{L} = \frac{P_{\text{out}}}{P_{\text{in}}} \Rightarrow g = (112.4 - 64)/2 = 24.2 \text{ dB} = 263$$

$$\text{so } \frac{4p(p r^2)(0.5 \times 10^9)^2}{(3 \times 10^5)^2} = 263 \Rightarrow r = 1.55 \times 10^{-3} \text{ km} = 1.55 \text{ m}$$

3.3-7

$$\frac{P_{out}}{P_{in}} = \frac{2 \times 10^{-6}}{5} = -64 \text{dB} \quad L = 92.4 - 14 + 20 = 98.4$$

$$\frac{g^2}{L} = \frac{P_{out}}{P_{in}} \Rightarrow g = \frac{98.4 - 64}{2} = 17.2 \text{dB} = 52.5$$

$$\text{so } \frac{4p(p r^2)(0.2 \times 10^9)^2}{(3 \times 10^5)^2} = 52.5 \Rightarrow r = 1.7 \times 10^{-3} \text{ km} = 1.7 \text{ m}$$

3.3-8

$$\text{With repeater } P_{out} = \frac{g_T g_R g_{rpt}}{L_1 L_2} P_{in} \quad \text{Without repeater } P_{out} = \frac{g_T g_R}{L} P_{in}$$

$$\frac{g_T g_R g_{rpt}}{L_1 L_2} = 1.2 \frac{g_T g_R}{L} \Rightarrow g_{rpt} = 1.2 \frac{L_1 L_2}{L}$$

$$L_{dB} = 92.4 - 20 \log f_{\text{GHz}} + 20 \log 25 \text{ km} = 120 + 20 \log f$$

$$L_2 = L_1$$

$$L = 92.4 - 20 \log f + 20 \log 50 = 126 + 2 - \log f$$

$$\text{Let } f = 1 \text{ GHz}$$

$$L_1 = L_2 = 120 \text{ dB} \Rightarrow 10^{12} \quad L = 126 \text{ dB} \Rightarrow 3.98 \times 10^{12}$$

$$g_{rpt} = 1.2 \frac{10^{12} \times 10^{12}}{3.98 \times 10^{12}} = 0.3 \times 10^{12} = 115 \text{ dB}$$

3.3-9

$$L_u = 92.4 + 20 \log 17 + 20 \log 3.6 \times 10^4 = 208$$

$$L_d = 92.4 + 20 \log 12 + 20 \log 3.6 \times 10^4 = 205$$

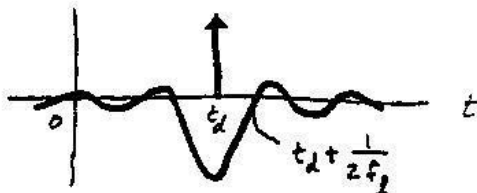
$$P_{in} = 30 \text{ dBW} \quad \text{so } P_{sat_{in}} = 30 + 55 - 208 + 20 = -103 \text{ dBW}$$

$$\text{based on parameters from Example 3.3-1 } g_{amp} = 18 + 144 = 162 \text{ dB}$$

$$P_{sat_{out}} = -103 + 162 = 59 \text{ dBW} \quad \text{so } P_{out} = 59 + 16 - 205 + 51 = -79 \text{ dBW} \Rightarrow 1.26 \times 10^{-8} \text{ W}$$

3.4-1

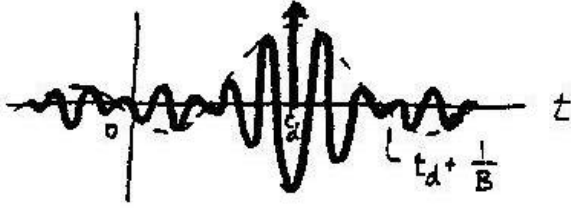
$$H(f) = K e^{-j\omega t_d} - K \Pi \left( \frac{f}{2f_\ell} \right) e^{-j\omega t_d} \quad h(t) = K d(t - t_d) - 2K f_\ell \text{ sinc } 2f_\ell (t - t_d)$$



3.4-2

$$H(f) = Ke^{-j\omega t_d} - H_{BP}(f) \text{ where } H_{BP}(f) = \text{Eq. (1)}$$

Thus, from Exercise 3.4-1,  $h(t) = K\delta(t - t_d) - 2BK \text{sinc } B(t - t_d) \cos \omega_c(t - t_d)$



3.4-3

$$|H(0.7B)|^2 = [1 + (0.7)^{2n}]^{-1} \geq 10^{-1/10} = 1/1.259$$

$$\text{so } 1 + (0.7)^{2n} \leq 1.259 \text{ or } (0.7)^{2n} \leq 0.259$$

$$\underline{n} \quad \underline{(0.7)^{2n}}$$

$$1 \quad 0.49 \quad \Rightarrow \text{select } n = 2$$

$$2 \quad 0.24$$

$$|H(3B)| = [1 + 3^{2n}]^{-1/2} = [1 + 3^4]^{-1/2} = 0.11 = -19\text{dB}$$

3.4-4

$$|H(0.9B)|^2 = [1 + (0.9)^{2n}]^{-1} \geq 10^{-1/10} = 1/1.259$$

$$\text{so } 1 + (0.9)^{2n} \leq 1.259 \text{ or } (0.9)^{2n} \leq 0.259$$

$$\underline{n} \quad \underline{(0.9)^{2n}}$$

$$6 \quad 0.282 \quad \Rightarrow \text{select } n = 7$$

$$7 \quad 0.229$$

$$|H(3B)| = [1 + 3^{2n}]^{-1/2} = [1 + 3^{14}]^{-1/2} = 4.6 \times 10^{-4} = -66.8\text{dB}$$

3.4-5

$$H(f) = \left[ 1 + j\sqrt{2} \frac{f}{B} - \left( \frac{f}{B} \right)^2 \right]^{-1} \text{ from Table 3.4-1}$$

$$\begin{aligned} H(s) = H(f)|_{f \Rightarrow s/j2p} &= \left[ 1 + j\sqrt{2} \frac{s}{2pB} - \left( \frac{s}{2pB} \right)^2 \right]^{-1} \\ &= \frac{2b^2}{(s+b)^2 + b^2} \quad b = 2pB/\sqrt{2} \end{aligned}$$

$$\text{so } h(t) = 2be^{-bt} \sin btu(t)$$

3.4-6

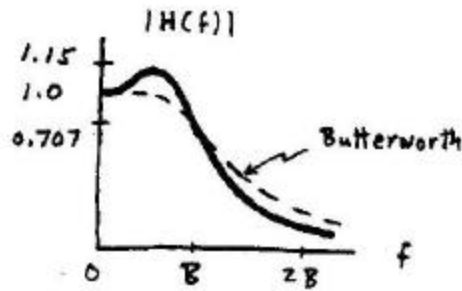
$$(a) H(f) = \frac{Z_{RC}}{Z_{RC} + j\omega L} \quad \text{where } Z_{RC} = \frac{R / j\omega C}{R + 1 / j\omega C} = \frac{\sqrt{LC}}{1 + j\omega\sqrt{LC}}$$

$$\text{Thus, } H(f) = \frac{1}{1 + j\omega\sqrt{LC} - \omega^2 LC}$$

$$\text{so } |H(f)|^2 = \left[ (1 - \omega^2 LC)^2 + (\omega^2 LC)^2 \right]^{-1} = 1 - (f/f_0)^2 + (f/f_0)^4 \quad \text{with } f_0 = \frac{1}{2p\sqrt{LC}}$$

$$(b) |H(B)|^2 = 1/2 \Rightarrow 1 - (B/f_0)^2 + (B/f_0)^4 = 1/2$$

$$\text{so } \left( \frac{B}{f_0} \right)^2 = \frac{1}{2}(1 + \sqrt{5}) \quad B = \sqrt{\frac{1}{2}(1 + \sqrt{5})} f_0 = 1.27 f_0$$



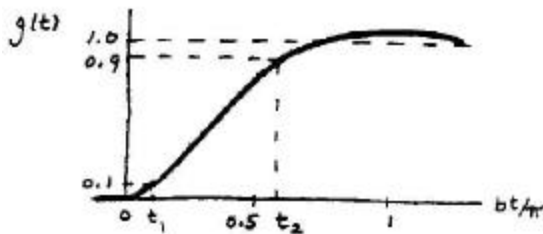
3.4-7

$$\left. \begin{aligned} 1 - e^{-2pBt_1} &= 0.1 \Rightarrow t_1 = 0.11/2pB \\ 1 - e^{-2pBt_2} &= 0.9 \Rightarrow t_2 = 2.30/2pB \end{aligned} \right\} t_r = t_2 - t_1 = \frac{2.30 - 0.11}{2pB} = \frac{1}{2.87B}$$

3.4-8

$$g(t) = \int_{-\infty}^t h(l) dl = 2b \int_0^t e^{-bl} \sin bl \, dl = 1 - e^{-bt} (\sin bt + \cos bt) \quad \text{for } t \geq 0$$

$$bt_1/p \approx 0.1, \quad bt_2/p \approx 0.6 \quad t_r \approx \frac{0.5p}{b} = \frac{0.5p\sqrt{2}}{2pB} = \frac{1}{2.8B}$$

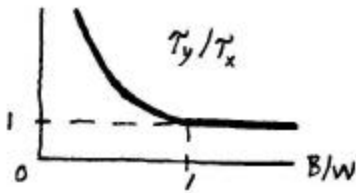


3.4-9

$$x(t) = A \text{sinc} 2Wt \Rightarrow \mathbf{t}_x = \frac{1}{W}, \quad X(f) = \frac{A}{2W} \Pi\left(\frac{f}{2W}\right)$$

$$Y(f) = \Pi\left(\frac{f}{2B}\right) X(f) = \begin{cases} \frac{A}{2W} \Pi\left(\frac{f}{2W}\right) & \text{for } B > W \\ \frac{A}{2W} \Pi\left(\frac{f}{2B}\right) & \text{for } B < W \end{cases}$$

$$y(t) = \begin{cases} A \text{sinc} 2Wt \Rightarrow \mathbf{t}_y = 1/W & \text{for } B > W \\ \frac{B}{W} A \text{sinc} 2Bt \Rightarrow \mathbf{t}_y = 1/B & \text{for } B < W \end{cases}$$



3.4-10

$$H(0) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \Big|_{f=0} = \int_{-\infty}^{\infty} h(t) dt$$

$$|h(t)| = \left| \int_{-\infty}^{\infty} H(f) e^{j\omega t} df \right| \leq \int_{-\infty}^{\infty} |H(f) e^{j\omega t}| df = \int_{-\infty}^{\infty} |H(f)| df$$

$$\text{Thus } \mathbf{t}_{\text{eff}} = \frac{H(0)}{|h(t)|_{\text{max}}} \geq \frac{H(0)}{\int_{-\infty}^{\infty} |H(f)| df} = \frac{1}{2B_{\text{eff}}}$$

3.4-11

$$(a) \quad H(f) = 2KB \int_0^{2t_d} \text{sinc} 2B(t - t_d) e^{-j\omega t} dt = 2KB e^{-j\omega t_d} \int_{-t_d}^{t_d} \text{sinc} 2B\mathbf{l} e^{-j\omega \mathbf{l}} d\mathbf{l}$$

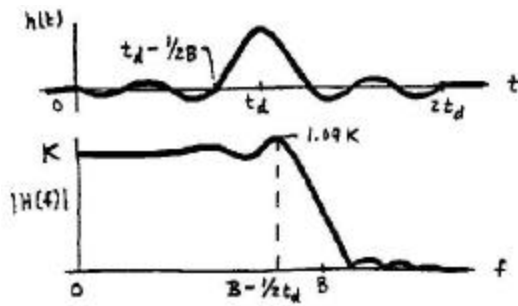
$$\text{where } \sin 2\mathbf{p} B \mathbf{l} \cos 2\mathbf{p} f \mathbf{l} = \frac{1}{2} [\sin 2\mathbf{p} (f + B) \mathbf{l} - \sin 2\mathbf{p} (f - B) \mathbf{l}]$$

$$\text{and } \int_0^{t_d} \frac{\sin 2\mathbf{p} (f \pm B) \mathbf{l}}{2\mathbf{p} B \mathbf{l}} d\mathbf{l} = \frac{1}{2\mathbf{p} B} \int_0^{2\mathbf{p} (f \pm B) t_d} \frac{\sin \mathbf{a}}{\mathbf{a}} d\mathbf{a} = \frac{1}{2\mathbf{p} B} \text{Si} [2\mathbf{p} (f \pm B) t_d]$$

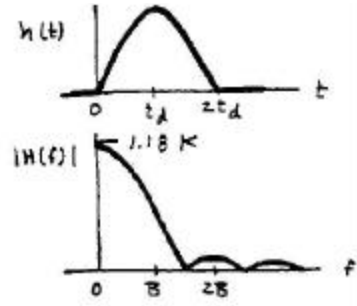
$$\text{Thus } H(f) = \frac{K}{\mathbf{p}} e^{-j\omega t_d} \{ \text{Si} [2\mathbf{p} (f + B) t_d] - \text{Si} [2\mathbf{p} (f - B) t_d] \}$$

(cont.)

$$(b) \quad t_d \ll \frac{1}{B}$$



$$t_d = \frac{1}{2B}$$



3.5-1

$$(a) \quad \hat{d}(t) = \frac{1}{p} \int_{-\infty}^{\infty} \frac{d(l)}{t-l} dl = \frac{1}{p} \frac{1}{t-l} \Big|_{l=0} = \frac{1}{pt}$$

$$\mathbf{F}[\hat{d}(t)] = (-j \operatorname{sgn} f) \mathbf{F}[d(t)] = -j \operatorname{sgn} f$$

$$\text{Thus, } \mathbf{F}^{-1}[-j \operatorname{sgn} f] = \hat{d}(t) = \frac{1}{pt}$$

$$(b) \quad \hat{d}(t) * \frac{1}{pt} = d(t) \quad \text{and} \quad \hat{d}(t) * \left( \frac{-1}{pt} \right) = -\frac{1}{pt} * \frac{1}{pt} = -\left( \frac{1}{pt} \right)$$

$$\text{Thus, } \left( \frac{1}{pt} \right) = -d(t)$$

3.5-2

$$A \Pi\left(\frac{t}{t}\right) = x(t + t/2) \quad \text{where} \quad x(t) = A[u(t) - u(t-t)]$$

$$\text{so } A \hat{\Pi}\left(\frac{t}{t}\right) = \hat{x}\left(t + \frac{t}{2}\right) = \frac{A}{p} \ln \left| \frac{t+t/2}{t+t/2-t} \right| = \frac{A}{p} \ln \left| \frac{2t+t}{2t-t} \right|$$

$$\text{Now let } v(t) = \lim_{t \rightarrow \infty} A \Pi\left(\frac{t}{t}\right) \quad \text{so} \quad \hat{v}(t) = \lim_{t \rightarrow \infty} \frac{A}{p} \ln \left| \frac{2t+t}{2t-t} \right| = \frac{A}{p} \ln 1 = 0$$

3.5-3

$$\begin{aligned} \mathbf{F}[\hat{x}(t)] &= (-j \operatorname{sgn} f) \frac{1}{2W} \Pi\left(\frac{f}{2W}\right) \\ &= \frac{j}{2W} \Pi\left(\frac{f+W/2}{W}\right) - \frac{j}{2W} \Pi\left(\frac{f-W/2}{W}\right) \end{aligned}$$

$$\text{Thus, } \hat{x}(t) = \frac{j}{2} \operatorname{sinc} Wt (e^{-jpWt} - e^{jpWt}) = \operatorname{sinc} Wt \operatorname{sinc} pWt = pWt \operatorname{sinc}^2 Wt$$

3.5-4

$$x(t) = \cos \mathbf{w}_0 t - \frac{1}{3} \cos 3\mathbf{w}_0 t + \frac{1}{5} \cos 5\mathbf{w}_0 t$$

$$\hat{x}(t) = \sin \mathbf{w}_0 t - \frac{1}{3} \sin 3\mathbf{w}_0 t + \frac{1}{5} \sin 5\mathbf{w}_0 t$$

3.5-5

$$x(t) = 4 \cos \mathbf{w}_0 t + \frac{4}{9} \cos 3\mathbf{w}_0 t + \frac{4}{25} \cos 5\mathbf{w}_0 t$$

$$\hat{x}(t) = 4 \sin \mathbf{w}_0 t + \frac{4}{9} \sin 3\mathbf{w}_0 t + \frac{4}{25} \sin 5\mathbf{w}_0 t$$

3.5-6

$$x(t) = \text{sinc} 2Wt \leftrightarrow X(f) = \frac{1}{2W} \Pi\left(\frac{f}{2W}\right) \quad |X(f)| = \frac{1}{2W} \Pi\left(\frac{f}{2W}\right)$$

$$\hat{x}(t) = \mathbf{p}Wt \text{sinc}^2 Wt = \text{sinc} Wt \sin \mathbf{p}Wt \leftrightarrow \hat{X}(f) = \frac{1}{2W} \Pi\left(\frac{f - \frac{W}{2}}{W}\right) e^{-j\mathbf{p}/2} + \frac{1}{2W} \Pi\left(\frac{f + \frac{W}{2}}{W}\right) e^{j\mathbf{p}/2}$$

$$|\hat{X}(f)| = \left| \frac{1}{2W} \Pi\left(\frac{f - \frac{W}{2}}{W}\right) e^{-j\mathbf{p}/2} \right| + \left| \frac{1}{2W} \Pi\left(\frac{f + \frac{W}{2}}{W}\right) e^{j\mathbf{p}/2} \right|$$

Note that the cross term is zero since there is no overlap. From the graph we see that the two rectangle functions form one larger function so

$$|\hat{X}(f)| = \frac{1}{2W} \Pi\left(\frac{f}{2W}\right) = |X(f)|$$

3.5-7

$$x(t) = A \cos \mathbf{w}_0 t \quad \hat{x}(t) = A \sin \mathbf{w}_0 t$$

$$\int_{-\infty}^{\infty} x(t) \hat{x}(t) dt = A^2 \int_{-\infty}^{\infty} \cos \mathbf{w}_0 t \sin \mathbf{w}_0 t dt$$

$$= \lim_{T \rightarrow \infty} \left[ \frac{A^2}{2} \int_{-T}^T \sin(\mathbf{w}_0 - \mathbf{w}_0)t dt + \frac{A^2}{2} \int_{-T}^T \sin(\mathbf{w}_0 + \mathbf{w}_0)t dt \right] = \lim_{T \rightarrow \infty} \left[ 0 + \frac{A^2}{2} \frac{1}{2\mathbf{w}_0} \cos 2\mathbf{w}_0 t \right]_{-T}^T$$

$$= \lim_{T \rightarrow \infty} \left[ \frac{A^2}{4\mathbf{w}_0} (\cos 2\mathbf{w}_0 T - \cos(-2\mathbf{w}_0 T)) \right] = 0$$



3.5-8

$$\begin{aligned}\mathbf{F}[h_e(t)] &= \int_{-\infty}^{\infty} \frac{1}{2} h(|t|) e^{-j\omega t} dt = 2 \int_0^{\infty} \frac{1}{2} h(|t|) \cos \omega t dt \\ &= \int_0^{\infty} h(t) \cos \omega t dt = \int_{-\infty}^{\infty} h(t) \cos \omega t dt = H_e(f)\end{aligned}$$

$$H(f) = \mathbf{F}\left[(1 + \operatorname{sgn} t) h_e(t)\right] = H_e(f) + \frac{1}{j2\pi f} * H_e(f) = H_e(f) - j \left[ H_e(f) * \frac{1}{\pi f} \right] = H_e(f) - j\hat{H}_e(f)$$

Thus,  $H_o(f) = -\hat{H}_e(f)$

3.6-1

$$\begin{aligned}R_{wv}(\mathbf{t}) &= \langle w(t) v^*(t - \mathbf{t}) \rangle = \langle w^*(t) v(t - \mathbf{t}) \rangle^* \\ &= \langle v[t + (-\mathbf{t})] w^*(t) \rangle^* = R_{vw}^*(-\mathbf{t})\end{aligned}$$

3.6-2

$$R_v(\mathbf{t} \pm mT_0) = \langle v(t + \mathbf{t} \pm mT_0) v^*(t) \rangle$$

$$\text{but } v(t + \mathbf{t} \pm mT_0) = v(t + \mathbf{t}) \quad \text{so } R_v(\mathbf{t} \pm mT_0) = \langle v(t + \mathbf{t}) v^*(t) \rangle = R_v(\mathbf{t})$$

3.6-3

$$P_w = \langle |v(t + \mathbf{t})|^2 \rangle = \langle |v(t)|^2 \rangle = P_v$$

$$|R_v(\mathbf{t})|^2 = \left| \langle v(t) w^*(t) \rangle \right|^2 \leq P_v P_v = R_v^2(0) \quad \text{so } |R_v(\mathbf{t})| \leq R_v(0)$$

3.6-4

$$x(t) = \cos 2\omega_0 t \quad \text{From Eq. (12) } R_x(\mathbf{t}) = \frac{1}{2} \cos 2\omega_0 \mathbf{t}$$

$$y(t) = \sin 2\omega_0 t = \cos(2\omega_0 t - 90^\circ) \Rightarrow R_y(\mathbf{t}) = \frac{1}{2} \cos 2\omega_0 \mathbf{t}$$

(Note that the phase delay does not appear in the autocorrelation)

Since  $R_y(\mathbf{t}) = R_x(\mathbf{t})$  we conclude that  $y(t)$  is similar to  $x(t)$ . This is the expected conclusion since  $y(t)$  is just a phase shifted version of  $x(t)$ .

3.6-5

$$V(f) = AD \operatorname{sinc} fD e^{-j\omega_d}$$

$$G_v(f) = (AD)^2 \operatorname{sinc}^2 fD \Rightarrow R_v(\mathbf{t}) = A^2 D \Lambda(\mathbf{t} / D), \quad E_v = R_v(0) = A^2 D$$

3.6-6

$$V(f) = \left( \frac{A}{4W} \right) \Pi \left( \frac{f}{4W} \right) e^{-j\omega t_d}$$

$$G_v(f) = \left( \frac{A}{4W} \right)^2 \Pi \left( \frac{f}{4W} \right) \Rightarrow R_v(t) = \frac{A^2}{4W} \text{sinc} 4Wt, \quad E_v = R_v(0) = \frac{A^2}{4W}$$

3.6-7

$$V(f) = \frac{A}{b + j2pf}$$

$$G_v(f) = \frac{A^2}{b^2 + (2pf)^2} \Rightarrow R_v(t) = \frac{A^2}{2b} e^{-b|t|}, \quad E_v = R_v(0) = \frac{A^2}{2b}$$

3.6-8

$$v(t) = A_0 + \frac{A_1}{2} e^{jf} e^{j\omega_0 t} + \frac{A_1}{2} e^{-jf} e^{-j\omega_0 t}$$

$$G_v(f) = A_0^2 d(f) + \frac{A_1^2}{4} [d(f - f_0) + d(f + f_0)]$$

$$R_v(t) = A_0^2 + \frac{A_1^2}{2} \cos \omega_0 t, \quad P_v = R_v(0) = A_0^2 + \frac{A_1^2}{2}$$

3.6-9

$$v(t) = \frac{A_1}{2} (e^{jf_1} e^{j\omega_0 t} + e^{-jf_1} e^{-j\omega_0 t}) + \frac{A_2}{2} (e^{-jp/2} e^{j2\omega_0 t} e^{jf_1} + e^{jp/2} e^{-j2\omega_0 t} e^{-jf_1})$$

$$G_v(f) = \frac{A_1^2}{4} [d(f - f_0) + d(f + f_0)] + \frac{A_2^2}{4} [d(f - 2f_0) + d(f + 2f_0)]$$

$$R_v(t) = \frac{A_1^2}{2} \cos \omega_0 t + \frac{A_2^2}{2} \cos 2\omega_0 t \quad P_v = R_v(0) = \frac{A_1^2}{2} + \frac{A_2^2}{2}$$

3.6-10

$$R_v(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 u(t) u(t-t) dt \quad \text{where } u(t) u(t-t) = \begin{cases} 0 & t < t \\ 1 & t > t \end{cases}$$

$$\text{Take } T/2 > t > 0, \text{ so } \int_{-T/2}^{T/2} u(t) u(t-t) dt = \int_t^{T/2} dt = \frac{T}{2} - t$$

$$\text{Thus } R_v(t) = \lim_{T \rightarrow \infty} \frac{A^2}{T} \left( \frac{T}{2} - t \right) = \frac{A^2}{2} \quad \text{for all } t$$

$$P_v = R_v(0) = \frac{A^2}{2} \quad G_v(f) = \frac{A^2}{2} d(f)$$

3.6-11

$$x(t) = \Pi(10t) = \Pi\left(\frac{t}{1/10}\right) \leftrightarrow X(f) = \frac{1}{10} \text{sinc} \frac{f}{10}$$

$$H(f) = Ke^{-j\omega t_d} \Pi\left(\frac{f}{2B}\right) = \left| 3e^{-j\omega 0.05} \Pi\left(\frac{f}{40}\right) \right|$$

$$G_y(f) = |H(f)|^2 G_x(f) = |H(f)|^2 |X(f)|^2 \quad \text{since } x(t) \text{ is an energy signal}$$

$$= \left| 3e^{-j\omega 0.05} \Pi\left(\frac{f}{40}\right) \right|^2 \left| \frac{1}{10} \text{sinc} \frac{f}{10} \right|^2 = \left[ 9 \Pi\left(\frac{f}{40}\right) \right] \left[ \frac{1}{100} \text{sinc}^2 \frac{f}{10} \right]$$

$$\text{so } R_y(t) = \int_{-20}^{20} \frac{9}{100} \text{sinc}^2 \frac{f}{10} e^{j2\pi f t} df$$

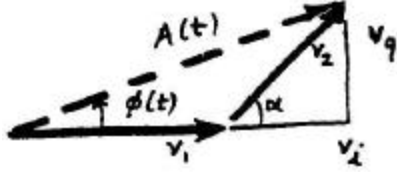
## Chapter 4

4.1-1

$$v_i(t) = v_1(t) + v_2(t) \cos \alpha \quad v_q(t) = v_2(t) \sin \alpha$$

$$A(t) = \sqrt{v_1^2(t) + 2v_1(t)v_2(t) \cos \alpha + v_2^2(t)} \approx v_1(t) + v_2(t) \cos \alpha$$

$$f(t) = \arctan \frac{v_2(t) \sin \alpha}{v_1(t) + v_2(t) \cos \alpha} \approx \frac{v_2(t) \sin \alpha}{v_1(t)}$$

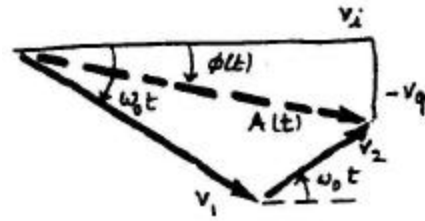


4.1-2

$$v_i(t) = [v_1(t) + v_2(t)] \cos \omega_0 t \quad v_q(t) = [v_2(t) - v_1(t)] \sin \omega_0 t$$

$$A(t) = \sqrt{v_1^2(t) + 2v_1(t)v_2(t) \cos 2\omega_0 t + v_2^2(t)} \approx v_1(t) + v_2(t) \cos 2\omega_0 t$$

$$f(t) = \arctan \frac{[v_2(t) - v_1(t)] \sin \omega_0 t}{[v_1(t) + v_2(t)] \cos \omega_0 t} \approx -\omega_0 t$$



4.1-3

$$(a) \int_{-\infty}^{\infty} v_{bp}(t) dt = \int_{-\infty}^{\infty} v_i(t) \cos \omega_c t dt - \int_{-\infty}^{\infty} v_q(t) \sin \omega_c t dt$$

$$\int_{-\infty}^{\infty} v_i(t) \cos \omega_c t dt = \int_{-\infty}^{\infty} V_i(f) \frac{1}{2} [\mathbf{d}(f - f_c) + \mathbf{d}(f + f_c)]^* df = \frac{1}{2} [V_i(f_c) + V_i(-f_c)] = 0$$

since  $f_c > W$  and  $V_i(f) = 0$  for  $|f| > W$

$$\begin{aligned} \int_{-\infty}^{\infty} v_q(t) \sin \omega_c t dt &= \int_{-\infty}^{\infty} V_q(f) \frac{1}{2} [e^{-jp/2} \mathbf{d}(f - f_c) + e^{jp/2} \mathbf{d}(f + f_c)]^* df \\ &= \frac{1}{2} [V_q(f_c) e^{-jp/2} + V_q(-f_c) e^{jp/2}] = 0 \end{aligned}$$

$$\text{Thus, } \int_{-\infty}^{\infty} v_{bp}(t) dt = 0$$

(cont).

$$(b) \ E_{bp} = \int_{-\infty}^{\infty} \left( v_i(t) \cos \mathbf{w}_c t - v_q \sin \mathbf{w}_c t \right)^2 dt$$

$$= \frac{1}{2} \left[ \int_{-\infty}^{\infty} v_i^2 dt + \int_{-\infty}^{\infty} v_q^2 dt + \int_{-\infty}^{\infty} v_i^2 \cos 2\mathbf{w}_c t dt + \int_{-\infty}^{\infty} v_q^2 \sin 2\mathbf{w}_c t dt + \int_{-\infty}^{\infty} v_i v_q \sin 2\mathbf{w}_c t dt \right]$$

but  $v_i^2, v_q^2$ , and  $v_i v_q$  are bandlimited in  $2W < 2f_c$  so, from the analysis in part (a)

$$\int_{-\infty}^{\infty} v_i^2 \cos 2\mathbf{w}_c t dt = \int_{-\infty}^{\infty} v_q^2 \sin 2\mathbf{w}_c t dt = \int_{-\infty}^{\infty} v_i v_q \sin 2\mathbf{w}_c t dt = 0$$

$$\text{Hence, } E_{bp} = \frac{1}{2} \left[ \int_{-\infty}^{\infty} v_i^2 dt + \int_{-\infty}^{\infty} v_q^2 dt \right] = \frac{1}{2} (E_i + E_q)$$

4.1-4

$$V_{\ell p}(f) = \Pi \left( \frac{f+100}{400} \right)$$

$$v_{\ell p}(t) = 400 \text{sinc} 400t \ e^{-j2\mathbf{p}100t}$$

$$= 400 \text{sinc} 400t (\cos 2\mathbf{p}100t + j \sin 2\mathbf{p}100t)$$

$$v_i(t) = 800 \text{sinc} 400t \cos 2\mathbf{p}100t \quad v_q(t) = -800 \text{sinc} 400t \sin 2\mathbf{p}100t$$

4.1-5

$$V_{\ell p}(f) = \frac{1}{2} \Pi \left( \frac{f-75}{100} \right) + \Pi \left( \frac{f+50}{150} \right)$$

$$v_{\ell p}(t) = \frac{150}{2} \text{sinc} 150t \ e^{j2\mathbf{p}75t} + 100 \text{sinc} 100t \ e^{-j2\mathbf{p}50t}$$

$$v_i(t) = 2 \text{Re} [v_{\ell p}(t)] = 150 \text{sinc} 150t \cos 2\mathbf{p}75t + 200 \text{sinc} 100t \cos 2\mathbf{p}50t$$

$$v_q(t) = 2 \text{Im} [v_{\ell p}(t)] = 150 \text{sinc} 150t \sin 2\mathbf{p}75t - 200 \text{sinc} 100t \sin 2\mathbf{p}50t$$

4.1-6

$$v_{bp}(t) = 2z(t) [\cos(\pm \mathbf{w}_0 t + \mathbf{a}) \cos \mathbf{w}_c t - \sin(\pm \mathbf{w}_0 t + \mathbf{a}) \sin \mathbf{w}_c t]$$

$$\text{so } v_i(t) = 2z(t) \cos(\pm \mathbf{w}_0 t + \mathbf{a}) \quad v_q(t) = 2z(t) \sin(\pm \mathbf{w}_0 t + \mathbf{a})$$

$$v_{\ell p}(t) = \frac{1}{2} 2z(t) [\cos(\pm \mathbf{w}_0 t + \mathbf{a}) + j \sin(\pm \mathbf{w}_0 t + \mathbf{a})] = z(t) e^{j(\pm \mathbf{w}_0 t + \mathbf{a})}$$

4.1-7

$$|H(f)|^2 = \left[ 1 + Q^2 \left( \frac{f}{f_0} - \frac{f_0}{f} \right)^2 \right]^{-1} = \frac{1}{2} \Rightarrow Q \left( \frac{f}{f_0} - \frac{f_0}{f} \right) = \pm 1$$

$$\text{so } \frac{Q}{f_0} f^2 \pm f = Q f_0 = 0 \Rightarrow f_\ell, f_u = \frac{f_0}{2Q} (\sqrt{1+4Q^2} \pm 1)$$

$$B = f_\ell - f_u = \frac{f_0}{2Q} (\sqrt{1+4Q^2} + 1) - \frac{f_0}{2Q} (\sqrt{1+4Q^2} - 1) = \frac{f_0}{Q}$$

4.1-8

$$\frac{f}{f_0} = 1 + \mathbf{d}, \quad \frac{f_0}{f} = (1 + \mathbf{d})^{-1} \approx 1 - \mathbf{d}, \text{ so}$$

$$H(f) \approx \left\{ 1 + jQ \left[ 1 + \mathbf{d} - (1 - \mathbf{d}) \right] \right\}^{-1} = \frac{1}{1 + j2Q\mathbf{d}}$$

$$\text{But } \mathbf{d} = \frac{f}{f_0} - 1 = \frac{f - f_0}{f_0} \text{ so}$$

$$H(f) \approx \frac{1}{1 + j2Q(f - f_0)/f_0} \quad \text{for } \begin{array}{l} f = f_0(1 + \mathbf{d}) > 0 \\ |f - f_0| = |\mathbf{d}|f_0 \leq f_0 \end{array}$$

4.1-9

$$H(f) \approx \frac{1}{\sqrt{1 + (f - f_c + b)^2 / b^2} \sqrt{1 + (f - f_c - b)^2 / b^2}} \quad \text{stagger-tuned}$$

$$\approx \frac{1}{\sqrt{1 + (f - f_c)^2 / 2b^2}} \quad \text{single tuned}$$

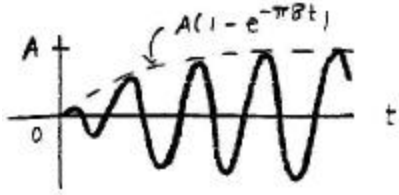
4.1-10

$$H_{\ell_p}(f) = \frac{1}{1 + j2f/B} = \frac{\mathbf{p}B}{\mathbf{p}B + j2\mathbf{p}f} \Rightarrow h_{\ell_p}(t) = \mathbf{p}B e^{-\mathbf{p}Bt} u(t)$$

$$x_{bp}(t) = 2\text{Re} \left[ \frac{A}{2} u(t) e^{j\omega_c t} \right] \Rightarrow x_{\ell_p}(t) = \frac{A}{2} u(t)$$

$$y_{\ell_p}(t) = h_{\ell_p} * x_{\ell_p}(t) = \frac{\mathbf{p}BA}{2} \int_0^t e^{-\mathbf{p}B(t-\tau)} d\tau = \frac{A}{2} (1 - e^{-\mathbf{p}Bt}) u(t)$$

$$y_{bp}(t) = 2\text{Re} \left[ y_{\ell_p}(t) e^{j\omega_c t} \right] = A (1 - e^{-\mathbf{p}Bt}) \cos \omega_c t u(t)$$



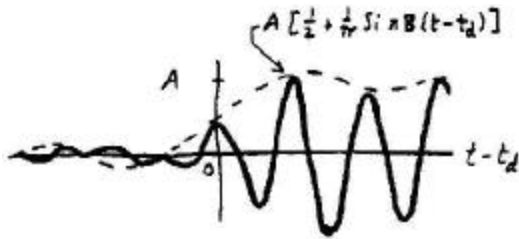
4.1-11

$$H_{\ell p}(f) = \Pi\left(\frac{f}{B}\right) e^{-j(\mathbf{w} + \mathbf{w}_c)t_d} \Rightarrow h_{\ell p}(t) = B e^{-j\mathbf{w}_c t_d} \text{sinc } B(t - t_d)$$

$$x_{bp}(t) = 2\text{Re}\left[\frac{A}{2}u(t)e^{j\mathbf{w}_c t}\right] \Rightarrow x_{\ell p}(t) = \frac{A}{2}u(t)$$

$$\begin{aligned} y_{\ell p}(t) &= h_{\ell p} * x_{\ell p}(t) = \frac{BA}{2} e^{-j\mathbf{w}_c t_d} \int_{-\infty}^t \text{sinc } B(\mathbf{l} - t_d) d\mathbf{l} \\ &= \frac{A}{2} e^{-j\mathbf{w}_c t_d} \left[ \int_{-\infty}^0 \text{sinc } \mathbf{m} d\mathbf{m} + \int_0^{B(t-t_d)} \text{sinc } \mathbf{m} d\mathbf{m} \right] \\ &= \frac{A}{2} e^{-j\mathbf{w}_c t_d} \left[ \frac{1}{2} + \frac{1}{\mathbf{p}} \text{Si } \mathbf{p} B(t - t_d) \right] \end{aligned}$$

$$y_{bp}(t) = 2\text{Re}\left[y_{\ell p}(t)e^{j\mathbf{w}_c t}\right] = A \left[ \frac{1}{2} + \frac{1}{\mathbf{p}} \text{Si } \mathbf{p} B(t - t_d) \right] \cos \mathbf{w}_c(t - t_d)$$



4.1-12

$$x_{\ell p}(t) = 2e^{ja}u(t)e^{\pm j\mathbf{w}_c t} \Rightarrow X_{\ell p}(f) = e^{ja} \left[ \frac{1}{j\mathbf{p}(f \mp f_0)} + \mathbf{d}(f \mp f_0) \right]$$

$$H_{\ell p}(f) = \Pi\left(\frac{f}{B}\right) \text{ with } \frac{B}{2} \ll f_0 \text{ so } \mathbf{d}(f \mp f_0) \text{ falls outside passband.}$$

$$\text{Thus, } Y_{\ell p}(f) = \frac{e^{ja}}{j\mathbf{p}(f \mp f_0)} \Pi\left(\frac{f}{B}\right) \approx \frac{e^{ja}}{\mp j\mathbf{p}f_0} \Pi\left(\frac{f}{B}\right) \text{ since } f_0 \ll f \text{ for } |f| < \frac{B}{2}$$

$$y_{\ell p}(t) \approx \pm j \left( \frac{e^{ja}}{\mathbf{p}f_0} \right) B \text{sinc } Bt$$

$$y_{bp}(t) \approx \frac{2B}{\mathbf{p}f_0} \text{sinc } Bt \text{ Re}\left[\pm j e^{ja} e^{j\mathbf{w}_c t}\right] = \mp \frac{2B}{\mathbf{p}f_0} \text{sinc } Bt \sin(\mathbf{w}_c t + a)$$

4.1-13

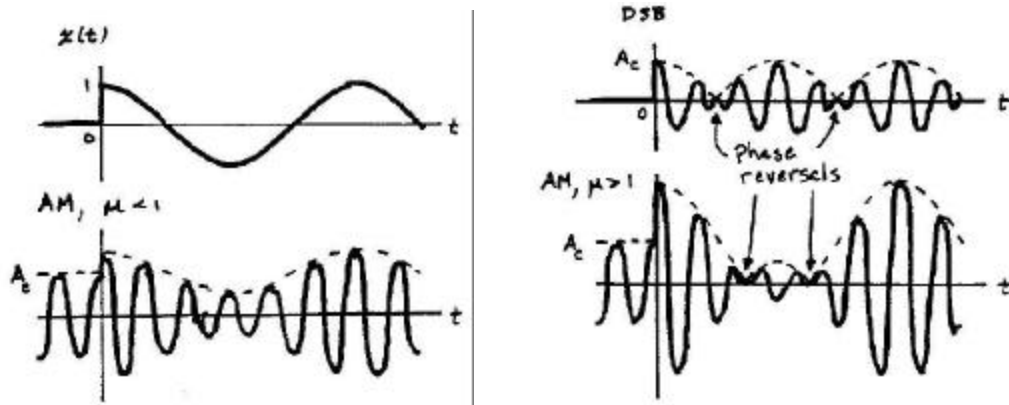
$$H_{lp}(f) = e^{j\frac{f^2}{b}} \quad X_{lp}(f) = \frac{1}{2}Z(f) = 0 \quad |f| \leq W \leq \frac{B}{2}$$

$$Y_{lp}(f) = e^{j\frac{f^2}{b}} \frac{1}{2}Z(f) \approx \frac{1}{2} \left[ 1 + j \frac{f^2}{b} \right] Z(f) \quad \text{since } \frac{f^2}{b} \ll 1 \text{ for } |f| \leq W$$

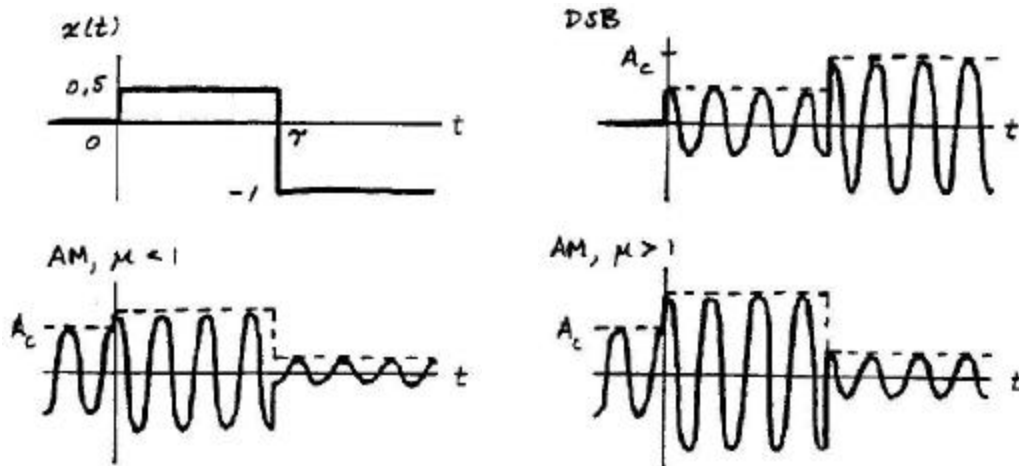
$$\approx \frac{1}{2} \left[ Z(f) - \frac{j}{4p^2b} (j2pf)^2 Z(f) \right] \Rightarrow y_{lp}(t) \approx \frac{1}{2} \left[ z(t) - \frac{j}{4p^2b} \frac{d^2}{dt^2} z(t) \right]$$

$$\text{Thus, } y_{bp}(t) \approx z(t) \cos \omega_c t - \frac{1}{4p^2b} \left[ \frac{d^2}{dt^2} z(t) \right] \sin \omega_c t$$

4.2-1



4.2-2



4.2-3

$$\text{AM: } B_T = 400\text{Hz} \quad S_T = \frac{1}{2} A_c^2 (1 + m^2 S_x) = \frac{100}{2} (1 + 0.6^2) = 68\text{W}$$

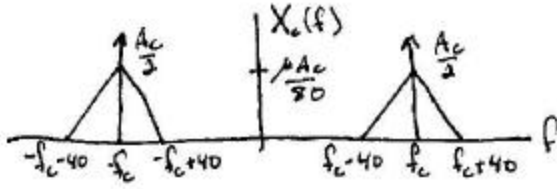
$$\text{DSB: } B_T = 400\text{Hz} \quad S_T = \frac{1}{2} A_c^2 S_x = \frac{100}{2} = 50\text{W}$$



4.2-4

$$\text{sinc}^2 40t \leftrightarrow \frac{1}{40} \Lambda\left(\frac{f}{40}\right)$$

$$B_T = 2W = 80 \text{ Hz}$$



4.2-5

$$A_{\max}^2 = (2A_c)^2 = 32 \text{ kW} \Rightarrow A_c^2 = 8 \text{ kW}$$

$$m=1, \quad S_x = \frac{1}{2} \Rightarrow S_T = \frac{1}{2} A_c^2 (1 + m^2 S_x) = 6 \text{ kW}$$

4.2-6

$$S_x = \frac{1}{2}, \quad S_T = \frac{1}{2} A_c^2 \left(1 + \frac{m^2}{2}\right) = 1 \text{ kW} \Rightarrow A_c^2 = \frac{4}{2 + m^2} \text{ kW}$$

$$A_{\max}^2 = (1 + m)^2 A_c^2 = 4 \frac{(1 + m)^2}{2 + m^2} \text{ kW} \leq 4 \text{ kW}$$

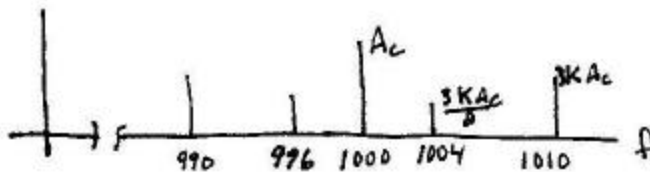
$$\text{so } 1 + 2m + m^2 \leq 2 + m^2 \Rightarrow m \leq 0.5$$

4.2-7

$$|x|_{\max} = x(0) = 3K(1 + 2) \leq 1 \Rightarrow K \leq 1/9$$

$$P_{sb} = \frac{1}{2} \left( \frac{3}{2} K A_c \right)^2 + \frac{1}{2} (3K A_c)^2 = \frac{45}{8} K^2 A_c^2 = \frac{45}{4} K^2 P_c$$

$$\frac{2P_{sb}}{S_T} = \frac{\frac{45}{2} K^2 P_c}{P_c + \frac{45}{2} K^2 P_c} = \frac{45K^2}{2 + 45K^2} \leq \frac{45}{207} \approx 22\%$$

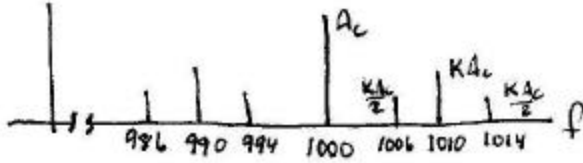


4.2-8

$$x(t) = 2K \cos 20\pi t + K \cos 12\pi t + K \cos 28\pi t \quad |x|_{\max} = x(0) = K(2+1+1) \leq 1 \Rightarrow K \leq 1/4$$

$$P_{sb} = \frac{1}{2} (KA_c)^2 + 2 \times \frac{1}{2} \left( \frac{1}{2} KA_c \right)^2 = \frac{3}{4} K^2 A_c^2 = \frac{3}{2} K^2 P_c$$

$$\frac{2P_{sb}}{S_T} = \frac{3K^2 P_c}{P_c + 3K^2 P_c} \leq \frac{3}{19} \approx 16\%$$



4.2-9

$$x(t) = 4 \sin \frac{B_T}{2} t = 4 \sin 2\pi \frac{1}{4} t \quad B_T = 2W = \frac{1}{2} \text{ kHz}$$

$$0.01 < \frac{B_T}{f_c} < 0.1 \Rightarrow 10B_T < f_c < 100B_T$$

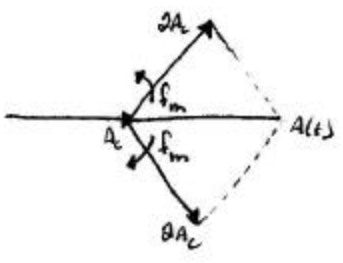
$$5 \text{ kHz} < f_c < 50 \text{ kHz}$$

4.2-10

$$x_c(t) = A_c [1 + x(t)] \cos \omega_c t \Rightarrow A(t) = A_c [1 + x(t)] \geq 0 \text{ for no phase reversals to occur}$$

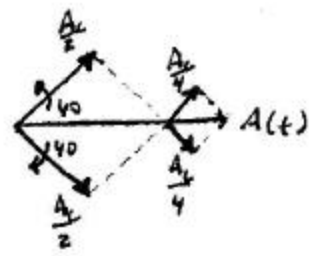
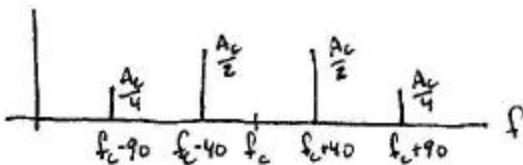
Since  $x(t)|_{\min} = -4$  there is no value of  $A_c$  that can keep  $A(t)$  from going negative.

Therefore phase reversals will occur whenever  $x(t)$  goes negative.



4.2-11

$$x_c(t) = A_c \left[ \cos 2\pi 40t + \frac{1}{2} \cos 2\pi 90t \right] \cos 2\pi f_c t$$



4.3-1

$$(a) \quad v_{out} = a_1 x(t) + a_2 x^2(t) + a_2 \cos^2 \omega_c t + \underbrace{a_1 \left[ 1 + \frac{2a_2}{a_1} x(t) \right] \cos \omega_c t}_{\text{desired term}}$$

Select a filter centered at  $f_c = 10$  kHz with a bandwidth of  $2W = 2 \times 120 = 240$  Hz.

$$(b) \quad a_1 \left[ 1 + \frac{2a_2}{a_1} x(t) \right] \cos \omega_c t = A_c [1 + m x(t)] \cos \omega_c t = 10 \left[ 1 + \frac{1}{2} x(t) \right] \cos \omega_c t$$

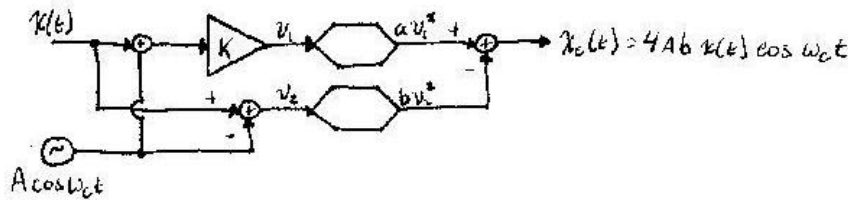
$$\Rightarrow a_1 = 10 \quad \frac{2a_2}{a_1} = \frac{1}{2} \Rightarrow a_2 = \frac{5}{2}$$

4.3-2

$$x_c(t) = aK^2 (x + A \cos \omega_c t)^2 - b(x - A \cos \omega_c t)^2$$

$$= (aK^2 - b)(x^2 + A^2 \cos^2 \omega_c t) + 2A(aK^2 + b)x \cos \omega_c t$$

$$= 4Abx(t) \cos \omega_c t \quad \text{if } K = \sqrt{\frac{b}{a}}$$

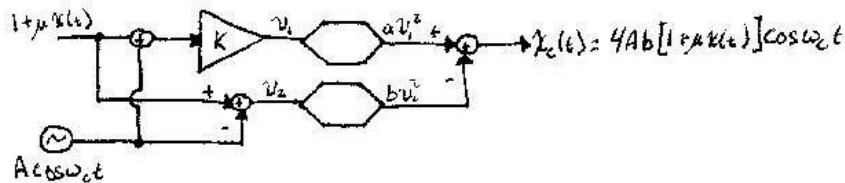


4.3-3

$$x_c(t) = aK^2 (v + A \cos \omega_c t)^2 - b(v - A \cos \omega_c t)^2$$

$$= (aK^2 - b)(v^2 + A^2 \cos^2 \omega_c t) + 2A(aK^2 + b)v \cos \omega_c t$$

$$= 4Ab[1 + m x(t)] \cos \omega_c t \quad \text{if } K = \sqrt{\frac{b}{a}} \quad \text{and } v(t) = 1 + m x(t)$$

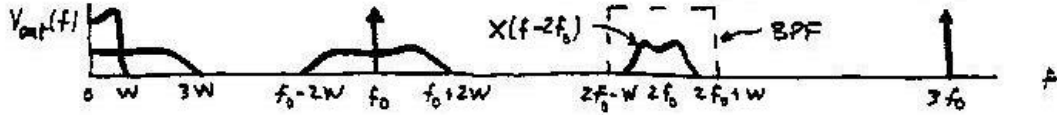


4.3-4

Take  $v_{in} = x + \cos \omega_0 t$  so

$$v_{out} = a_1(x + \cos \omega_0 t) + a_3(x^3 + 3x^2 \cos \omega_0 t + 3x \cos^3 \omega_0 t + \cos^3 \omega_0 t)$$

$$= \left(a_1 + \frac{3}{2}a_3\right)x + a_3x^3 + \left(a_1 + \frac{3}{4}a_3 + 3a_3x^2\right)\cos \omega_0 t + \frac{3}{2}a_3x \cos 2\omega_0 t + \frac{1}{4}a_3 \cos 3\omega_0 t$$



Take  $f_c = 2f_0$  where  $f_0 + 2W < 2f_0 - W$  so  $f_c > 6W$

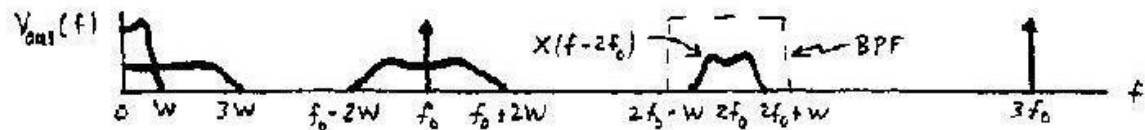


4.3-5

Take  $v_{in} = y + \cos \omega_0 t$ , where  $y = Kx(t)$ , so

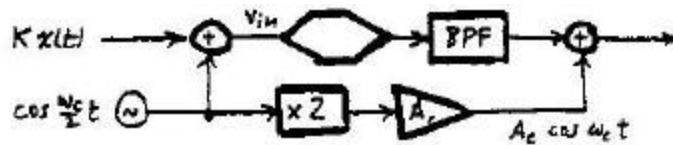
$$v_{out} = a_1(y + \cos \omega_0 t) + a_3(y^3 + 3y^2 \cos \omega_0 t + 3y \cos^2 \omega_0 t + \cos^3 \omega_0 t)$$

$$= \left(a_1 + \frac{3}{2}a_3\right)y + a_3y^3 + \left(a_1 + \frac{3}{4}a_3 + 3a_3y^2\right)\cos \omega_0 t + \frac{3}{2}a_3y \cos 2\omega_0 t + \frac{1}{4}a_3 \cos 3\omega_0 t$$



Take  $f_c = 2f_0$  where  $f_0 + 2W < 2f_0 - W$  so  $f_c > 6W$

$$x_c(t) = \left[\frac{3}{2}a_3Kx(t) + A_c\right] \cos \omega_c t = A_c \left[1 + \frac{3a_3K}{2A_c}x(t)\right] \cos \omega_c t$$



4.3-6

$$\text{Let } v_{out+} = a_1 \left( A_c \cos \mathbf{w}_c t + \frac{1}{2} x \right) + a_2 \left( A_c \cos \mathbf{w}_c t + \frac{1}{2} x \right)^2 + a_3 \left( A_c \cos \mathbf{w}_c t + \frac{1}{2} x \right)^3$$

$$v_{out-} = b_1 \left( A_c \cos \mathbf{w}_c t - \frac{1}{2} x \right) + b_2 \left( A_c \cos \mathbf{w}_c t - \frac{1}{2} x \right)^2 + b_3 \left( A_c \cos \mathbf{w}_c t - \frac{1}{2} x \right)^3$$

$$\text{Expanding using } \cos^2 \mathbf{w}_c t = \frac{1}{2} + \frac{1}{2} \cos 2\mathbf{w}_c t, \quad \cos^3 \mathbf{w}_c t = \frac{3}{4} \cos \mathbf{w}_c t + \frac{1}{4} \cos 3\mathbf{w}_c t$$

Since BPFs reject components outside  $f_c - W < |f| < f_c + W$ ,

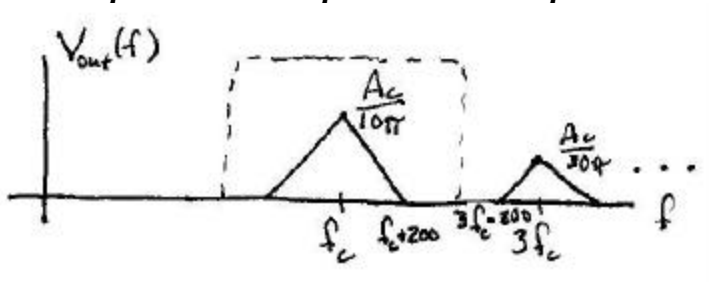
$$\begin{aligned} x_c(t) &= v_{out+}|_{BPF} - v_{out-}|_{BPF} \\ &= \left( a_1 + \frac{3}{4}a_3 - b_1 - \frac{3}{4}b_3 \right) \cos \mathbf{w}_c t + 2(a_2 + b_2)x(t) \cos \mathbf{w}_c t + 3(a_3 - b_3)x^2(t) \cos \mathbf{w}_c t \end{aligned}$$

so there's unsuppressed carrier and 2<sup>nd</sup> harmonic distortion

4.3-7

$$x(t) = 20 \text{sinc}^2 400t \quad \leftrightarrow \quad X(f) = \frac{20}{400} \Lambda \left( \frac{f}{400} \right) = \frac{1}{20} \Lambda \left( \frac{f}{400} \right)$$

$$v_{out}(t) = \frac{4}{p} x(t) \cos \mathbf{w}_c t - \frac{4}{3p} x(t) \cos 3\mathbf{w}_c t + \frac{4}{5p} x(t) \cos 5\mathbf{w}_c t - \dots$$



$$\text{need } f_c + 200 < 3f_c - 200 \Rightarrow f_c > 100 \text{ Hz}$$

But  $f_c$  must meet fractional bandwidth requirements as well

$$\text{so } 400 < 0.1f_c \Rightarrow f_c > 4000 \text{ Hz which meets the earlier requirements as well.}$$

4.4-1

$$\begin{aligned} x_c(t) &= 2 \text{Re} \left\{ \frac{1}{4} A_c [x(t) \pm j\hat{x}(t)] e^{j\mathbf{w}_c t} \right\} \\ &= \frac{A_c}{2} \text{Re} \left\{ [x(t) \cos \mathbf{w}_c t \pm (-1)\hat{x}(t) \sin \mathbf{w}_c t] + j[x(t) \sin \mathbf{w}_c t \pm \hat{x}(t) \cos \mathbf{w}_c t] \right\} \\ &= \frac{A_c}{2} [x(t) \cos \mathbf{w}_c t \mp \hat{x}(t) \sin \mathbf{w}_c t] \end{aligned}$$

4.4-2

$$x(t) \cos \omega_c t \leftrightarrow \frac{1}{2} X(f - f_c) + \frac{1}{2} X(f + f_c)$$

$$\sin \omega_c t = \frac{1}{j2} (e^{j\omega_c t} - e^{-j\omega_c t}) \quad \text{and} \quad \hat{X}(f) = (-j \operatorname{sgn} f) X(f) \quad \text{so}$$

$$\hat{x}(t) \sin \omega_c t \leftrightarrow -\frac{1}{2} \operatorname{sgn}(f - f_c) X(f - f_c) + \frac{1}{2} \operatorname{sgn}(f + f_c) X(f + f_c)$$

$$\text{Thus, } X_c(f) = \frac{A_c}{4} \{ [1 \pm \operatorname{sgn}(f - f_c)] X(f - f_c) + [1 \mp \operatorname{sgn}(f + f_c)] X(f + f_c) \}$$

4.4-3

Upper signs for USSB, so

$$1 + \operatorname{sgn}(f - f_c) = \begin{cases} 2 & f > f_c \\ 0 & f < f_c \end{cases}, \quad 1 - \operatorname{sgn}(f + f_c) = \begin{cases} 0 & f > -f_c \\ 2 & f < -f_c \end{cases},$$

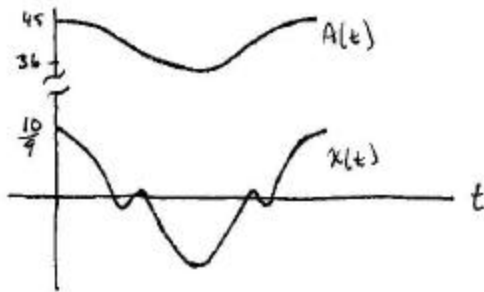
$$X_c(f) = \begin{cases} \frac{A_c}{2} X(f - f_c) & f > f_c \\ 0 & |f| < f_c \\ \frac{A_c}{2} X(f + f_c) & f < -f_c \end{cases}$$

4.4-4

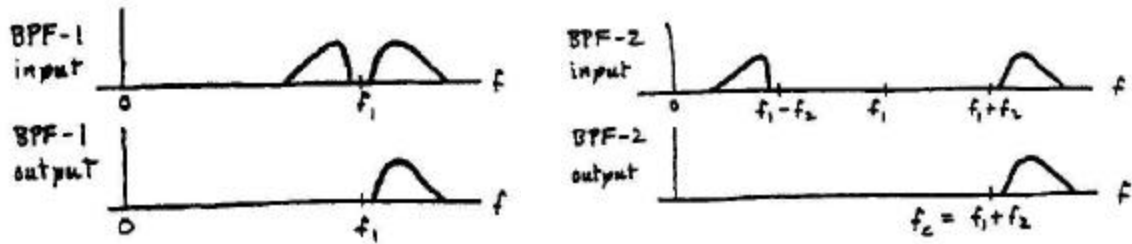
$$\text{Let } \mathbf{q} = \omega_m t \quad \text{so} \quad \hat{x}(t) = \sin \mathbf{q} + \frac{1}{9} \sin 3\mathbf{q}$$

$$\begin{aligned} x^2 + \hat{x}^2 &= \left( \cos \mathbf{q} + \frac{1}{9} \cos 3\mathbf{q} \right)^2 + \left( \sin \mathbf{q} + \frac{1}{9} \sin 3\mathbf{q} \right)^2 \\ &= 1 + \frac{1}{81} + \frac{2}{9} \cos \mathbf{q} \cos 3\mathbf{q} + \frac{2}{9} \sin \mathbf{q} \sin 3\mathbf{q} = \frac{82}{81} + \frac{2}{9} \cos 2\mathbf{q} \end{aligned}$$

$$A(t) = \frac{1}{2} A_c \sqrt{x^2(t) + \hat{x}^2(t)} = \frac{1}{2} \times 81 \times \frac{1}{9} \sqrt{82 + 18 \cos 2\mathbf{q}} = \frac{9}{2} \sqrt{82 + 18 \cos 2\mathbf{q}}$$



4.4-5



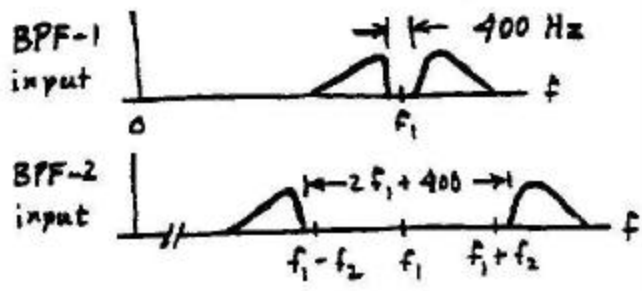
For LSSB, upper cutoffs of BPFs should be  $f_1$  and  $f_2$ , respectively.

4.4-6

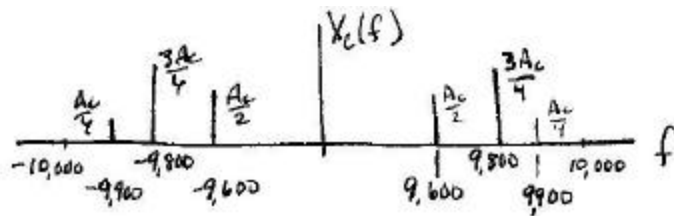
$$2b = 400 \geq 0.01f_1 \Rightarrow f_1 \leq 40\text{kHz}$$

$$0.01f_2 \leq 2f_1 + 400 \leq 80.4\text{kHz}$$

$$f_2 \leq 8.04\text{MHz} \text{ and } f_c = f_1 + f_2 \leq 8.08\text{MHz}$$



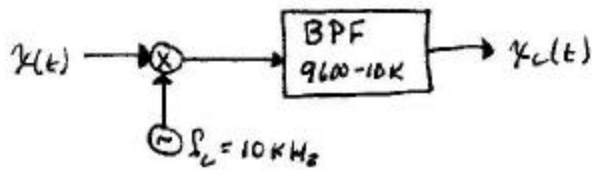
4.4-7



$$S_T = \frac{1}{4} A_c^2 S_x = \frac{1}{4} A_c^2 \left[ \frac{1}{2} (1)^2 + \frac{1}{2} (3)^2 + \frac{1}{2} (2)^2 \right] = \frac{7}{4} A_c^2 \quad \text{or calculate directly from the line spectrum}$$

$$B_T = W = 400 \text{ Hz}$$

4.4-8



Check to make sure BPF meets requirements:

$$0.01 < \frac{B}{f_c} < 0.1 \Rightarrow \frac{B}{f_c} = \frac{10,000 - 9600}{10^4} \Rightarrow 0.01 < 0.04 < 0.1 \checkmark$$

$$\text{Also } f_c < 200b = 200 \times 100 = 20 \text{ kHz } \checkmark$$

Note that a LPF at 10 kHz would have violated the fractional bandwidth requirements so a BPF must be used.

4.4-9

$$\cos(\mathbf{w}_c t - 90^\circ + \mathbf{d}) = \sin(\mathbf{w}_c t + \mathbf{d}) = \cos \mathbf{d} \sin \mathbf{w}_c t + \sin \mathbf{d} \cos \mathbf{w}_c t \approx \sin \mathbf{w}_c t + \mathbf{d} \cos \mathbf{w}_c t$$

$$\text{Thus, } x_c(t) \approx \frac{A_c}{2} \{ [x(t) + \mathbf{d} \hat{x}(t)] \cos \mathbf{w}_c t + \hat{x}(t) \sin \mathbf{w}_c t \}$$

$$A(t) \approx \frac{A}{2} [x^2(t) + \hat{x}^2(t) + 2\mathbf{d} \hat{x}(t)x(t)]^{1/2}$$

4.4-10

$$(1 - \mathbf{e}) \cos(\mathbf{w}_m t - 90^\circ + \mathbf{d}) = (1 - \mathbf{e}) [\cos \mathbf{d} \sin \mathbf{w}_m t + \sin \mathbf{d} \cos \mathbf{w}_m t] \approx (1 - \mathbf{e}) \sin \mathbf{w}_m t + \mathbf{d} \cos \mathbf{w}_m t$$

$$\begin{aligned} x_c(t) &\approx \frac{A_c}{2} [\cos \mathbf{w}_m \cos \mathbf{w}_c t - (1 - \mathbf{e}) \sin \mathbf{w}_m t \sin \mathbf{w}_c t - \mathbf{d} \cos \mathbf{w}_m t \sin \mathbf{w}_c t] \\ &= \frac{A_c}{4} \{ 2 \cos(\mathbf{w}_c + \mathbf{w}_m)t + \mathbf{e} [\cos(\mathbf{w}_c - \mathbf{w}_m)t - \cos(\mathbf{w}_c + \mathbf{w}_m)t] \\ &\quad - \mathbf{d} [\sin(\mathbf{w}_c - \mathbf{w}_m)t + \sin(\mathbf{w}_c + \mathbf{w}_m)t] \} \end{aligned}$$

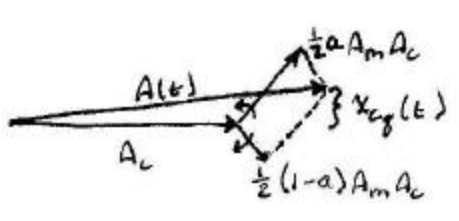
$$\text{But } \mathbf{e} \cos \mathbf{q} - \mathbf{d} \sin \mathbf{q} = \sqrt{\mathbf{e}^2 + \mathbf{d}^2} \cos(\mathbf{q} + \arctan(\mathbf{d}/\mathbf{e}))$$

$$\begin{aligned} (2 - \mathbf{e}) \cos \mathbf{q} - \mathbf{d} \sin \mathbf{q} &= \sqrt{(2 - \mathbf{e})^2 + \mathbf{d}^2} \cos\left(\mathbf{q} + \arctan \frac{\mathbf{d}}{2 - \mathbf{e}}\right) \\ &\approx 2\sqrt{1 - \mathbf{e}/2} \cos(\mathbf{q} + \mathbf{d}/2) \end{aligned}$$

$$\text{Thus } x_c(t) \approx \frac{A_c}{2} \sqrt{1 - \mathbf{e}/2} \cos[(\mathbf{w}_c + \mathbf{w}_m)t + \mathbf{d}/2] + \frac{A_c}{4} \sqrt{\mathbf{e}^2 + \mathbf{d}^2} \cos\left[(\mathbf{w}_c - \mathbf{w}_m)t + \arctan \frac{\mathbf{d}}{\mathbf{e}}\right]$$



4.4-11



The easiest way to find the quadrature component is graphically from the phasor diagram.

$$x_{cq}(t) = \frac{1}{2} a A_m A_c \sin 2\mathbf{p} f_m t - \frac{1}{2} (1-a) A_m A_c \sin 2\mathbf{p} f_m t = \left( a - \frac{1}{2} \right) A_m A_c \sin 2\mathbf{p} f_m t$$

4.4-12

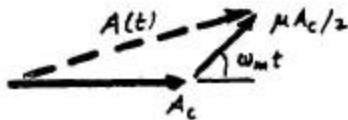
$$\begin{aligned} x_c(t) &= \frac{A_c}{2} \left[ (0.5 + a) \cos(\mathbf{w}_c + \mathbf{w}_m)t + (0.5 - a) \cos(\mathbf{w}_c - \mathbf{w}_m)t \right] \\ &= \frac{A_c}{2} \left\{ \frac{1}{2} [\cos(\mathbf{w}_c + \mathbf{w}_m)t + \cos(\mathbf{w}_c - \mathbf{w}_m)t] + 2a \frac{1}{2} [\cos(\mathbf{w}_c + \mathbf{w}_m)t - \cos(\mathbf{w}_c - \mathbf{w}_m)t] \right\} \\ &= \frac{A_c}{2} [\cos \mathbf{w}_m t \cos \mathbf{w}_c t - 2a \sin \mathbf{w}_m t \sin \mathbf{w}_c t] \end{aligned}$$

$$a = 0 \Rightarrow x_c(t) = \frac{A_c}{2} \cos \mathbf{w}_m t \cos \mathbf{w}_c t \quad \text{DSB}$$

$$a = \pm 0.5 \Rightarrow x_c(t) = \frac{A_c}{2} [\cos \mathbf{w}_m t \cos \mathbf{w}_c t \mp \sin \mathbf{w}_m t \sin \mathbf{w}_c t] = \frac{A_c}{2} \cos(\mathbf{w}_c \pm \mathbf{w}_m)t \quad \text{SSB}$$

4.4-13

$$\begin{aligned} x_c(t) &= A_c \left[ \cos \mathbf{w}_c t + \frac{\mathbf{m}}{2} \cos(\mathbf{w}_c + \mathbf{w}_m)t \right] \\ A(t) &= A_c \left[ \left( 1 + \frac{\mathbf{m}}{2} \cos \mathbf{w}_m t \right)^2 + \left( \frac{\mathbf{m}}{2} \sin \mathbf{w}_m t \right)^2 \right]^{1/2} \\ &= A_c \left[ 1 + \mathbf{m} \cos \mathbf{w}_m t + \frac{\mathbf{m}^2}{4} \right]^{1/2} \end{aligned}$$

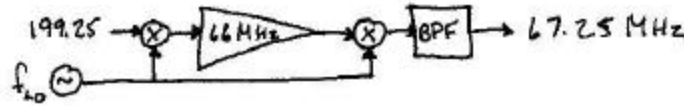


4.5-1

$$|f_1 \pm 199.25| = 66 \text{ MHz} \Rightarrow f_1 = 265.25 \text{ or } 133.25$$

$$|f_2 \pm 66| = 67.25 \text{ MHz} \Rightarrow f_2 = 133.25 \text{ or } 1.25$$

Take  $f_{LO} = 133.25 \text{ MHz}$

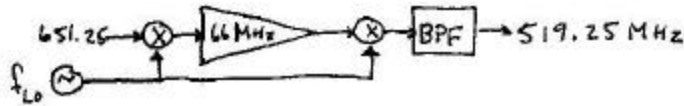


4.5-2

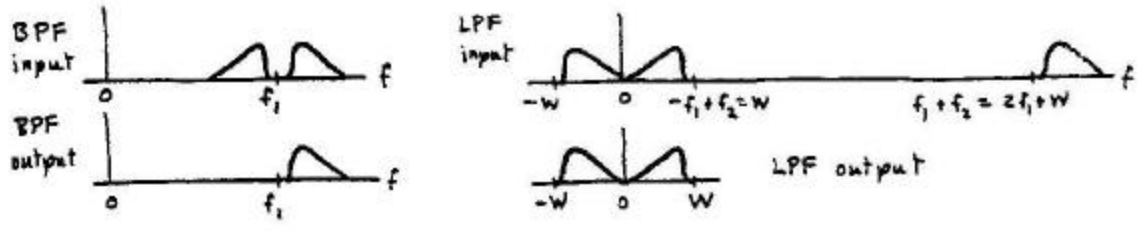
$$|f_1 \pm 651.25| = 66 \text{ MHz} \Rightarrow f_1 = 717.25 \text{ or } 585.25$$

$$|f_2 \pm 66| = 519.25 \text{ MHz} \Rightarrow f_2 = 585.25 \text{ or } 453.25$$

Take  $f_{LO} = 585.25 \text{ MHz}$



4.5-3



Output is unintelligible because spectrum is reversed, so low-frequency components become high frequencies, and vice versa.

Output signal can be unscrambled by passing it through a second, identical scrambler which again reverses the spectrum.

4.5-4

$$\begin{aligned} \text{LPF input} &= \left[ (K_c + K_m x) \cos w_c t - K_m x_q \sin w_c t \right] \cos (w_c t + f) \\ &= (K_c + K_m x) \cos f + (K_c + K_m x) \cos (2w_c t + f) + K_m x_q \sin f - K_m x_q \sin (2w_c t + f) \\ y_D(t) &= [K_c + K_m x(t)] \cos f + K_m x_q(t) \sin f \end{aligned}$$

Modulation	$K_c$	$K_m$	$x_q(t)$	$y_D(t)$
AM	$A_c$	$m A_c$	0	$A_c [1 + m x(t)] \cos f$
DSB	0	$A_c$	0	$A_c x(t) \cos f$
SSB	0	$A_c / 2$	$\mp \hat{x}(t)$	$A_c / 2 [x(t) \cos f \mp \hat{x}(t) \sin f]$
VSB	0	$A_c / 2$	$\hat{x}(t) + x_b(t)$	$A_c / 2 \{ x(t) \cos f + [\hat{x}(t) + x_b(t)] \sin f \}$

4.5-5

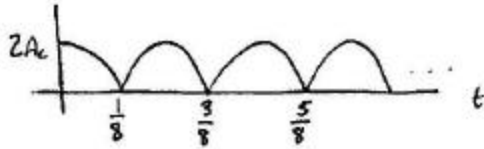
From equation for  $x_c(t)$  we see that

$a = \frac{1}{2}$  will produce standard AM with no distortion at the output.

$a = 1$  will produce USSB + C  
 $a = 0$  will produce LSSB + C } maximum distortion from envelope detector.

4.5-6

Envelope detector follows the shape of the positive amplitude portions of  $x_c(t)$ .



Envelope detector output is proportional to  $|x(t)|$ .

4.5-7

A square wave, like any other periodic signal, can be written as a Fourier series of harmonically spaced sinusoids. If the square wave has even symmetry and a fundamental of  $f_c$ , it will have terms like  $a_1 \cos \omega_c t + a_3 \cos 3\omega_c t + a_5 \cos 5\omega_c t + \dots$ . This will cause signals at  $f_c, 3f_c, 5f_c \dots$  to be shifted to the origin. If  $f_c$  is large enough, and our desired signal can be isolated, our synchronous detector will work fine. Otherwise there may be noise or intelligible crosstalk. Note that any phase shift will cause amplitude distortion. For any periodic signal in general, as long as the Fourier series has a term at  $f_c$  and our signal can be isolated, this can also serve as our local oscillator signal.

4.5-8

Between peaks  $v(t) \approx A_c [1 + \cos 2pWt_1] e^{-(t-t_1)/t}$ ,  $t_1 < t < t_1 + 1/f_c$

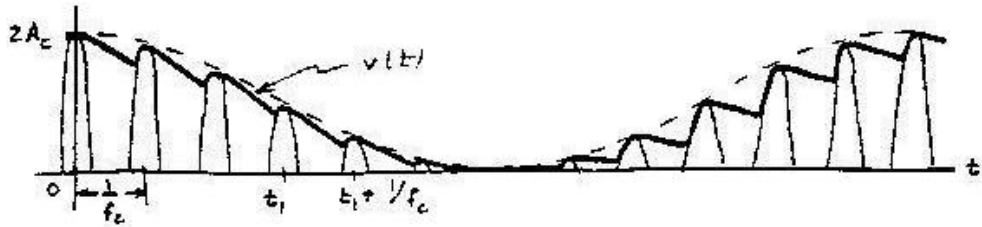
Maximum negative envelope slope occurs at  $t_1 = \frac{1}{4W}$  and we want

$$v\left(t_1 + \frac{1}{f_c}\right) \approx A_c e^{-1/tf_c} < A_c \left[1 + \cos 2pW\left(t_1 + \frac{1}{f_c}\right)\right] = A_c \left(1 - \sin \frac{2pW}{f_c}\right)$$

so  $1 - \frac{1}{tf_c} < 1 - \frac{2pW}{f_c}$  if  $t \leq \frac{1}{f_c}$  and  $f_c \leq W$

We also want  $t \leq \frac{1}{f_c}$  for linear decay between peaks.

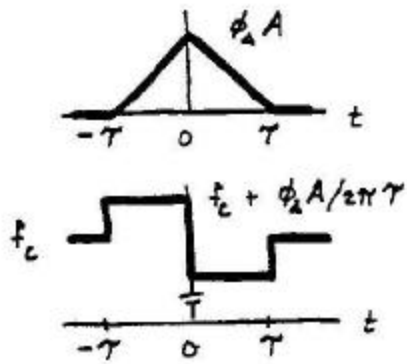
Thus  $2pW \leq R_1 C_1 \leq f_c$  and  $f_c / W \geq 2p \times 10 \approx 60$



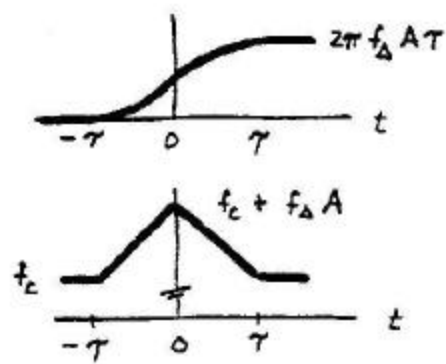
## Chapter 5

5.1-1

PM

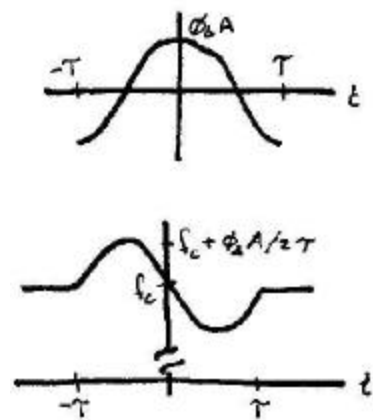


FM

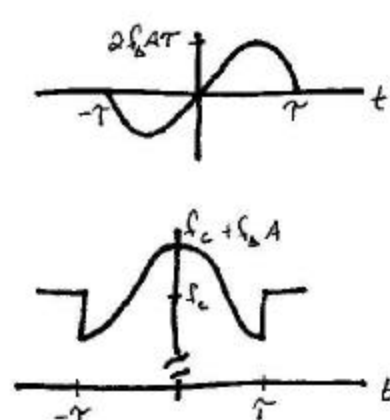


5.1-2

PM



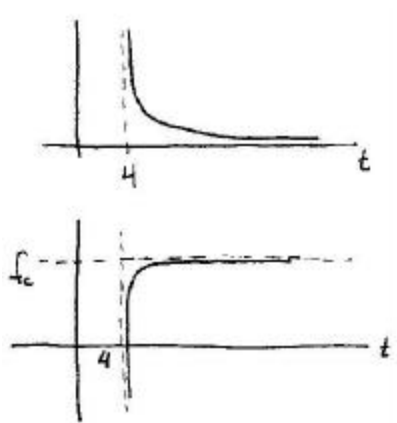
FM



5.1-3

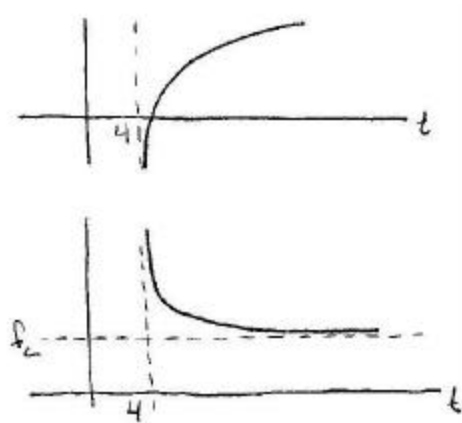
$$\frac{dx(t)}{dt} = -4A \frac{t^2 + 16}{(t^2 - 16)^2} \quad t > 4,$$

PM



$$\int' x(l) dl = \frac{4A}{2} \log(t^2 - 16) \quad t > 4$$

FM



5.1-4

$$f(t) = a + bt \quad \text{for } 0 < t < T$$

$$f(0) = a = f_1, \quad f(T) = a + bT = f_2 \Rightarrow b = \frac{f_2 - f_1}{T}$$

$$\mathbf{q}_c(t) = 2\mathbf{p} \int_0^t f(\mathbf{l}) d\mathbf{l} = 2\mathbf{p} \int_0^t \left( f_1 + \frac{f_2 - f_1}{T} \mathbf{l} \right) d\mathbf{l} = 2\mathbf{p} \left( f_1 t + \frac{f_2 - f_1}{T} t^2 \right)$$

5.1-5

Type	$\underline{f(t)}$	$\underline{f(t)}$	$\underline{f_{\max}}$	$\underline{f_{\max}}$
Phase-integral	$K \frac{dx(t)}{dt}$	$f_c + \frac{K}{2\mathbf{p}} \frac{d^2x(t)}{dt^2}$	$K 2\mathbf{p} f_m$	$f_c + K 2\mathbf{p} f_m^2$
PM	$\mathbf{f}_{\Delta} x(t)$	$f_c + \frac{\mathbf{f}_{\Delta}}{2\mathbf{p}} \frac{dx(t)}{dt}$	$\mathbf{f}_{\Delta}$	$f_c + \mathbf{f}_{\Delta} f_m$
FM	$2\mathbf{p} f_{\Delta} \int^t x(\mathbf{l}) d\mathbf{l}$	$f_c + f_{\Delta} x(t)$	$\frac{f_{\Delta}}{f_m}$	$f_c + f_{\Delta}$
Phase-accel.	$2\mathbf{p} K \int^t \left[ \int^m x(\mathbf{l}) d\mathbf{l} \right] d\mathbf{m}$	$f_c + K \int^t x(\mathbf{l}) d\mathbf{l}$	$\frac{K}{2\mathbf{p} f_m^2}$	$f_c + \frac{K}{2\mathbf{p} f_m}$

5.1-6

$$\begin{aligned} x_c(t) &= A_c \left[ \cos(\mathbf{b} \sin \mathbf{w}_m t) \cos \mathbf{w}_c t - \sin(\mathbf{b} \sin \mathbf{w}_m t) \sin \mathbf{w}_c t \right] \\ &= A_c \left[ J_0(\mathbf{b}) \cos \mathbf{w}_c t + \sum_{n \text{ even}} 2J_n(\mathbf{b}) \cos n\mathbf{w}_m t \cos \mathbf{w}_c t - \sum_{n \text{ odd}} 2J_n(\mathbf{b}) \sin n\mathbf{w}_m t \sin \mathbf{w}_c t \right] \end{aligned}$$

$$\text{where } \cos n\mathbf{w}_m t \cos \mathbf{w}_c t = \frac{1}{2} \left[ \cos(\mathbf{w}_c - n\mathbf{w}_m)t + \cos(\mathbf{w}_c + n\mathbf{w}_m)t \right]$$

$$\sin n\mathbf{w}_m t \sin \mathbf{w}_c t = \frac{1}{2} \left[ \cos(\mathbf{w}_c - n\mathbf{w}_m)t - \cos(\mathbf{w}_c + n\mathbf{w}_m)t \right]$$

$$\begin{aligned} \text{so } x_c(t) &= A_c J_0(\mathbf{b}) \cos \mathbf{w}_c t + \sum_{n \text{ even}} J_n(\mathbf{b}) \left[ \cos(\mathbf{w}_c + n\mathbf{w}_m)t + \cos(\mathbf{w}_c - n\mathbf{w}_m)t \right] \\ &\quad + \sum_{n \text{ odd}} J_n(\mathbf{b}) \left[ \cos(\mathbf{w}_c + n\mathbf{w}_m)t - \cos(\mathbf{w}_c - n\mathbf{w}_m)t \right] \end{aligned}$$

5.1-7

$$e^{j\mathbf{b} \sin \mathbf{w}_m t} = \sum_{n=-\infty}^{\infty} c_n e^{jn\mathbf{w}_m t} \quad \text{with period } T_m = 2\mathbf{p} / \mathbf{w}_m$$

$$\text{so } c_n = \frac{1}{T_m} \int_{T_m} e^{j\mathbf{b} \sin \mathbf{w}_m t} e^{-jn\mathbf{w}_m t} dt = \frac{1}{2\mathbf{p}} \int_{-\mathbf{p}}^{\mathbf{p}} e^{j(\mathbf{b} \sin \mathbf{l} - n\mathbf{l})} d\mathbf{l} = J_n(\mathbf{b})$$

$$\text{Thus, } \cos(\mathbf{b} \sin \mathbf{w}_m t) = \text{Re} \left[ e^{j\mathbf{b} \sin \mathbf{w}_m t} \right] = \text{Re} \left[ \sum_{n=-\infty}^{\infty} J_n(\mathbf{b}) e^{jn\mathbf{w}_m t} \right]$$

(cont.)

$$\begin{aligned}
&= \sum_{n=-\infty}^{\infty} J_n(\mathbf{b}) \cos n\mathbf{w}_m t = J_0(\mathbf{b}) + \sum_{n=1}^{\infty} [J_n(\mathbf{b}) + J_{-n}(\mathbf{b})] \cos n\mathbf{w}_m t \\
\sin(\mathbf{b} \sin \mathbf{w}_m t) &= \text{Im} \left[ e^{j\mathbf{b} \sin \mathbf{w}_m t} \right] = \text{Im} \left[ \sum_{n=-\infty}^{\infty} J_n(\mathbf{b}) e^{jn\mathbf{w}_m t} \right] \\
&= \sum_{n=-\infty}^{\infty} J_n(\mathbf{b}) \sin n\mathbf{w}_m t = 0 + \sum_{n=1}^{\infty} [J_n(\mathbf{b}) - J_{-n}(\mathbf{b})] \sin n\mathbf{w}_m t
\end{aligned}$$

But  $J_{-n}(\mathbf{b}) = (-1)^n J_n(\mathbf{b})$  so

$$J_n + J_{-n} = \begin{cases} 2J_n & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \quad J_n - J_{-n} = \begin{cases} 0 & n \text{ even} \\ 2J_n & n \text{ odd} \end{cases}$$

Hence,  $\cos(\mathbf{b} \sin \mathbf{w}_m t) = J_0(\mathbf{b}) + \sum_{n \text{ even}} [2J_n(\mathbf{b})] \cos n\mathbf{w}_m t$

$$\sin(\mathbf{b} \sin \mathbf{w}_m t) = \sum_{n \text{ odd}} [2J_n(\mathbf{b})] \sin n\mathbf{w}_m t$$

5.1-8

$\mathbf{b} = f_{\Delta} A_m$  for PM,  $\mathbf{b} = A_m f_{\Delta} / f_m$  for FM

- (a) Line spacing remains fixed, while line amplitudes change in the same way since  $\mathbf{b}$  is proportional to  $A_m$ .
- (b) Line spacing changes in the same way but FM line amplitudes also change while PM line amplitudes remain fixed.
- (c) Line spacing changes in the same way but PM line amplitudes also change while FM line amplitudes remain fixed.

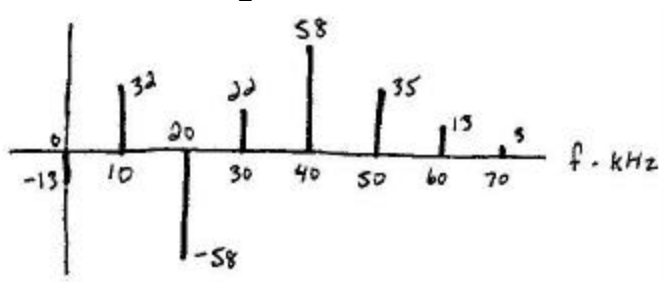
5.1-9

(a)  $f(t) = f_c + f_{\Delta} x(t) = f_c + \frac{\mathbf{b} f_m}{A_m} \cos \mathbf{w}_m t$

Assuming  $A_m = 1$   $f(t) = 30 + 20 \cos \mathbf{w}_m t$  kHz

(b) "Folded" component at  $|f_c - 4f_m| = 10$  kHz

$$S_T = (-13)^2 + \frac{1}{2} [(35-3)^2 + (-58)^2 + 22^2 + 58^2 + 35^2 + 13^2 + 3^2] = 4988.5 < \frac{100^2}{2}$$



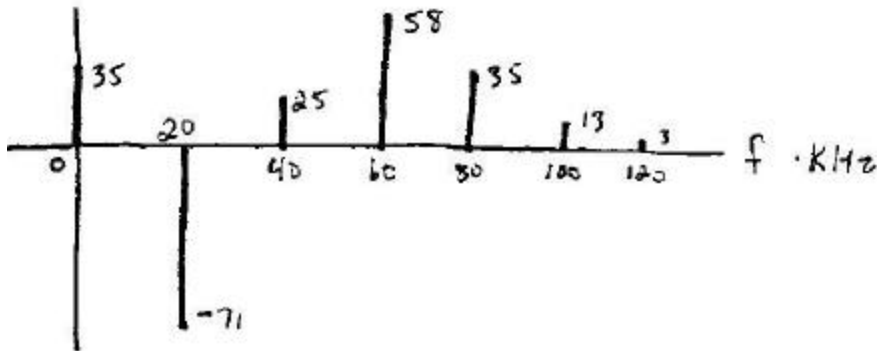
5.1-10

$$(a) f(t) = f_c + f_{\Delta}x(t) = f_c + \frac{b f_m}{A_m} \cos w_m t$$

$$\text{Assuming } A_m = 1 \quad f(t) = 40 + 40 \cos w_m t \text{ kHz}$$

$$(b) \text{ "Folded" components at } |f_c - 3f_m| = 20 \text{ kHz and } |f_c - 4f_m| = 40 \text{ kHz}$$

$$S_T = 35^2 + \frac{1}{2} [(-58-13)^2 + (22+3)^2 + 58^2 + 35^2 + 13^2 + 3^2] = 6441.5 > \frac{100^2}{2}$$

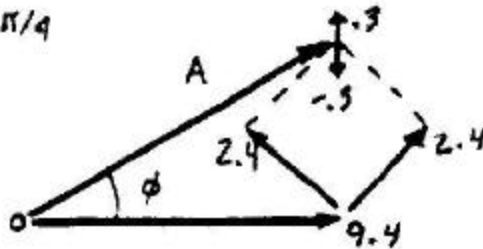


5.1-11

$$w_m t = 0 \quad A = 9.4 + 2 \times 3 = 10 \quad f = 0$$



$$w_m t = \pi/4$$

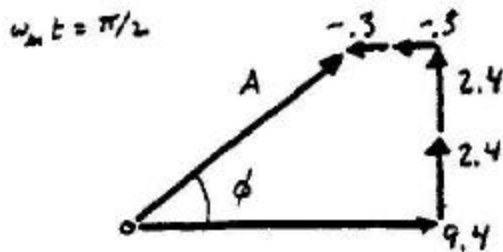


$$w_m t = \frac{p}{4} \quad A = \sqrt{9.4^2 + (2.4\sqrt{2})^2} = 9.99 \quad f = \arctan \frac{2.4\sqrt{2}}{9.4} = 0.347 \text{ rad}$$

$$0.5 \text{ rad} \times \sin \frac{p}{4} = 0.356 \text{ rad}$$

(cont.)



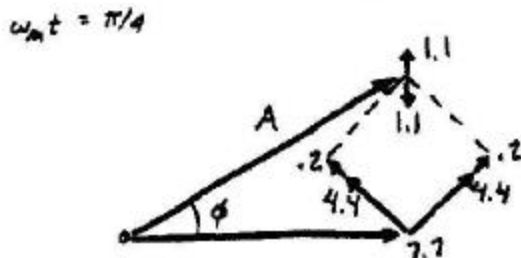
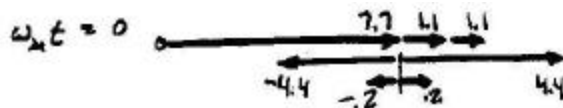


$$w_m t = \frac{p}{2} \quad A = \sqrt{(9.4 - .6)^2 + (2 \times 2.4)^2} = 10.02 \quad f = \arctan \frac{2 \times 2.4}{9.4 - .6} = 0.499 \text{ rad}$$

$$0.5 \text{ rad} \times \sin \frac{p}{2} = 0.5 \text{ rad}$$

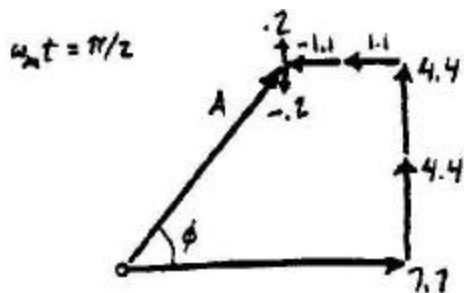
5.1-12

$$w_m t = 0 \quad A = 7.7 + 2 \times 1.1 = 9.9 \quad f = 0$$



$$w_m t = \frac{p}{4} \quad A = \sqrt{7.7^2 + (4.6\sqrt{2})^2} = 10.08 \quad f = \arctan \frac{4.6\sqrt{2}}{7.7} = 0.702 \text{ rad}$$

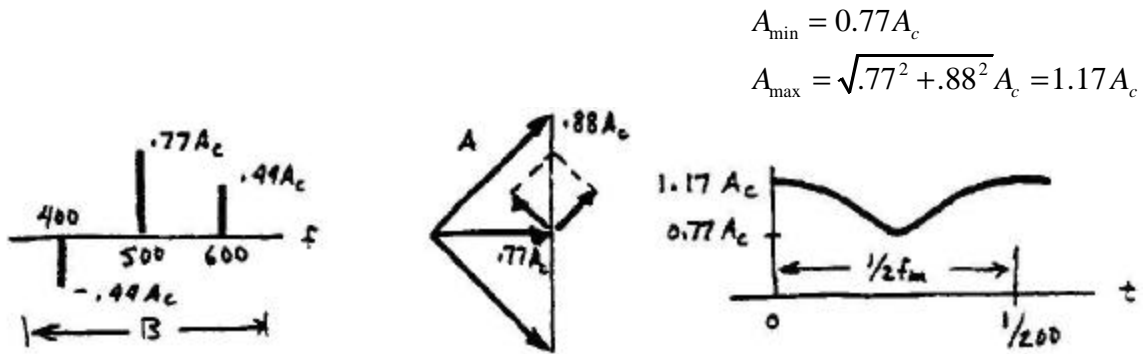
$$1 \text{ rad} \times \sin \frac{p}{4} = 0.707 \text{ rad}$$



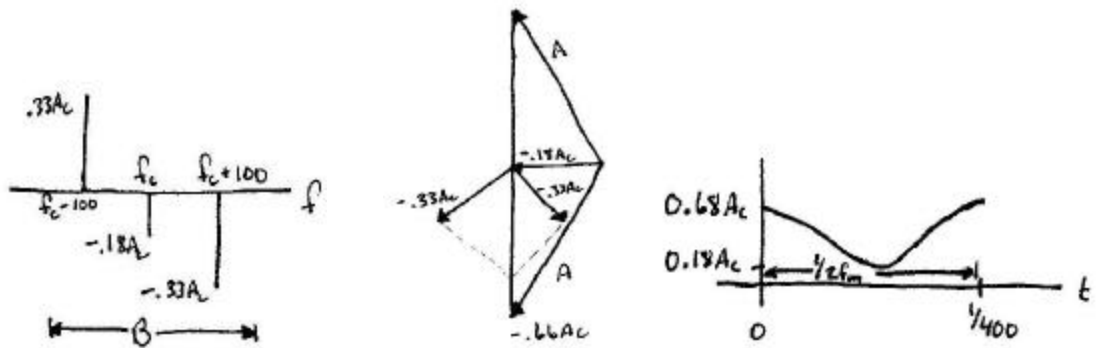
$$w_m t = \frac{p}{2} \quad A = \sqrt{(7.7 - 2.2)^2 + (2 \times 4.4)^2} = 10.02 \quad f = \arctan \frac{2 \times 4.4}{7.7 - 2.2} = 1.012 \text{ rad}$$

$$1 \text{ rad} \times \sin \frac{p}{2} = 1 \text{ rad}$$

5.1-13



5.1-14



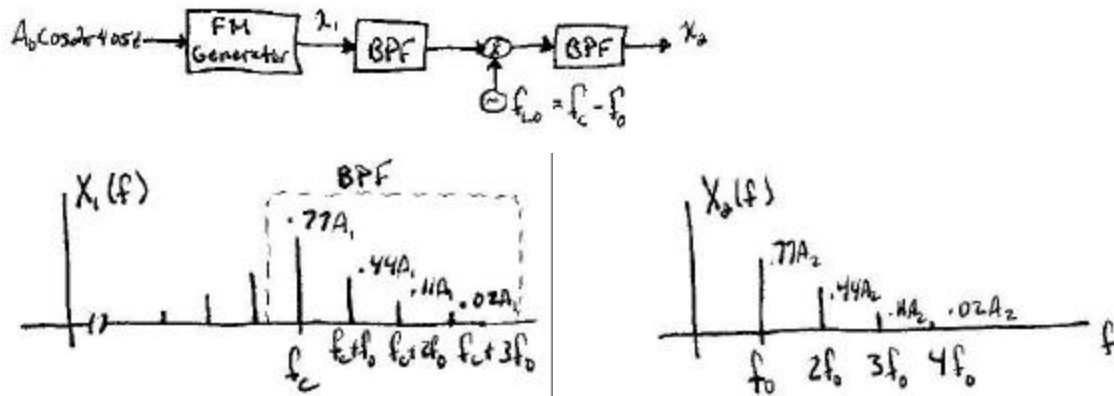
$$A_{\min} = 0.18A_c$$

$$A_{\max} = \sqrt{.18^2 + .66^2} A_c = .68A_c$$

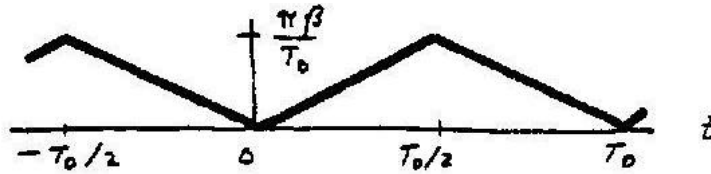
5.1-15

Want  $f_0$  plus 3 harmonics  $\Rightarrow$  select  $b = 1.0$

Generate FM signal with  $24,300 < f_c < 243,000$  to meet fractional bandwidth requirements since  $B_T = 6 \times 405 = 2,430$  Hz. Apply BPF to select carrier plus 3 sidebands. Use frequency converter at  $f_{LO} = f_c - f_0$ .



5.1-16



(a) For  $0 < t < T/2$   $f(t) = 2p f_{\Delta} t + f(0) = 2p f_{\Delta} t$

For  $|t| < \frac{T}{2}$   $f(t) = 2p f_{\Delta} |t| = \frac{2pb}{T_0} |t|$

$$\begin{aligned} c_n &= \frac{1}{T_0} \int_{-T_0/2}^0 e^{-j2p(b+n)t/T_0} dt + \frac{1}{T_0} \int_0^{T_0/2} e^{j2p(b-n)t/T_0} dt \\ &= \frac{\sin p(b+n)/2}{p(b+n)} e^{jp(b+n)/2} + \frac{\sin p(b-n)/2}{p(b-n)} e^{jp(b-n)/2} \\ &= \frac{1}{2} e^{jpb} \left[ \text{sinc}\left(\frac{n+b}{2}\right) e^{jpn/2} + \text{sinc}\left(\frac{n-b}{2}\right) e^{-jpn/2} \right] \end{aligned}$$

(b)  $b f_0 = f_{\Delta}$   $|c_n| \approx \frac{1}{2} \left[ \left| \text{sinc}\left(\frac{n+b}{2}\right) \right| + \left| \text{sinc}\left(\frac{n-b}{2}\right) \right| \right]$  when  $b \ll 1$



5.2-1

$f_{\Delta}$ , kHz	$D$	Eq.	$B_T$ , kHz
0.1	0.01	5	$(0.01+1)(30) = 30.3$
0.5	0.03	5	$(0.03+1)(30) = 31$
1	0.07	5	$(0.07+1)(30) = 32$
5	0.33	4	$(1.8)(30) = 54$
10	0.67	4	$(2.2)(30) = 66$
50	3.33	6	$(3.33+2)(30) \approx 160$
100	6.67	6	$(6.67+2)(30) \approx 260$
500	33	5 or 6	$(33+1)(30) = 1030$

5.2-2

$f_{\Delta}$ , kHz	$D$	Eq.	$B_T$ , kHz
0.1	0.02	5	$(0.02+1)(10) = 10.2$
0.5	0.1	5	$(0.1+1)(10) = 11$
1	0.2	5	$(0.2+1)(10) = 12$
5	1	4	$(2.7)(10) = 27$
10	2	4	$(3.8)(10) = 38$
50	10	6	$(10+2)(10) = 120$
100	20	5 or 6	$(20+1)(10) = 210$
500	100	5	$(100+1)(10) = 1010$

5.2-3

	$\underline{W}$	$\underline{D}$	$\underline{M(D)}$	$\underline{B_T}$
(a)	2 kHz	5	6 (lower curve)	2? 6? 2=24 kHz
(b)	3.2 kHz	3.1	3.1+2 (middle curve)	2? 5.1? 3.2? 33 kHz
(c)	20 kHz	0.5	2.5 (upper curve)	2? 2.5? 20=100kHz

5.2-4

FM:

$$D = \frac{f_{\Delta}}{W} = \frac{25}{5} = 5 \quad B_T = 2(4+2)5 = 60 \text{ MHz} \quad f_c > 10B_T = 600 \text{ MHz}$$

DSB:

$$B_T = 2W = 10 \text{ MHz} \quad f_c > 10B_T = 100 \text{ MHz}$$

5.2-5

$$W = 15 \quad f_c = 5 \times 10^{14}$$

$$0.01 < \frac{B_T}{f_c} < 0.1 \Rightarrow (0.01)(5 \times 10^{14}) < B_T < (0.1)(5 \times 10^{14})$$

$$5 \times 10^{12} < B_T < 5 \times 10^{13}$$

$$B_T \approx 2(D+1)W = 2(f_{\Delta} + W) \approx 2f_{\Delta} \Rightarrow 2.5 \times 10^{12} < f_{\Delta} < 2.5 \times 10^{13}$$

5.2-6

$$\text{For CD: } B_T = 2(5+2)15 = 210 \text{ kHz}$$

$$\text{For talk show: } B_T = 2(5+2)5 = 70 \text{ kHz}$$

Since station must broadcast at CD bandwidth, the fraction of the available bandwidth used during the talk show is

$$\frac{B_{T_{\text{used}}}}{B_{T_{\text{available}}}} = \frac{B_{T_{\text{talk}}}}{B_{T_{\text{CD}}}} = \frac{70}{210} \times 100 = 33.3\%$$

5.2-7

$$D = f_{\Delta} = 30/10 = 3 \Rightarrow B_T = 2M(3)W \approx 100 \text{ kHz}$$

$f_m$ , kHz	FM			PM	
	$b$	$B = 2M(b)f_m$	$B/B_T$	$B = 2M(b)f_m$	$B/B_T$
0.1	300	600×0.1	60%	1	1%
1.0	30	62×1.0	62%	10	10%
5.0	6	14×5.0	70%	50	50%

5.2-8

Take  $x(t) = A_m \cos \omega_m t$ ,  $b = f_{\max}$ , and  $B \approx 2(b+1)f_m$

	<u>Phase-integral modulation</u>	<u>Phase-acceleration modulation</u>
$f(t)$	$-2pKA_m f_m \sin \omega_m t$	$-(KA_m / 2p f_m^2) \cos \omega_m t$
$b$	$2pKA_m f_m$	$KA_m / 2p f_m^2$
$B$	$2(2pKA_m f_m^2 + f_m)$	$2(KA_m / 2p f_m + f_m)$
$B_T$	$\begin{cases} 4pKW^2 & 2pKW \ll 1 \\ 2W & 2pKW \gg 1 \end{cases}$	$\begin{cases} K/p f_{m_{\min}} & K/2p \ll f_{m_{\min}} W \\ 2W & K/2p \gg f_{m_{\min}} W \end{cases}$

In both cases, spectral lines are spaced by  $f_m$  and  $B$  increases with  $A_m$ . However, in phase-integral modulation, tones at  $f_m \ll W$  occupy much less than  $B_T$  if  $2pKW \ll 1$ . In phase-acceleration modulation, mid-frequency tones may occupy the most bandwidth and will determine  $B_T$  when  $K/2p \approx f_{m_{\min}} W$ .

5.2-9

$$x_{lp}(t) = \frac{1}{2} A_c e^{j f(t)} \approx \frac{1}{2} A_c [1 + j f(t)], \quad f(t) = f_\Delta x(t)$$

$$Y_p(f) = \frac{1}{1 + j2Qf/f_c} \frac{1}{2} A_c [d(f) + j f_\Delta X(f)] = \frac{1}{2} A_c \left[ d(f) + j f_\Delta \frac{p f_c}{Q} \frac{1}{\frac{p f_c}{Q} + j 2p f} X(f) \right]$$

$$y_{lp}(t) = \frac{1}{2} A_c \left[ 1 + j f_\Delta \frac{p f_c}{Q} \tilde{x}(t) \right] \quad \text{where} \quad \tilde{x}(t) = [e^{-p f_c t/Q} u(t)] * x(t)$$

$$y_c(t) = A_c \operatorname{Re} \left\{ e^{j \omega_c t} + j f_\Delta \frac{p f_c}{Q} \tilde{x}(t) [\cos \omega_c t + j \sin \omega_c t] \right\}$$

$$= A_c \left[ \cos \omega_c t - f_\Delta \frac{p f_c}{Q} \tilde{x}(t) \sin \omega_c t \right]$$

$$\text{so } f(t) = \arctan \left[ f_\Delta \frac{p f_c}{Q} \tilde{x}(t) \right]$$

5.2-10

$$Y_{\ell p}(f) = [K_0 - K_2 f^2] X_{\ell p}(f) = K_0 X_{\ell p}(f) - \frac{K_2}{(j2\mathbf{p}f)^2} X_{\ell p}(f)$$

$$y_{\ell p}(t) = K_0 x_{\ell p}(t) + \frac{K_2}{4\mathbf{p}^2} \ddot{x}_{\ell p}(t)$$

$$\text{where } x_{\ell p}(t) = \frac{1}{2} A_c e^{j\mathbf{f}(t)} \quad \text{and} \quad \ddot{x}(t) = \frac{j}{2} A_c [\ddot{\mathbf{f}}(t) e^{j\mathbf{f}(t)} + j\dot{\mathbf{f}}^2(t) e^{j\mathbf{f}(t)}]$$

$$\text{with } \dot{\mathbf{f}}(t) = 2\mathbf{p} f_{\Delta} \dot{x}(t), \quad \ddot{\mathbf{f}}(t) = 2\mathbf{p} f_{\Delta} \ddot{x}(t)$$

$$\begin{aligned} y_c(t) &= A_c \operatorname{Re} \left\{ K_0 e^{j[\mathbf{w}_c t + \mathbf{f}(t)]} + \frac{K_2}{4\mathbf{p}^2} [j\dot{\mathbf{f}}(t) - \dot{\mathbf{f}}^2(t)] e^{j[\mathbf{w}_c t + \mathbf{f}(t)]} \right\} \\ &= A_c \left\{ \left[ K_0 - \frac{K_2}{4\mathbf{p}^2} 4\mathbf{p}^2 f_{\Delta}^2 x^2(t) \right] \cos[\mathbf{w}_c t + \mathbf{f}(t)] - \frac{K_2}{4\mathbf{p}^2} 2\mathbf{p} f_{\Delta} \dot{x}(t) \sin[\mathbf{w}_c t + \mathbf{f}(t)] \right\} \end{aligned}$$

$$\text{so } A(t) = A_c \left\{ \left[ K_0 - K_2 f_{\Delta}^2 x^2(t) \right]^2 + \left[ \frac{K_2 f_{\Delta}}{2\mathbf{p}} \dot{x}(t) \right]^2 \right\}$$

5.2-11

$$y_c(t) = A_c \cos[\mathbf{w}_c t + \mathbf{f}_y(t)] \quad \text{with } \mathbf{f}_y(t) = \mathbf{f}(t) + \arg H[f(t)]$$

$$f(t) - f_c = f_{\Delta} x(t) \Rightarrow \arg H[f(t)] = \mathbf{a}_1 f_{\Delta} x(t) + \mathbf{a}_3 f_{\Delta}^3 x^3(t)$$

$$f_y(t) = f_c + \frac{1}{2\mathbf{p}} \dot{\mathbf{f}}_y(t) = f_c + f_{\Delta} \dot{x}(t) + \frac{\mathbf{a}_1 f_{\Delta}}{2\mathbf{p}} \dot{x}(t) + \frac{3\mathbf{a}_3 f_{\Delta}^3}{2\mathbf{p}} x^2(t) \dot{x}(t)$$

5.2-12

$$H[f(t)] = \left[ 1 + j \frac{2Q f_{\Delta} x(t)}{f_c} \right]^{-1} = [1 + j\mathbf{a}x(t)]^{-1}, \quad \mathbf{a}^2 \ll 1$$

$$|H[f(t)]| = [1 + \mathbf{a}^2 x^2(t)]^{-1/2} \approx 1 - \frac{1}{2} \mathbf{a}^2 x^2(t), \quad \arg H[f(t)] = -\arctan \mathbf{a}x(t) \approx -\mathbf{a}x(t)$$

$$\text{Thus, } y_c(t) \approx A_c \left[ 1 - \frac{1}{2} \mathbf{a}^2 x^2(t) \right] \cos \left[ \mathbf{w}_c t + 2\mathbf{p} f_{\Delta} \int^t x(l) dl - \mathbf{a}x(t) \right]$$

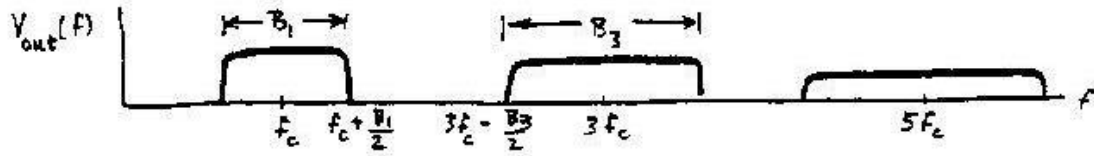
$$f_y(t) = f_c + \frac{1}{2\mathbf{p}} \dot{\mathbf{f}}_y(t) = f_c + f_{\Delta} \dot{x}(t) - \frac{\mathbf{a}}{2\mathbf{p}} \dot{x}(t)$$

5.2-13

$$B_1 \approx 2(D+1)W, \quad B_3 \approx 2(3D+1)W \quad \text{since} \quad 3f(t) = 2p(3f_\Delta) \int^t x(1) d1$$

$$\text{We want } 3f_c - \frac{B_3}{2} > f_c + \frac{B_1}{2} \Rightarrow 2f_c > (D+1)W + (3D+1)W = 4f_\Delta + 2W$$

$$\text{hence } f_\Delta < \frac{(f_c - W)}{2}$$



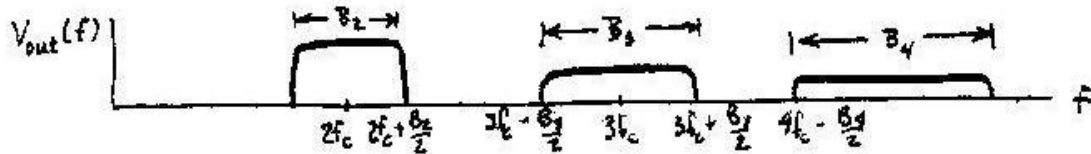
5.2-14

$$nf(t) = 2p(nf_\Delta) \int^t x(1) d1 \Rightarrow B_n \approx 2(nD+1)W$$

$$\text{We want } 2f_c + \frac{B_2}{2} < 3f_c + \frac{B_3}{2} \quad \text{and} \quad 3f_c + \frac{B_3}{2} < 4f_c + \frac{B_4}{2}$$

$$\text{so } f_c > \frac{1}{2}(B_3 + B_4) = (3D+1 + 4D+1)W = 7f_\Delta + 2W$$

$$\text{and } f_\Delta < \frac{f_c - 2W}{7}$$



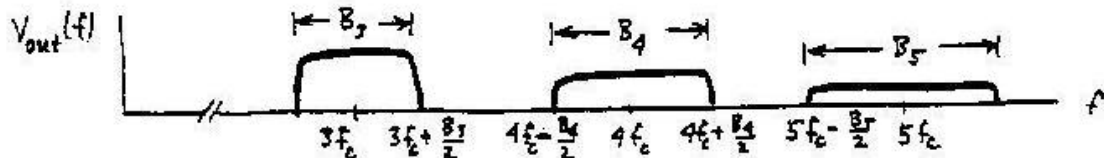
5.2-15

$$nf(t) = 2p(nf_\Delta) \int^t x(1) d1 \Rightarrow B_n \approx 2(nD+1)W$$

$$\text{We want } 3f_c + \frac{B_3}{2} < 4f_c - \frac{B_4}{2} \quad \text{and} \quad 4f_c + \frac{B_4}{2} < 5f_c - \frac{B_5}{2}$$

$$\text{so } f_c > \frac{1}{2}(B_4 + B_5) = (4D+1 + 5D+1)W = 9f_\Delta + 2W$$

$$\text{and } f_\Delta < \frac{(f_c - 2W)}{9}$$



5.3-1

Let  $\mathbf{a} = 1/NV_B \ll 1$

$$c(t) = c_1 + \frac{c_2}{\sqrt{V_B}}(1 + \mathbf{a}x)^{-1/2} = c_1 + \frac{c_2}{\sqrt{V_B}} \left( 1 - \frac{1}{2}\mathbf{a}x + \frac{3}{8}\mathbf{a}^2x^2 + \dots \right)$$

Since  $|x| \leq 1$ , we want  $\frac{3}{8}\mathbf{a}^2 \leq \frac{1}{100} \frac{\mathbf{a}}{2} \Rightarrow NV_B \geq \frac{300}{4}$

Then  $c(t) \approx c_0 - cx(t)$  with

$$c_0 = c_1 + \frac{c_2}{\sqrt{V_B}} \quad c = \frac{c_2}{\sqrt{V_B}} \frac{\mathbf{a}}{2} = \frac{c_2}{2NV_B\sqrt{V_B}}$$

Thus,  $c \leq \frac{c_2}{150\sqrt{V_B}}$ ,  $c_0 > \frac{c_2}{\sqrt{V_B}}$

so  $\frac{f_\Delta}{f_c} = \frac{c}{2c_0} < \frac{1}{300}$

5.3-2

$$f_\Delta = 150 \text{ kHz} \quad B_T \approx 2(f_\Delta + 2W)$$

$$0.01 < \frac{B_T}{f_c} < 0.1 \Rightarrow 0.01f_c < 2f_\Delta + 4W < 0.1f_c$$

If  $f_c = 10 \text{ MHz}$   $\frac{(0.01 \times 10 \text{ MHz}) - (2 \times 150 \text{ kHz})}{4} < W < \frac{(0.1 \times 10 \text{ MHz}) - (2 \times 150 \text{ kHz})}{4}$

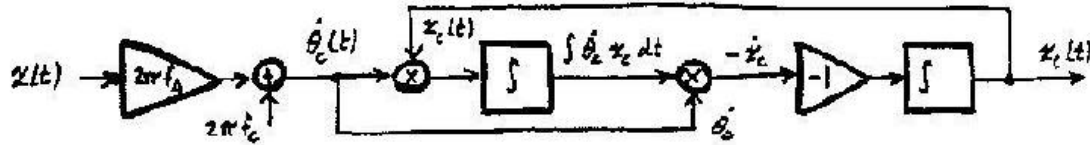
Since  $W$  cannot be negative,  $W < 175 \text{ kHz}$

5.3-3

$$\dot{x}_c(t) = -A_c \dot{q}_c(t) \sin q_c(t), \quad \int \dot{q}_c(t) x_c(t) dt = A_c \int \cos q_c(t) \frac{dq_c}{dt} dt = A_c \sin q_c(t)$$

Thus,  $-\dot{q}_c(t) \int \dot{q}_c(t) x_c(t) dt = -\dot{q}_c(t) A_c \sin q_c(t) = \dot{x}_c(t)$

For FM, we want  $\dot{q}_c(t) = 2\pi f(t) = 2\pi [f_c + f_\Delta x(t)]$



5.3-4

The frequency modulation index is proportional to the message amplitude and inversely proportional to the message frequency, whereas the phase modulation index is proportional to amplitude only. Therefore the output of an FM detector tends to boost higher frequencies, resulting in the higher frequencies in the output message signal being boosted relative to the original message signal.



## 5.3-5

The lower frequencies would have much more phase deviation than a PM modulator would have given them. Since the output from a PM demodulator is proportional to the phase deviation, the lower frequencies in the output message signal would be boosted relative to the original message signal.

## 5.3-6

$$D = \frac{f_{\Delta}}{W} \Rightarrow f_{\Delta} = DW = 5 \times 15 = 75 \text{ kHz}$$

$$f_{\Delta} = n \left( \frac{f_{\Delta}}{2pT} \right) \text{ so } 75,000 < n \times 20 \Rightarrow n > 3750$$

Since we are using triplers we need  $n = 3^m > 3750$

$$\text{For } m = \begin{cases} 7 & 3^7 = 2187 \\ 8 & 3^8 = 6561 \end{cases} \text{ therefore } m = 8 \text{ triplers are needed.}$$

If the local oscillator is placed at the end,  $6581f_{c_1} - f_{LO} = 915 \times 10^6$

$$\text{Thus, } f_{LO} = 6581 \times (500 \times 10^3) - 915 \times 10^6 = 2.37 \times 10^9 \text{ Hz}$$

## 5.3-7

$$f_{\Delta} = DW = 25 \text{ kHz, } n > \frac{25 \text{ kHz}}{20 \text{ Hz}} = 1250$$

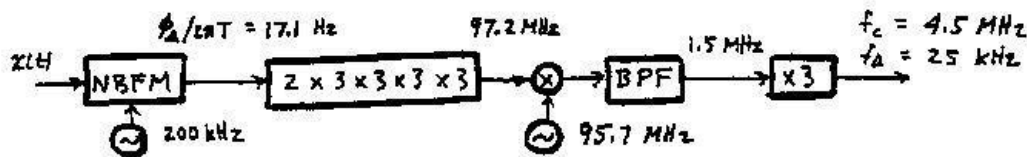
One doubler and 6 triplers yield  $n = 2 \times 3^6 = 1458$

$$\text{so } \frac{f_{\Delta}}{2pT} = \frac{25 \text{ kHz}}{1458} = 17.1 \text{ Hz}$$

$$200 \text{ kHz} \times 1458 = 291.6 \text{ MHz} > 100 \text{ MHz}$$

Use down-converter before last tripler, where  $291.6/3 = 97.2 \text{ MHz}$

$$\text{so } f_{LO} = 97.2 - (4.5/3) = 95.7 \text{ MHz}$$



## 5.3-8

$$f_{\Delta} = n \frac{f_{\Delta}}{2pT} \Rightarrow n = \frac{f_{\Delta}}{\frac{f_{\Delta}}{2pT}} > 120$$

Using doublers only  $2^7 = 128 > 120 \Rightarrow 7$  doublers

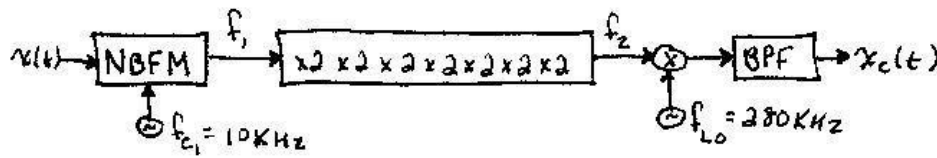
$$nf_{c_1} = 128 \times 10 \text{ kHz} = 1.28 \text{ MHz}$$

Since this doesn't exceed 10 MHz, the down converter can be located at any point.

$\Rightarrow$  Choose to place it after the last doubler.

$$f_n(t) = nf_{c_1} + n \frac{f_{\Delta}}{2pT} x(t) \text{ at the end of the last doubler}$$

$$f_c = |nf_{c_1} \pm f_{LO}| \Rightarrow 1 \text{ MHz} = |128 \times 10 \text{ kHz} \pm f_{LO}| \Rightarrow f_{LO} = 280 \text{ kHz}$$



5.3-9

$$(a) \frac{f_{\Delta}}{T} \int A_m \cos \mathbf{w}_m t dt = \mathbf{b} \sin \mathbf{w}_m t$$

$$\text{NBFM output} = A_c \cos \mathbf{w}_{c_1} t - A_c \mathbf{b} \sin \mathbf{w}_m t \sin \mathbf{w}_{c_1} t = A(\mathbf{b}) \cos [\mathbf{w}_{c_1} t + \arctan(\mathbf{b} \sin \mathbf{w}_m t)]$$

$$f_1(t) = f_{c_1} + \frac{1}{2p} \frac{d}{dt} [\arctan(\mathbf{b} \sin \mathbf{w}_m t)] = f_{c_1} + \mathbf{b} f_m \frac{\cos \mathbf{w}_m t}{1 + \mathbf{b}^2 \sin^2 \mathbf{w}_m t}$$

$$\frac{\cos \mathbf{w}_m t}{1 + \mathbf{b}^2 \sin^2 \mathbf{w}_m t} = \cos \mathbf{w}_m t [1 - \mathbf{b}^2 \sin^2 \mathbf{w}_m t + \mathbf{b}^4 \sin^4 \mathbf{w}_m t + \dots]$$

$$\approx \cos \mathbf{w}_m t \left[ 1 - \mathbf{b}^2 \left( \frac{1}{2} - \frac{1}{2} \cos 2\mathbf{w}_m t \right) \right], \quad \mathbf{b} \ll 1$$

$$= \left( 1 - \frac{\mathbf{b}^2}{2} \right) \cos \mathbf{w}_m t + \frac{\mathbf{b}^2}{4} (\cos \mathbf{w}_m t + \cos 3\mathbf{w}_m t)$$

$$\text{Thus, } f_1(t) \approx f_{c_1} + \mathbf{b} f_m \left[ \left( 1 - \frac{\mathbf{b}^2}{4} \right) \cos \mathbf{w}_m t + \frac{\mathbf{b}^2}{4} \cos 3\mathbf{w}_m t \right]$$

$$\approx f_{c_1} + \mathbf{b} f_m \left[ \cos \mathbf{w}_m t + \left( \frac{\mathbf{b}}{2} \right)^2 \cos 3\mathbf{w}_m t \right]$$

(cont.)

$$(b) \text{ 3}^{\text{rd}} \text{ harmonic distortion} = \left(\frac{b}{2}\right)^2 \times 100 = 25 \left(\frac{f_{\Delta}}{2pT}\right)^2 \frac{A_m^2}{f_m^2} \leq 1\%$$

Worst case occurs with  $A_m$  maximum and  $f_m$  minimum, so

$$5 \left(\frac{f_{\Delta}}{2pT}\right)_{30 \text{ Hz}} \leq 1 \Rightarrow \frac{f_{\Delta}}{2pT} \leq 6 \text{ Hz}$$

5.3-10

$$f(t) = f_c + f_{\Delta}x(t) = f_0 - b + f_{\Delta}x(t)$$

$$\begin{aligned} |H[f(t)]| &= \left[1 + \left(\frac{2Q}{f_0}\right)^2 (b - f_{\Delta}x)^2\right]^{-1/2} = \left[1 + \mathbf{a}^2 \left(1 - \frac{f_{\Delta}x}{b}\right)^2\right]^{-1/2} \quad \text{where } \mathbf{a} = \frac{2Qb}{f_0} \\ &\approx 1 - \frac{1}{2} \mathbf{a}^2 \left(1 - \frac{f_{\Delta}x}{b}\right)^2 \quad \text{for } \mathbf{a}^2 \left(1 - \frac{f_{\Delta}x}{b}\right)^2 \ll 1 \end{aligned}$$

$$A(t) = A_c |H[f(t)]| \approx A_c \left(1 - \frac{\mathbf{a}^2}{2}\right) + A_c \left(\frac{\mathbf{a}^2 f_{\Delta}}{b}\right) x(t)$$

$$\text{so } y_D(t) \approx K_D f_{\Delta} x(t) \quad \text{where } K_D = A_c \frac{\mathbf{a}^2}{b} = A_c \left(\frac{2Q}{f_0}\right)^2 b$$

5.3-11

$$f(t) = f_c + f_{\Delta}x(t) \quad \text{and} \quad H(f) = \left[1 + j \left(\frac{f}{f_c}\right)\right]^{-1}$$

Let  $\mathbf{a} = f_{\Delta} / f_c$  so  $|\mathbf{a}x| \ll 1$

$$\begin{aligned} |H[f(t)]| &= \left[1 + (1 + \mathbf{a}x)^2\right]^{-1/2} = \frac{1}{\sqrt{2}} \left(1 + \mathbf{a}x + \frac{\mathbf{a}^2}{2} x^2\right)^{-1/2} \\ &= \frac{1}{\sqrt{2}} \left[1 - \frac{1}{2} \left(\mathbf{a}x + \frac{\mathbf{a}^2 x^2}{2}\right) + \frac{3}{8} \left(\mathbf{a}x + \frac{\mathbf{a}^2 x^2}{2}\right)^2 + \dots\right] \\ &\approx \frac{1}{\sqrt{2}} \left(1 - \frac{1}{2} \mathbf{a}x + \frac{1}{8} \mathbf{a}^2 x^2\right) \end{aligned}$$

$$A(t) = A_c |H[f(t)]| \approx \frac{A_c}{\sqrt{2}} \left[1 - \frac{f_{\Delta}}{2f_c} x(t) + \frac{1}{8} \left(\frac{f_{\Delta}}{f_c}\right)^2 x^2(t)\right]$$

$$\text{so } y_D(t) \approx -K_1 f_{\Delta} x(t) + K_2 f_{\Delta}^2 x^2(t) \quad \text{with } K_1 = \frac{A_c}{2\sqrt{2}f_c}, \quad K_2 = \frac{A_c}{8\sqrt{2}f_c^2}$$

(cont.)

If  $x(t) = \cos \mathbf{w}_m t$ , then  $x^2(t) = \frac{1}{2} + \frac{1}{2} \cos 2\mathbf{w}_m t$

$$\begin{aligned} \text{So } 2^{\text{nd}} \text{ harmonic distortion} &= \frac{K_2 f_\Delta^2 / 2}{K_1 f_\Delta - K_2 f_\Delta^2 / 2} \times 100 \approx 100 \frac{K_2}{2K_1} f_\Delta \\ &= 100 \frac{f_\Delta}{8f_c} < 1\% \Rightarrow \frac{f_\Delta}{f_c} < 0.08 \end{aligned}$$

5.3-12

$$\begin{aligned} y_D(t) &= A_c \left\{ \left| H_u[f(t)] \right| - \left| H_\ell[f(t)] \right| \right\} \quad f(t) = f_c + f_\Delta x(t) \\ &= A_c \left\{ \left[ 1 + \frac{\mathbf{a}^2}{b^2} (f_c + f_\Delta x - f_c - b)^2 \right]^{-1/2} - \left[ 1 + \frac{\mathbf{a}^2}{b^2} (f_c + f_\Delta x - f_c + b)^2 \right]^{-1/2} \right\} \\ \frac{y_D(t)}{A_c} &= \left[ 1 + \mathbf{a}^2 \left( 1 - \frac{f_\Delta x}{b} \right)^2 \right]^{-1/2} - \left[ 1 + \mathbf{a}^2 \left( 1 + \frac{f_\Delta x}{b} \right)^2 \right]^{-1/2} \\ &= \left[ 1 - \frac{\mathbf{a}^2}{2} \left( 1 - \frac{f_\Delta x}{b} \right)^2 + \frac{3}{8} \mathbf{a}^4 \left( 1 - \frac{f_\Delta x}{b} \right)^4 + \dots \right] - \left[ 1 - \frac{\mathbf{a}^2}{2} \left( 1 + \frac{f_\Delta x}{b} \right)^2 + \frac{3}{8} \mathbf{a}^4 \left( 1 + \frac{f_\Delta x}{b} \right)^4 + \dots \right] \\ &\approx \frac{1}{2} \frac{\mathbf{a}^2 4 f_\Delta x}{b} - \frac{3}{8} \mathbf{a}^4 \left[ 8 \left( \frac{f_\Delta x}{b} \right) + 8 \left( \frac{f_\Delta x}{b} \right)^3 \right] \quad \text{when } \mathbf{a}^2 \left( 1 \pm \frac{f_\Delta x}{b} \right)^2 \ll 1 \end{aligned}$$

$$\text{so } y_D(t) \approx A_c \left[ \left( 2\mathbf{a}^2 - 3\mathbf{a}^4 \right) \frac{f_\Delta x}{b} - 3\mathbf{a}^4 \left( \frac{f_\Delta x}{b} \right)^3 \right] = K_1 x(t) - K_3 x^3(t)$$

$$\text{where } K_1 = A_c \frac{(2\mathbf{a}^2 - 3\mathbf{a}^4) f_\Delta}{b} \approx A_c \frac{2\mathbf{a}^2}{b} f_\Delta \quad \text{and} \quad K_3 = A_c \frac{3\mathbf{a}^4}{b^3} f_\Delta^3$$

$$\frac{K_3}{K_1} = \frac{3}{2} \left( \frac{\mathbf{a} f_\Delta}{b} \right)^2 = \frac{2}{3} \mathbf{a}^2 \left( \frac{\mathbf{a} f_\Delta}{b} \right)^2 \ll 1 \quad \text{since } \mathbf{a}^2 \ll 1 \text{ and } b \leq f_\Delta$$

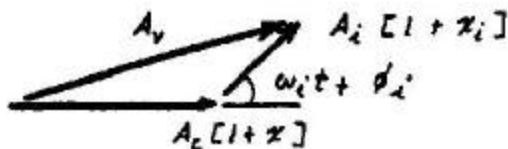
5.4-1

$$v(t) = A_c [1 + x(t)] \cos \mathbf{w}_c t + A_i [1 + x_i(t)] \cos [(\mathbf{w}_c + \mathbf{w}_i)t + \mathbf{f}_i] \quad \text{where } A_i \ll A_c$$

$$A_v(t) \approx A_c [1 + x] + A_i [1 + x_i] \cos(\mathbf{w}_i t + \mathbf{f}_i) = A_c \{1 + x(t) + \mathbf{r} [1 + x_i(t)] \cos(\mathbf{w}_i t + \mathbf{f}_i)\}$$

$$y_D(t) \approx K_D [x(t) + \mathbf{r} \cos(\mathbf{w}_i t + \mathbf{f}_i) + \mathbf{r} x_i(t) \cos(\mathbf{w}_i t + \mathbf{f}_i)]$$

$x_i(t)$  will be unintelligible if  $\mathbf{w}_i \neq 0$

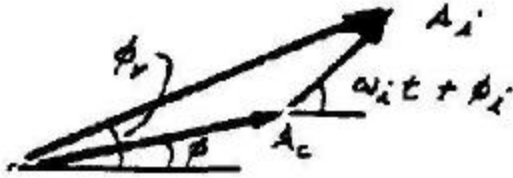


#### 5.4-2

$$v(t) = A_c \cos[\omega_c t + f(t)] + A_i \cos[(\omega_c + \omega_i)t + f_i(t)] \quad \text{where } f(t) = f_\Delta x(t) \ll 1, \quad A_i \ll A_c$$

$$f_v(t) = \arctan \frac{A_c \sin f + A_i \sin(\omega_i t + f_i)}{A_c \cos f + A_i \cos(\omega_i t + f_i)} \approx \arctan \left[ f + \frac{A_i}{A_c} \sin(\omega_i t + f_i) \right]$$

$$\approx f_\Delta x(t) + r \sin[\omega_i t + f_i(t)] \quad f_i(t) \text{ will be unintelligible if } \omega_i \neq 0.$$



#### 5.4-3

$$v(t) = A_c [1 + m x(t)] \cos \omega_c t + a A_c [1 + m x(t - t_d)] \cos(\omega_c t - \omega_c t_d)$$

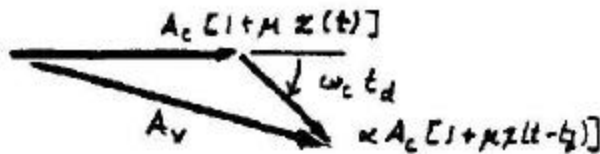
$$\text{Envelope detection: } y_D(t) = K_D [A_v(t) - \langle A_v(t) \rangle]$$

$$\text{where } A_v(t) = A_c \left( \{1 + m x(t) + a [1 + m x(t - t_d)] \cos \omega_c t_d\}^2 + \{a [1 + m x(t - t_c)] \sin \omega_c t_d\}^2 \right)^{1/2}$$

$$\text{Synchronous detection: } y_D(t) = K_D [v_i(t) - \langle v_i(t) \rangle]$$

$$\text{where } v_i(t) = A_c \{1 + m x(t) + a [1 + m x(t - t_d)] \cos \omega_c t_d\}$$

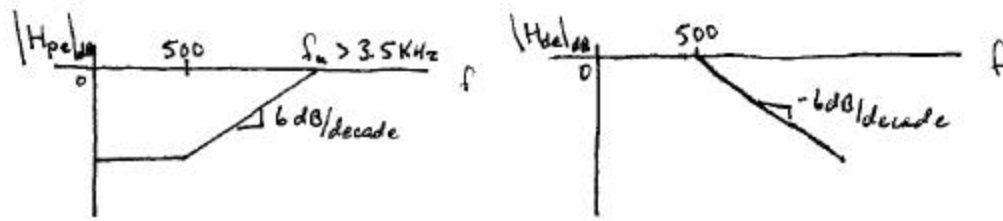
Thus,  $A_v(t)$  always has the same or more distortion than  $v_i(t)$ . If  $\omega_c t_d \approx \pi/2$  then  $\cos \omega_c t_d = 0$  and  $v_i(t)$  is distortionless. If  $\omega_c t_d \approx \pi$  then  $\sin \omega_c t_d = 0$  and  $A_v(t) \approx v_i(t)$ .



#### 5.4-4

Motorized electric appliances generate electromagnetic waves that can interfere with the amplitude of the AM signals. FM signals do not suffer in quality when the amplitude of the transmitted signal is corrupted. In addition, most FM cordless phones are above the frequencies of these interfering signals and other household remote-controlled devices such as garage door openers.

5.4-5



Preemphasis increases the energy above 500 Hz so  $S_x$  will increase.

$$S_T = P_c + 2P_{sb} \quad \text{for AM} \quad S_T = 2P_{sb} \quad \text{for DSB}$$

$$\text{but } \frac{P_{sb}}{A_{\max}^2} = \begin{cases} \frac{S_x}{4} & \text{DSB} \\ \frac{S_x}{16} & \text{AM} \end{cases}$$

Assuming the peak envelope power allowed by the system is the same for both AM and DSB

$$S_T = P_c + 2\frac{S_x}{16}A_{\max}^2 \quad \text{for AM} \quad S_T = 2\frac{S_x}{4}A_{\max}^2 \quad \text{for DSB}$$

Thus, the transmitted power for DSB is increased much more than it is for AM.

5.4-6

Transmitted power is the same in both cases since it depends only on the carrier amplitude.

Transmitted bandwidth is greater if preemphasis is done prior to transmission since the frequency deviation is increased by a factor of  $W/B_{de}$ . However, since speech has very little energy at high frequencies, the bandwidth is driven by the higher amplitude lower frequencies that are not affected by the preemphasis.

Preemphasis after transmission will amplify any noise or interference signals along with the signal of interest. Therefore preemphasis prior to transmission is less susceptible to interference.

Overall, the greater difference is in susceptibility to interference since  $B_T$  is not much larger with preemphasis before transmission. Therefore preemphasis at the microphone end is better than at the receiver end.

5.4-7

$$G_{x_{pe}}(f) = |H_{pe}(f)|^2 G_x(f) \approx \begin{cases} G_x(f) & |f| < B_{de} \\ \left(\frac{f}{B_{de}}\right)^2 G_x(f) & |f| > B_{de} \end{cases}$$

Thus, for  $|f| < B_{de}$ ,  $G_{x_{pe}}(f) \leq G_x(f)|_{\max} = G_{\max}$

while for  $|f| > B_{de}$ ,  $G_{x_{pe}}(f) = \left(\frac{f}{B_{de}}\right)^2 G_x(f) \leq G_{\max}$  if  $G_x(f) \leq \left(\frac{B_{de}}{f}\right)^2 G_{\max}$

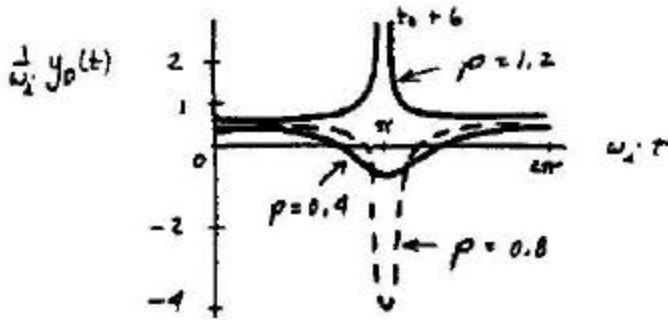
Since  $B_T$  is essentially determined by the combination of maximum amplitude and maximum-frequency sinusoidal components in the modulating signal,  $B_T$  is not increased if  $G_{x_{pe}}(f) \leq G_{\max}$ .

5.4-8

$$y_D(t) = \mathbf{a}(\mathbf{r}, \mathbf{w}_i t) \mathbf{w}_i$$

where

$$\mathbf{a} = \begin{cases} \frac{\mathbf{r}}{1+\mathbf{r}} & \mathbf{w}_i t = 0 \\ \frac{\mathbf{r}^2}{1+\mathbf{r}^2} & \mathbf{w}_i t = \frac{p}{2} \\ \frac{-\mathbf{r}}{1-\mathbf{r}} & \mathbf{w}_i t = p \end{cases}$$



5.4-9

$$\mathbf{a}(1 \pm \mathbf{e}, p) = \frac{(1 \pm \mathbf{e})^2 - (1 \pm \mathbf{e})}{1 + (1 \pm \mathbf{e})^2 - 2(1 \pm \mathbf{e})} = \frac{\mathbf{e}^2 \pm \mathbf{e}}{\mathbf{e}^2} = 1 \pm \frac{1}{\mathbf{e}}$$

Thus,  $\mathbf{a}(1 \pm \mathbf{e}, p) \rightarrow \pm \infty$  as  $\mathbf{e} \rightarrow 0$

5.4-10

$$v(t) = A_c \cos[\mathbf{w}_c t + \mathbf{f}(t)] + \mathbf{r} A_c [\mathbf{w}_c + \mathbf{q}_i(t)]$$

$$\text{so } \mathbf{f}_v(t) = \arctan \frac{\sin \mathbf{f} + \mathbf{r} \sin \mathbf{q}_i}{\cos \mathbf{f} + \mathbf{r} \cos \mathbf{q}_i}$$

$$\begin{aligned} y_D(t) &= \frac{1}{2\mathbf{p}} \dot{\mathbf{f}}_v(t) = \frac{1}{2\mathbf{p}} \left[ 1 + \left( \frac{\sin \mathbf{f} + \mathbf{r} \sin \mathbf{q}_i}{\cos \mathbf{f} + \mathbf{r} \cos \mathbf{q}_i} \right)^2 \right]^{-1} \frac{d}{dt} \left[ \frac{\sin \mathbf{f} + \mathbf{r} \sin \mathbf{q}_i}{\cos \mathbf{f} + \mathbf{r} \cos \mathbf{q}_i} \right] \\ &= \frac{1}{2\mathbf{p}} \frac{(\cos \mathbf{f} + \mathbf{r} \cos \mathbf{q}_i)(\dot{\mathbf{f}} \cos \mathbf{f} + \mathbf{r} \dot{\mathbf{q}}_i \cos \mathbf{q}_i) - (\sin \mathbf{f} + \mathbf{r} \sin \mathbf{q}_i)(-\dot{\mathbf{f}} \sin \mathbf{f} - \mathbf{r} \dot{\mathbf{q}}_i \sin \mathbf{q}_i)}{(\cos \mathbf{f} + \mathbf{r} \cos \mathbf{q}_i)^2 + (\sin \mathbf{f} + \mathbf{r} \sin \mathbf{q}_i)^2} \\ &= \frac{\{1 + \mathbf{r} \cos[\mathbf{f}(t) - \mathbf{q}_i(t)]\} \dot{\mathbf{f}}(t) / 2\mathbf{p} + \{\mathbf{r} + \cos[\mathbf{f}(t) - \mathbf{q}_i(t)]\} \mathbf{r} \dot{\mathbf{q}}_i}{1 + \mathbf{r}^2 + 2\mathbf{r} \cos[\mathbf{f}(t) - \mathbf{q}_i(t)]} \end{aligned}$$

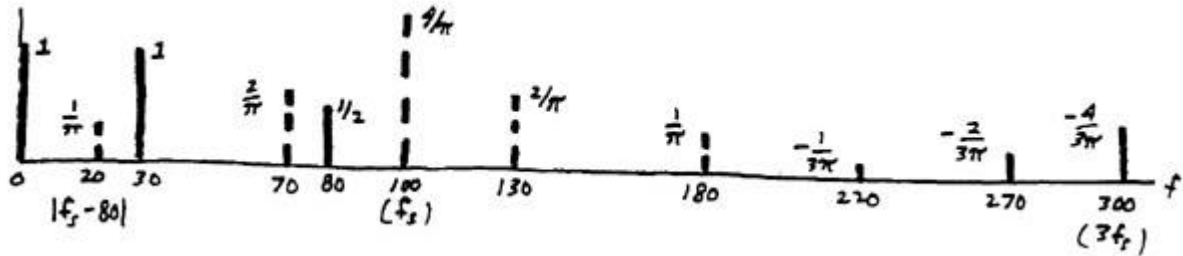


## Chapter 6

6.1-1

$$c_n = \frac{1}{2} \text{sinc} \frac{n}{2} \Rightarrow c_0 = 1/2, \quad 2c_1 = 2/\pi, \quad 2c_2 = 0, \quad 2c_3 = -2/3\pi$$

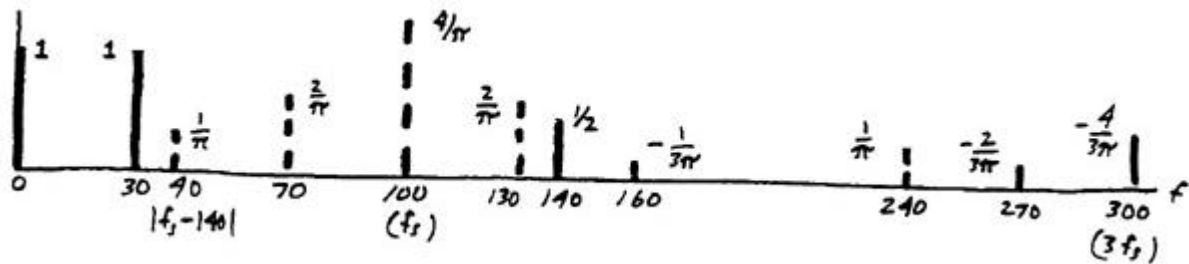
$$y(t) = 1 + \frac{1}{\pi} \cos 2\pi 20t + \cos 2\pi 30t + \frac{2}{\pi} \cos 2\pi 70t$$



6.1-2

$$c_n = \frac{1}{2} \text{sinc} \frac{n}{2} \Rightarrow c_0 = 1/2, \quad 2c_1 = 2/\pi, \quad 2c_2 = 0, \quad 2c_3 = -2/3\pi$$

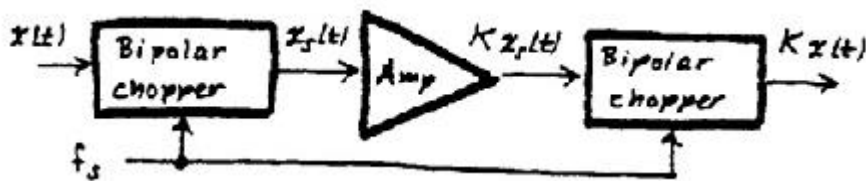
$$y(t) = 1 + \cos 2\pi 30t + \frac{1}{\pi} \cos 2\pi 40t + \frac{2}{\pi} \cos 2\pi 70t$$



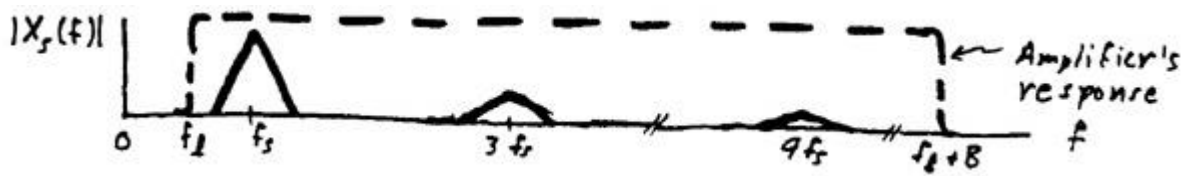
6.1-3

Take  $f_s = f_l + W + \epsilon$ . Amplifier then passes  $x_s(t)$  since  $f_l > f_s - W$  and  $f_l + B \gg f_s$ .

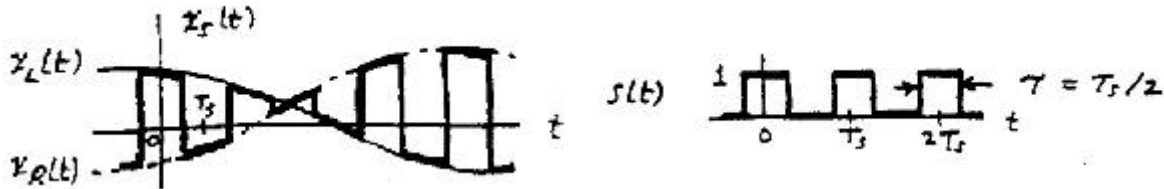
Second chopper with synchronization yields  $Kx_s(t)s(t) = Kx(t)s^2(t) = Kx(t)$  since  $s^2(t) = 1$ .



6.1-3 continued



6.1-4



$$x_s(t) = x_L(t)s(t) + x_R(t)[1 - s(t)]$$

$$c_n = \frac{1}{2} \text{sinc} \frac{n}{2} = \frac{1}{\pi n} \sin \frac{\pi n}{2} \Rightarrow s(t) = \frac{1}{2} + \frac{2}{\pi} \cos \omega_s t - \frac{2}{3\pi} \cos 3\omega_s t + \dots$$

$$x_s(t) = x_L(t) + \left[ \frac{1}{2} + \frac{2}{\pi} \cos \omega_s t - \frac{2}{3\pi} \cos 3\omega_s t + \dots \right] + x_R(t) \left[ \frac{1}{2} - \frac{2}{\pi} \cos \omega_s t + \frac{2}{3\pi} \cos 3\omega_s t + \dots \right]$$

Since LPF rejects  $|f| \geq 99 \text{ kHz}$  and  $3f_s - W = 3 \times 38 - 15 = 99 \text{ kHz}$ ,

$$x_b(t) = \frac{K_1}{2} [x_L(t) + x_R(t)] + \frac{2K_2}{\pi} [x_L(t) - x_R(t)] + A \cos \frac{\omega_s}{2} t$$

so we want  $K_1 = 2$  and  $K_2 = \pi/2$

6.1-5

$$c_n = \frac{1}{2} \text{sinc} \frac{n}{2} = \frac{1}{\pi n} \sin \frac{\pi n}{2} \Rightarrow s(t) = \frac{1}{2} + \frac{2}{\pi} \cos \omega_s t - \frac{2}{3\pi} \cos 3\omega_s t + \dots$$

$$x_1(t) = \frac{1}{2} [x_L(t) + x_R(t)] + \frac{1}{2} \times \frac{2}{\pi} [x_L(t) - x_R(t)] + \text{high-frequency terms}$$

$$x_2(t) = \frac{1}{2} [x_L(t) + x_R(t)] + \frac{1}{2} \times \left( -\frac{2}{\pi} \right) [x_L(t) - x_R(t)] + \text{high-frequency terms}$$

### 6.1-5 continued

$$(a) \quad v_L(t) = \left[ \left( \frac{1}{2} + \frac{1}{\pi} \right) - K \left( \frac{1}{2} - \frac{1}{\pi} \right) \right] x_L(t) + \left[ \left( \frac{1}{2} - \frac{1}{\pi} \right) - K \left( \frac{1}{2} + \frac{1}{\pi} \right) \right] x_R(t) + \dots$$

$$v_R(t) = \left[ \left( \frac{1}{2} - \frac{1}{\pi} \right) - K \left( \frac{1}{2} + \frac{1}{\pi} \right) \right] x_L(t) + \left[ \left( \frac{1}{2} + \frac{1}{\pi} \right) - K \left( \frac{1}{2} - \frac{1}{\pi} \right) \right] x_R(t) + \dots$$

we want  $\left( \frac{1}{2} - \frac{1}{\pi} \right) - K \left( \frac{1}{2} + \frac{1}{\pi} \right) = 0 \Rightarrow K = \frac{\pi - 2}{\pi + 2} = 0.222$

so lowpass filtering yields  $v'_L(t) = 0.778x_L(t)$ ,  $v'_R(t) = 0.778x_R(t)$

(b) If  $K = 0$ , then lowpass filtering yields

$$v'_L(t) = 0.818x_L(t) + 0.182x_R(t), \quad v'_R(t) = 0.182x_L(t) + 0.818x_R(t)$$

So there's incomplete separation of left and right channels at output.

### 6.1-6

Let  $v(t) = s_\delta(t) = \sum_k \delta(t - kT_s)$  with period  $T_s = 1/f_s$  so

$$c(nf_0) = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} s_\delta(t) dt = f_s \int_{-T_s/2}^{T_s/2} \delta(t) dt = f_s$$

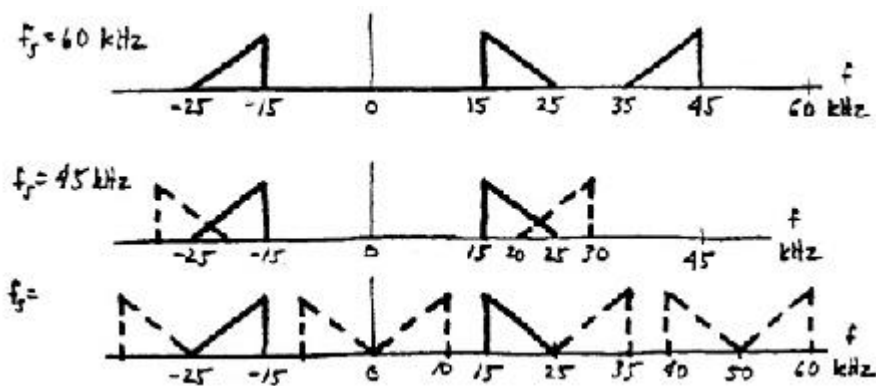
$$\text{Thus } S_\delta(f) = V(f) = \sum_n f_s \delta(f - nf_s) = f_s \sum_n \delta(f - nf_s)$$

### 6.1-7

$f_s = 60$  kHz Recover using LPF  $25 \leq B \leq 35$  kHz

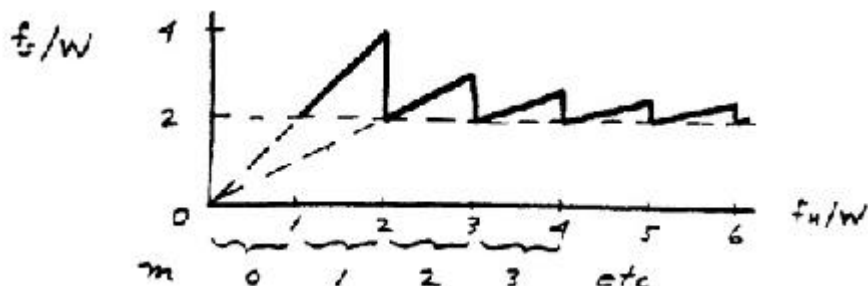
$f_s = 45$  kHz Can't recover by filtering

$f_s = 25$  kHz Recover using BPF over  $f_l \leq |f| \leq 25$  kHz with  $10 < f_l < 15$  kHz

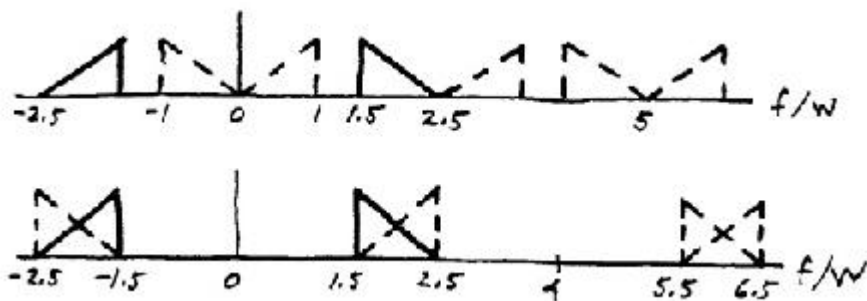


6.1-8

$$(a) \frac{f_s}{W} = \frac{1}{M} 2(f_u / W)$$



$$(b) m = 2, f_s = \frac{1}{2} \times 2 \times 2.5 W = 2.5 W \text{ and } f_s = 4W$$



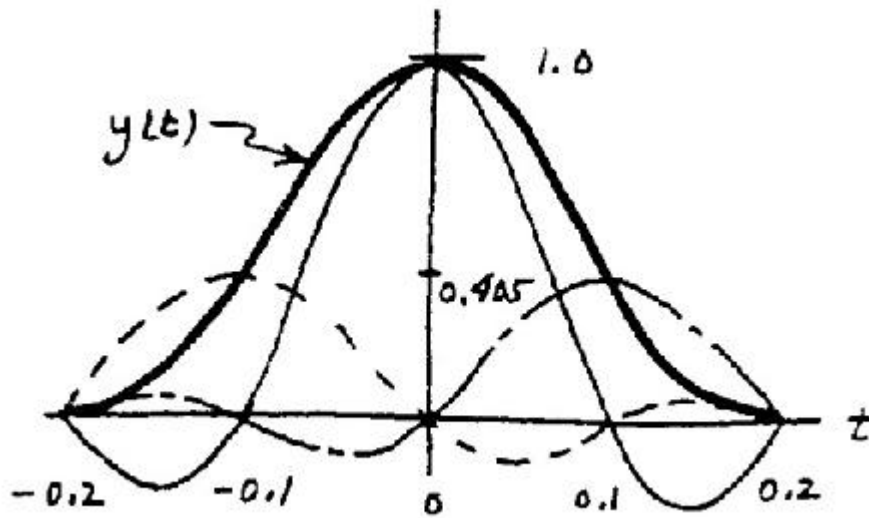
6.1-9

$$x(kT_s) = \text{sinc}^2 5(0.1k) = \text{sinc}^2 0.5k$$

$$\text{since } \text{sinc}^2 0.5k \approx 1 \text{ for } |k| \geq 2,$$

$$y(t) \approx 0.405 \text{sinc} 10(t + 0.1) + \text{sinc} 10t + 0.405 \text{sinc} 10(t - 0.1) \approx \text{sinc}^2 5t$$

6.1-9 continued

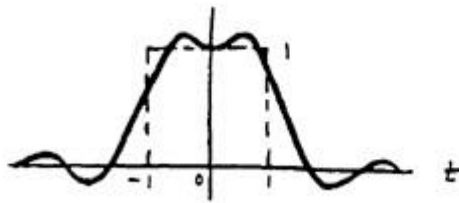


6.1-10

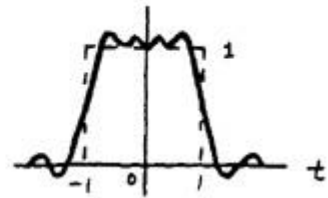
$$x(kT_s) = \Pi\left(\frac{kT_s}{2}\right) = \begin{cases} 1, & |kT_s| \leq 1 \\ 0, & |kT_s| > 1 \end{cases}$$

Take  $K = \frac{1}{f_s}$  and  $t_d = 0$  so  $y(t) = \sum_{k=-M}^M \text{sinc } f_s(t - kT_s)$  where  $\frac{1}{T_s} - 1 < M \leq \frac{1}{T_s}$

$T_s = 0.8$   
 $M = 1$



$T_s = 0.4$   
 $M = 2$

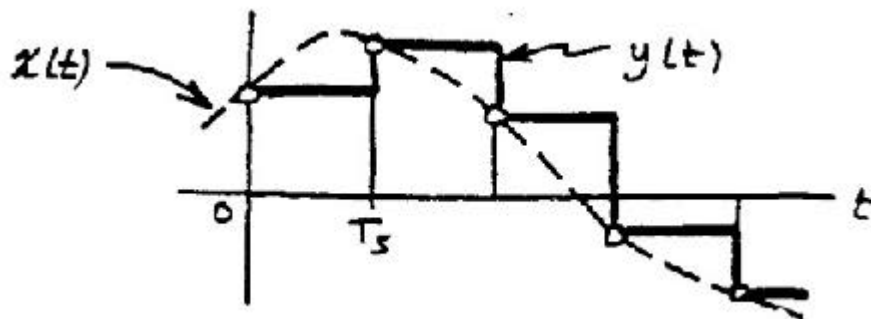


6.1-11

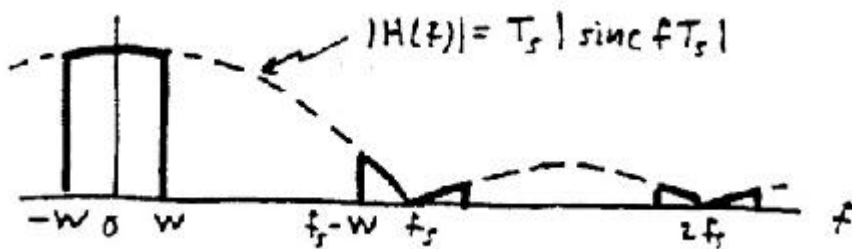
(a)  $h(t) = u(t) - u(t - T_s)$

$$y(t) = h(t) * x_\delta(t) = \sum_k x(kT_s) [u(t - kT_s) - u(t - kT_s - T_s)]$$

6.1-11 continued



(b)



$$|H(f)| = T_s |\text{sinc } fT_s|$$

since  $W \ll f_s$ ,  $|Y(f)| \approx T_s |X(f)|$  for  $|f| \leq W$

so  $x(t)$  can be recovered using a simple LPF to remove  $|f| \geq f_s - W$

6.1-12

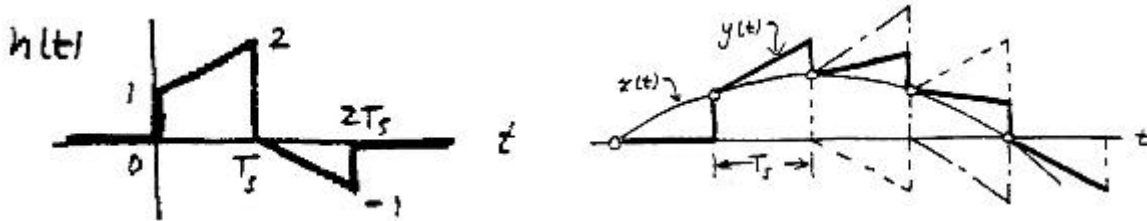
(a) Let  $h_z(t) =$  impulse response of a ZOH  $= u(t) - u(t - T_s)$

$$\text{Then } h(t) = \frac{1}{T_s} h_z(t) * h_z(t) + h_z(t) - h_z(t - T_s)$$

$$\text{where } \frac{1}{T_s} h_z(t) * h_z(t) = \Lambda\left(\frac{t - T_s}{T_s}\right)$$

$y(t)$  is a linear piecewise approximation obtained by extrapolating forward from the two previous values.

6.1-12 continued

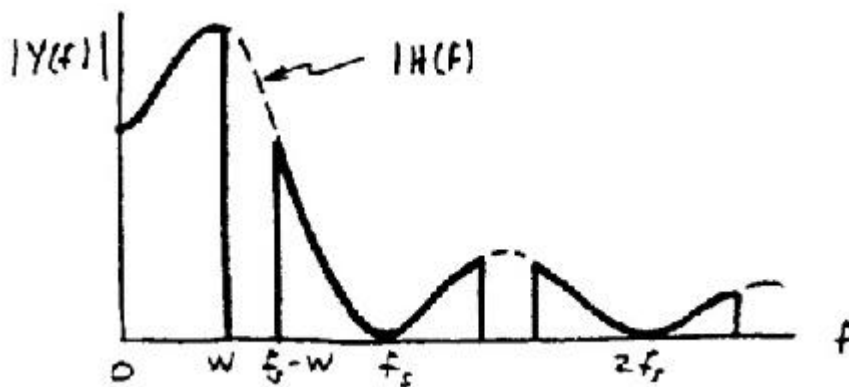


(b) Let  $H_z(f) = \mathfrak{F}[h_z(t)] = T_s \text{sinc } fT_s e^{-j\omega T_s/2}$

$$\begin{aligned}
 H(f) &= \frac{1}{T_s} H_z^2(f) + H_z(f) - H_z(f) e^{-j\omega T_s} \\
 &= T_s \text{sinc}^2 fT_s e^{-j\omega T_s} + T_s \text{sinc } fT_s (e^{j\omega T_s/2} - e^{-j\omega T_s}) e^{-j\omega T_s} \\
 &= T_s \left( 1 + \frac{1}{\text{sinc } fT_s} j2 \sin \pi f T_s \right) \text{sinc}^2 fT_s e^{-j\omega T_s} \\
 &= T_s (1 + j2\pi f T_s) \text{sinc}^2 fT_s e^{-j\omega T_s}
 \end{aligned}$$

$$|H(f)| = T_s \sqrt{(1 + (j2\pi f T_s)^2) \text{sinc}^2 fT_s}$$

Note that high frequency components of  $X(f)$  are accentuated.



6.1-13

$$\begin{aligned}
 X(f) &= \mathfrak{F} \left[ \sum_k x \left( \frac{k}{2W} \right) \text{sinc } 2Wt \left( t - \frac{k}{2W} \right) \right] = \sum_{k=-\infty}^{\infty} x \left( \frac{k}{2W} \right) \frac{1}{2W} \Pi \frac{f}{2W} e^{-j\omega k / 2W} \\
 E &= \int_{-\infty}^{\infty} X(f) X^*(f) df = \int_{-W}^{+W} \left[ \sum_k x \left( \frac{k}{2W} \right) \frac{1}{2W} e^{-j\omega k / 2W} \right] \left[ \sum_m x^* \left( \frac{m}{2W} \right) \frac{1}{2W} e^{+j\omega m / 2W} \right] df \\
 &= \frac{1}{2W} \sum_k \sum_m x \left( \frac{k}{2W} \right) x^* \left( \frac{m}{2W} \right) \frac{1}{2W} \int_{-W}^W e^{+j\pi(m-k)f / W} df \\
 &= \frac{1}{2W} \sum_k \sum_m x \left( \frac{k}{2W} \right) x^* \left( \frac{m}{2W} \right) \text{sinc}(m-k) \\
 &= \frac{1}{2W} \sum_{k=-\infty}^{\infty} \left| x \left( \frac{k}{2W} \right) \right|^2 \quad \text{since } \text{sinc}(m-k) = \begin{cases} 1 & m=k \\ 0 & m \neq k \end{cases}
 \end{aligned}$$

6.1-14

$$v(t) = \sum_{n=-\infty}^{\infty} c_v(nf_0) e^{jn\omega_0 t} \Rightarrow V(f) = \sum_{n=-\infty}^{\infty} c_v(nf_0) \delta(f - nf_0)$$

where

$$c_v(nf_0) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{-T}^T x(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{-\infty}^{\infty} x(t) e^{-jn\omega_0 t} dt = f_0 X(nf_0)$$

But  $x(t) = v(t) \Pi(t/2T) \Rightarrow X(f) = V(f) * (2T \text{ sinc } 2Tf)$  so

$$\begin{aligned}
 X(f) &= \left[ \sum_n f_0 X(nf_0) \delta(f - nf_0) \right] * (2T \text{ sinc } 2Tf) \\
 &= 2Tf_0 \sum_{n=-\infty}^{\infty} X(nf_0) \text{ sinc } 2T(f - nf_0)
 \end{aligned}$$

Hence,  $X(f)$  is completely determined by the sample values of  $X(nf_0)$ .

6.1-15

$$B = f_s / 2 \Rightarrow f_s \leq 2B = 12 \text{ MHz}$$

$$f_x = 1/T_x = 12.5 \text{ MHz}, \quad 1 - \alpha = f_s / f_x = 0.96 \Rightarrow \alpha = 0.04$$

$$2m+1 < 1/\alpha = 25 \Rightarrow m_{\max} = 11$$

$$\text{Presampling bandwidth} \leq 11 \times 12.5 = 137.5 \text{ MHz}$$



6.1-16

$$B = f_s / 2 \Rightarrow f_s \leq 2B$$

$$1 - \alpha = f_s / f_x \leq 2BT_x < 2/3 \Rightarrow \alpha = 1/3$$

$$2m+1 < 1/\alpha < 3 \Rightarrow m_{\max} = 0$$

so only the dc component could be displayed

6.1-17

$$W = 15 \text{ kHz}, f_s = 150 \text{ kHz}$$

with

$$|H_{ZOH}(f)| = |T_s \text{sinc}(fT_s)| \text{ and } |H_{FOH}(f)| = T_s \sqrt{1 + (2\pi fT_s)^2} \text{sinc}^2(fT_s)$$

(a) For a ZOH, the maximum aperture error in the signal passband occurs at  $f = 15 \text{ kHz}$  and thus:

$$|H_{ZOH}(f)|_{f=15 \text{ kHz}} = 0.9836 \text{ and } |H_{ZOH}(f)|_{f=0 \text{ kHz}} = 1$$

$$\Rightarrow \% \text{ aperture error} = \frac{1 - 0.9836}{1} \times 100\% = 1.640\%$$

(b) For a FOH, the maximum aperture error in the signal passband occurs at  $f = 15 \text{ kHz}$  and thus:

$$|H_{FOH}(f)|_{f=15 \text{ kHz}} = 1.1427 \text{ and } |H_{FOH}(f)|_{f=0 \text{ kHz}} = 1$$

$$\Rightarrow \% \text{ aperture error} = \frac{1 - 1.1427}{1} \times 100\% = -14.27\%$$

6.1-18

$$W = 15 \text{ kHz}, f_s = 150 \text{ kHz}, \text{ Error}\% = \frac{1/0.707}{\sqrt{1 + (f_a/B)^2}} \times 100\% \text{ and } B = W$$

$$\Rightarrow f_a = 150 - 15 = 135 \text{ kHz}, \Rightarrow \text{Error}\% = \frac{1/0.707}{\sqrt{1 + (135/15)^2}} \times 100\% = 15.61\%$$

6.1-19

If  $x(t)$  is a sinusoid with period  $2T_0$  with its zero crossings occurring at  $t = T_0$  and the sampling function has period  $T_s = 2T_0$ . It is possible for the sampler to sample  $x(t)$  at  $t = T_0$ . Therefore, the output of the sampler is always = 0.

6.1-20

$$(a) \text{ sinc}(100t) = \text{sinc}(2 \times 50t) \Leftrightarrow \frac{1}{100} \Pi\left(\frac{f}{100}\right) \Rightarrow \text{to sample, } f_s \geq 100$$

$$(b) \text{ sinc}^2(100t) = \text{sinc}^2(2 \times 50t) \Leftrightarrow \frac{1}{100} \Lambda\left(\frac{f}{100}\right) \Rightarrow \text{to sample, } f_s \geq 200$$

$$(c) 10\cos^3 2\pi \times 10^5 t = \frac{10}{4} (3\cos 2\pi \times 10^5 t + \cos 2\pi \times 3 \times 10^5 t)$$

$$\text{Its bandwidth} = (3 - 1) \times 10^5 = 2 \times 10^5 \text{ Hz} \Rightarrow f_s > 4 \times 10^5 \text{ Hz.}$$

6.1-21

At  $f = 159 \text{ kHz}$  the signal level is down -3 dB and we want aliased components down -40 dB.

$\Rightarrow$  at  $f = 159 \text{ kHz}$ , aliased components should be down -43 dB =  $5 \times 10^{-5}$ .

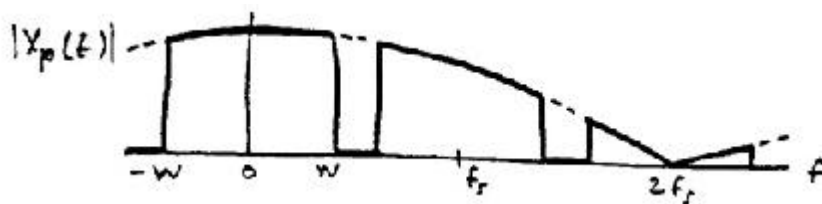
$$|H(f)| = \frac{1}{\sqrt{1 + (f/B)^2}} \Rightarrow 5 \times 10^{-5} = \frac{1}{\sqrt{1 + (f/159)^2}} \Rightarrow f = 3172 \text{ MHz.}$$

6.2-1

$$P(f) = \tau \text{sinc } f \tau = \frac{T_s}{2} \text{sinc } \frac{f}{2f_s}$$

$$H_{eq}(f) = \frac{K}{\text{sinc}(f/2f_s)} = \frac{K}{\text{sinc}(f/5fW)} \text{ for } |f| \leq W$$

$$H_{eq}(0) = K, H_{eq}(W) = 1.07K, \text{ so equalization is not essential}$$



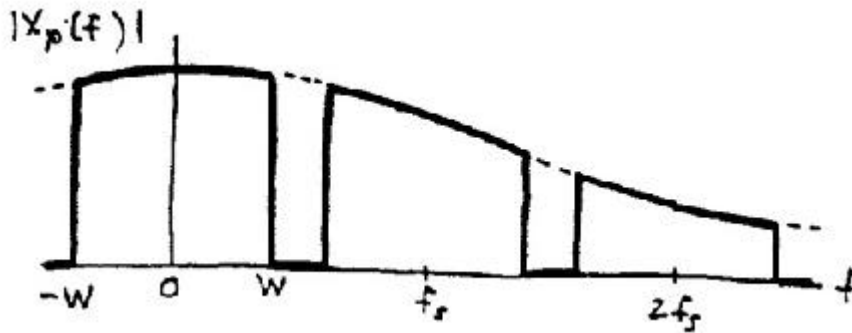
6.2-2

$$P(f) = 2 \int_0^{\tau/2} \cos \frac{\pi t}{\tau} \cos 2\pi f t \, dt$$

$$= \frac{\tau}{2} \left[ \text{sinc} \left( f\tau - \frac{1}{2} \right) + \text{sinc} \left( f\tau + \frac{1}{2} \right) \right] = \frac{T_s}{4} \left[ \text{sinc} \left( \frac{f - f_s}{2f_s} \right) + \text{sinc} \left( \frac{f + f_s}{2f_s} \right) \right]$$

$$H_{eq}(f) = K \left[ \text{sinc} \left( \frac{f - 2.5W}{5W} \right) + \text{sinc} \left( \frac{f + 2.5W}{5W} \right) \right]^{-1} \quad \text{for } |f| \leq W$$

$H_{eq}(0) = 0.785K$ ,  $H_{eq}(W) = 0.816K$  so equalization is not essential.



6.2-3

(a) Let  $\bar{x}(t) = \frac{1}{\tau} \int_{t-\tau}^t x(\lambda) d\lambda = x(t) * h(t)$  where

$$h(t) = \frac{1}{\tau} \int_{t-\tau}^t \delta(\lambda) d\lambda = \frac{1}{\tau} [u(t) - u(t - \tau)] \Rightarrow H(f) = \text{sinc } f\tau e^{-j\omega\tau/2}$$

Averaging filter  $\bar{X}(f)$   $\bar{X}_\delta(f)$   $X_p(f)$

$X(f) \rightarrow H(f) = \text{sinc } f\tau e^{-j\omega\tau/2} \rightarrow \text{Ideal sampler} \rightarrow P(f) \rightarrow$

$X_p(f) = P(f) \bar{X}_\delta(f)$

where  $\bar{X}_\delta(f) = f_s \sum_n \bar{X}(f - nf_s) = f_s \sum_n H(f - nf_s) X(f - nf_s)$

(b)  $X_p(f) = P(f) f_s H(f) X(f)$  for  $|f| \leq W$ , where  $P(f) = \tau \text{sinc } f\tau$

Thus,  $H_{eq}(f) = K e^{-j\omega(t_d - \tau/2)} / \text{sinc}^2 f\tau$ ,  $|f| \leq W$

6.2-4

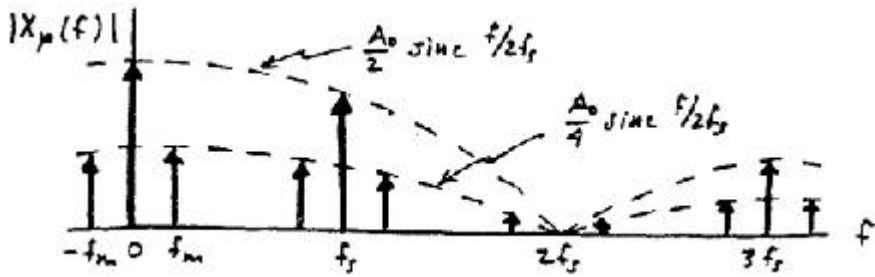
(a) Let  $v(t) = A_0 [1 + \mu x(t)] \leftrightarrow V(f) = A_0 [\delta(f) + \mu X(f)]$

$$x_p(t) = \sum_k v(kT_s) p(t - kT_s) = p(t) * v_\delta(t)$$

$$X_p(f) = P(f) V_\delta(f) = A_0 f_s P(f) \left\{ \sum_n [\delta(f - nf_s) + \mu X(f - nf_s)] \right\}$$

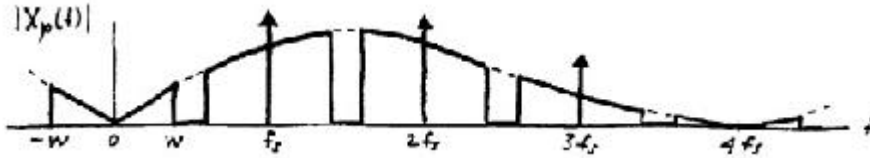
(b)  $P(f) = \tau \text{sinc } f\tau = \frac{1}{2f_s} \text{sinc } \frac{f}{2f_s}$

$$\mu X(f) = \frac{1}{2} [\delta(f - f_m) + \delta(f + f_m)]$$



6.2-5

(a)  $P(f) = \tau \text{sinc } f\tau (e^{-j\omega\tau/2} - e^{j\omega\tau/2}) = \tau \text{sinc } f\tau (-2j \sin \pi f\tau) = -\frac{j\pi\tau}{8f_s^2} \text{sinc}^2 \frac{f}{4f_s}$



(b)  $X_p(f) = -A_0 \frac{j\pi f}{8f_s} \text{sinc}^2 \frac{f}{4f_s} \mu X(f) \text{ for } |f| \leq W$

$$H_{eq}(f) = \frac{K e^{-j\omega\tau_d}}{-j\pi f \text{sinc}^2(f/4f_s)} \quad f_l \leq |f| \leq W$$

If  $f_l \rightarrow 0$  then  $|H_{eq}(0)| \rightarrow \infty$  and equalization is not possible.

6.2-6

The spectrum of a PAM signal is like that of the chopper sampled signal of Fig. 6.1-4 and can be written as  $X_s(f) = c_0 X(f) + c_1 [X(f - f_s) + X(f + f_s)] + \dots$

With product detection  $\Rightarrow x_s(t) \times \cos 2\pi f_s t$  giving a frequency domain expression of

$$\begin{aligned} & \frac{c_0}{2} X(f - f_s) + \frac{c_0}{2} X(f + f_s) \\ & + \frac{c_1}{2} X(f - f_s + f_s) + \frac{c_1}{2} X(f - f_s - f_s) + \frac{c_1}{2} X(f + f_s + f_s) + \frac{c_1}{2} X(f + f_s - f_s) \end{aligned}$$

Combining terms and using a LPF the output spectra from the product detector gives

$$\frac{c_1}{2} X(f) + \frac{c_1}{2} X(f) = c_1 X(f)$$

6.3-1

$$\tau_{\min} = \frac{T_s}{5} (1 - 0.8) = \frac{1}{25f_s} \Rightarrow t_r \leq \frac{1}{100f_s}$$

$$\text{so } B_T \geq 1/t_r \geq 50f_s = 400 \text{ kHz}$$

6.3-2

$$\tau_{\min} = \tau_0 (1 - 0.8) \geq 3t_r \text{ and } t_r \geq 1/2B_T \Rightarrow 0.2\tau_0 \geq 3/2B_T \Rightarrow \tau_0 \geq 15 \mu\text{s}$$

$$\tau_{\max} = \tau_0 (1 + 0.8) \leq T_s/3 \Rightarrow \tau_0 \leq \frac{1}{1.8 \times 3f_s} = 23.1 \mu\text{s}$$

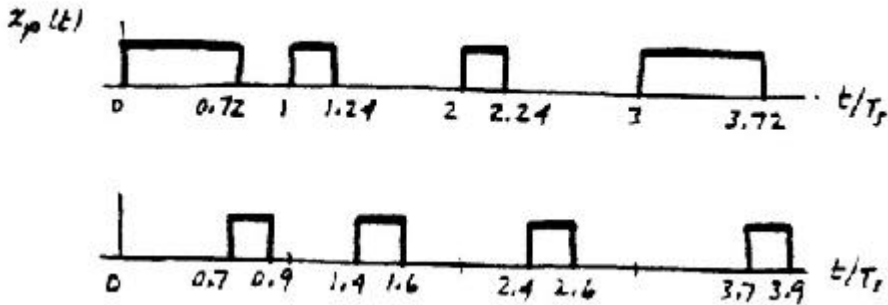
Thus,  $15 \leq \tau_0 \leq 23.1 \mu\text{s}$

6.3-3

$$(a) \tau_k = 0.4T_s \left[ 1 + 0.8 \cos \left( \frac{2\pi k}{3} \right) \right]$$

$$(b) t_k = T_s \left[ k + 0.5 + 0.2 \cos \left( \frac{2\pi k}{3} \right) \right]$$

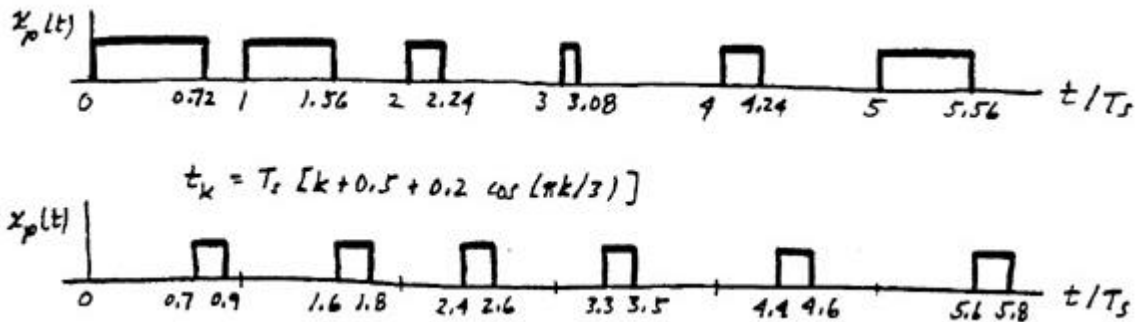
6.3-3 continued



6.3-4

(a)  $\tau_k = 0.4T_s \left[ 1 + 0.8 \cos\left(\frac{\pi k}{3}\right) \right]$

(b)  $t_k = T_s \left[ k + 0.5 + 0.2 \cos\left(\frac{\pi k}{3}\right) \right]$



6.3-5

Take  $t_0 \ll T_s$  so that  $|t_0 \dot{x}(t)| < 1$ . Apply  $-x(t)$  to PPM generator to get

$$x_p(t) = A(t) \left\{ 1 + \sum_n 2 \cos[n\omega_s t + n\omega_s t_0 x(t)] \right\}$$

where  $A(t) = Af_s [1 + t_0 \dot{x}(t)] > 0$

6.3-5 continued

$$\begin{array}{ccccccc}
 -x(t) & & x_p(t) & \text{BPF} & v(t) & & \\
 \rightarrow \text{PPM} & \rightarrow & f_0 = mf_s & & > \text{Lim} & \rightarrow & \text{BPF} \rightarrow x_c(t) \\
 & & & & & & f_0 = mf_s
 \end{array}$$

First BPF yields  $v(t) = 2A(t) \cos[\omega_c t + \phi_\Delta x(t)]$

with  $f_c = mf_s$ ,  $\phi_\Delta = 2\pi mf_s t_0 \square \pi$  if  $m \square \frac{T_s}{2t_0}$

Limiter and second BPF give  $x_c(t) = A_c \cos[\omega_c t + \phi_\Delta x(t)]$

6.3-6

$$(a) \quad s_\delta(t) = \mathfrak{I}^{-1}[S_\delta(f)] = f_s \sum_n e^{-j2\pi n f_s t} \quad \text{and} \quad s_\delta(t) = \sum_k \delta(t - kT_s)$$

Thus,  $\sum_{n=-\infty}^{\infty} e^{\pm j2\pi n t / L} = L \sum_{k=-\infty}^{\infty} \delta(t - kL)$  where  $L = 1/f_s = T_s$

$$(b) \quad S_\delta(f) = \mathfrak{I}[s_\delta(t)] = \sum_k e^{-j2\pi k T_s f} \quad \text{and} \quad S_\delta(f) = f_s \sum_n \delta(f - n f_s)$$

Thus,  $\sum_{k=-\infty}^{\infty} e^{\pm j2\pi k f / L} = L \sum_{n=-\infty}^{\infty} \delta(f - nL)$  where  $L = 1/T_s = f_s$

6.3-7

$$g(t) = v \Rightarrow t = g^{-1}(v) \quad \text{and} \quad \lambda = g^{-1}(0)$$

$$\dot{g}(t) = \frac{dv}{dt} \Rightarrow dt = \frac{1}{\dot{g}(t)} dv = \frac{1}{\dot{g}[g^{-1}(v)]} dv$$

$$\text{Thus, } \int_a^b d[g(t)] dt = \int_{v=g^{-1}(a)}^{v=g^{-1}(b)} \frac{\delta(v)}{\dot{g}[g^{-1}(v)]} dv$$

If  $\dot{g}(\lambda) > 0$ , then  $g(b) > 0 > g(a)$  so

$$\int_{g^{-1}(a)}^{g^{-1}(b)} \frac{\delta(v)}{\dot{g}[g^{-1}(v)]} dv = \frac{1}{\dot{g}[g^{-1}(0)]} = \frac{1}{\dot{g}(\lambda)}$$

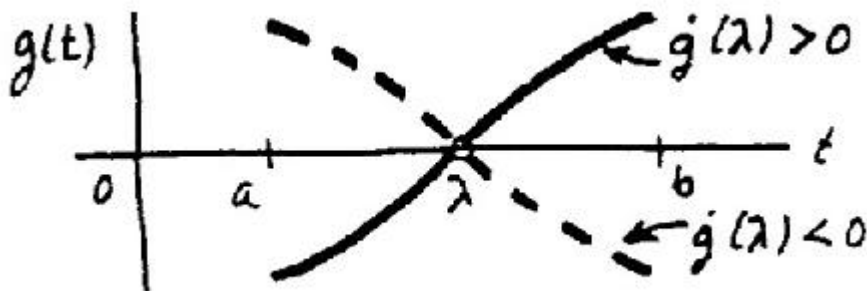
6.3-7 continued

If  $\dot{g}(\lambda) < 0$ , then  $g(b) < 0 < g(a)$  so

$$\int_{g(a)}^{g(b)} \frac{\delta(v)}{\dot{g}[g^{-1}(v)]} dv = - \int_{g(b)}^{g(a)} \frac{\delta(v)}{\dot{g}[g^{-1}(v)]} dv = - \frac{1}{\dot{g}(\lambda)}$$

$$\text{Hence } \int_a^b \delta[g(t)] dt = \frac{1}{|\dot{g}(\lambda)|} = \int_a^b \frac{1}{|\dot{g}(t)|} \delta(t - \lambda) dt$$

$$\text{so } \delta[g(t)] = \delta(t - \lambda) / |\dot{g}(t)|$$





## Chapter 7

7.1-1

$$f_c' = f_c + 2f_{IF} \geq 1600 + \frac{10}{2} \text{ kHz} \Rightarrow f_{IF} \geq (1605 - 540)/2 = 532.5 \text{ kHz}$$

$$f_{LO} = f_c + f_{IF} = 1072.5 \text{ to } 2132.5 \text{ kHz}, B_T = 10 \text{ kHz} < B_{RF} < 2f_{IF} = 1065 \text{ kHz}$$

7.1-2

$$f_c' = f_c - 2f_{IF} \leq 88,100 - \frac{250}{2} \text{ kHz} \Rightarrow f_{IF} \geq (107.9 - 87.975)/2 = 9.9625 \text{ MHz}$$

$$f_{LO} = f_c - f_{IF} = 78.1375 \text{ to } 97.9375 \text{ MHz}, B_T = 250 \text{ kHz} < B_{RF} < 2f_{IF} = 19.925 \text{ kHz}$$

7.1-3

$$C = 1/4\pi^2 L f_{lo}^2 = 2.533 \times 10^4 / f_{lo}^2$$

$$f_{lo} = f_c + f_{IF} = 995 - 2055 \text{ kHz} \Rightarrow C = 6.0 - 25.6 \text{ nF}$$

$$f_{lo} = f_c - f_{IF} = 85 - 1145 \text{ kHz} \Rightarrow C = 19.3 - 3,506 \text{ nF}$$

7.1-4

$$f_c = 1/2\pi\sqrt{LC} \Rightarrow C = 1/4\pi^2 L f_c^2 = 9.9 - 86.9 \text{ nF}$$

$$Q = R\sqrt{\frac{C}{L}} = \frac{f_c}{B_{RF}} = \frac{1}{2\pi\sqrt{LC}B_{RF}} \Rightarrow R = \frac{1}{2\pi B_{RF} C}$$

$$B_{RF} > B_T \Rightarrow R < \frac{1}{2\pi \times 10 \text{ kHz} \times 9.9 \text{ nF}} = 1.6 \text{ k}\Omega,$$

$$B_{RF} > 2f_{IF} \Rightarrow R > \frac{1}{2\pi \times 910 \text{ kHz} \times 86.9 \text{ nF}} = 2.0 \Omega$$

7.1-5

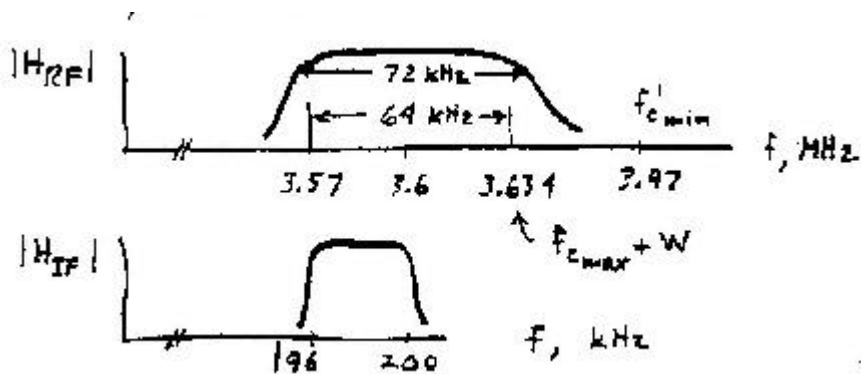
$$f_{IF} \approx B_T / 0.02 = 200 \text{ kHz since } B_T = W$$

$$f_{LO} = f_c + f_{IF} = 3.77 - 3.83 \text{ MHz}, f_c' = f_{LO} + f_{IF} = 3.97 - 4.03 \text{ MHz}$$

$$\text{Take } B_{RF} \approx 0.02 \times 3.6 \text{ MHz} = 72 \text{ kHz centered at } 3.6 \text{ MHz}$$

$$\text{IF must pass } f_{IF} - W \leq f \leq f_{IF}$$

7.1-5 continued



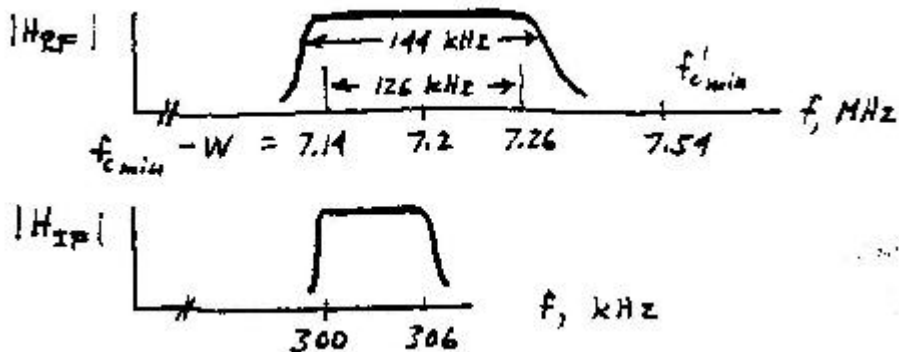
7.1-6

$$f_{IF} \approx B_T / 0.02 = 300 \text{ kHz since } B_T = W$$

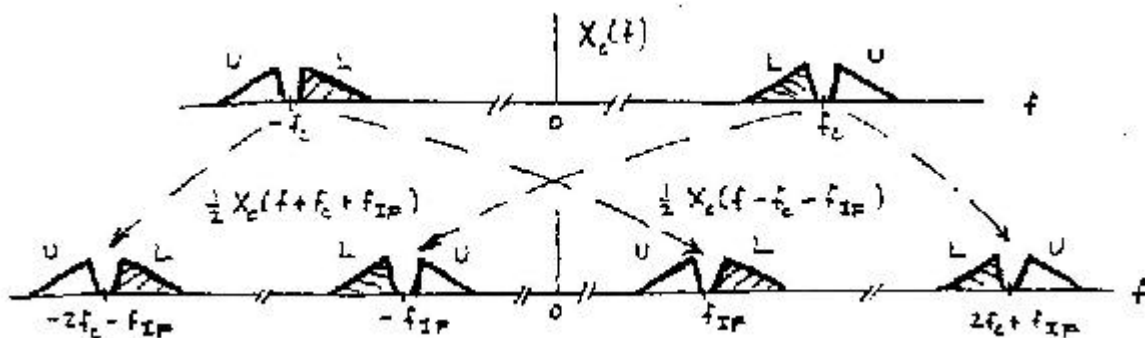
$$f_{LO} = f_c + f_{IF} = 7.34 - 7.46 \text{ MHz, } f'_c = f_{LO} + f_{IF} = 7.54 - 7.66 \text{ MHz}$$

$$\text{Take } B_{RF} \approx 0.02 \times 7.2 \text{ MHz} = 144 \text{ kHz centered at 7.2 MHz}$$

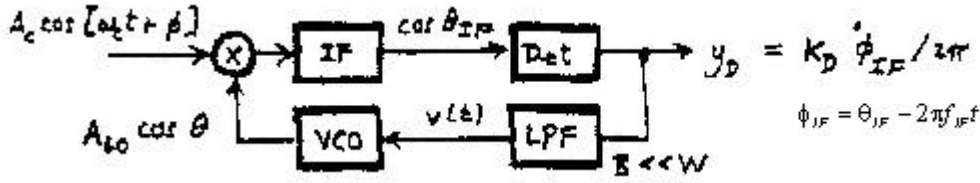
$$\text{IF must pass } f_{IF} \leq f \leq f_{IF} + W$$



7.1-7



7.1-8



$$\theta_{IF} = \omega_c t + \phi - \theta \quad \text{where } \phi = 2\pi f_{\Delta} x(t), \quad \dot{\theta} = 2\pi [f_c - f_{IF} + kv(t) + \varepsilon(t)]$$

$$y_D = \frac{K_D}{2\pi} (\dot{\theta}_{IF} - 2\pi f_{IF}) = K_D [f_c + f_{\Delta} x - f_c + f_{IF} - Kv - \varepsilon - f_{IF}]$$

$$= K_D [f_{\Delta} x(t) - Kv(t) - \varepsilon(t)]$$

$$\text{so } v(t) = K_D [-Kv(t) - \varepsilon(t)]$$

$$\text{Thus, } v(t) = -K_D \varepsilon(t) / (1 + K_D K)$$

$$\text{and } y_D(t) = K_D \left[ f_{\Delta} x(t) - \frac{-K_D K \varepsilon(t)}{1 + K_D K} - \varepsilon(t) \right] = K_D \left[ f_{\Delta} x(t) - \frac{1}{1 + K_D K} \varepsilon(t) \right]$$

$$\approx K_D f_{\Delta} x(t) \quad \text{if } K_D K \gg 1$$

7.1-9

$$\text{(a) With } f_c = 50 \rightarrow 54 \text{ MHz and } f_{IF} = 455 \text{ kHz} \Rightarrow f_{LO} = 50.455 \rightarrow 54.455 \text{ MHz.}$$

$$\Rightarrow f'_c = f_c + 2f_{IF} = 50.910 \rightarrow 54.910 \text{ MHz.}$$

$$\text{(b) With } f_c = 50 \rightarrow 54 \text{ MHz and } f_{IF} = 7 \text{ MHz} \Rightarrow f_{LO} = 57 \rightarrow 61 \text{ MHz.}$$

$$\Rightarrow f'_c = f_c + 2f_{IF} = 64 \rightarrow 68 \text{ MHz.}$$

7.1-10

If  $W$  = signal bandwidth, then the incoming signal is  $50 + W \rightarrow 54 + W$  MHz.

With  $f_{IF} = 100$  MHz, to avoid sideband reversal use  $f_{LO} = 150 \rightarrow 154$  MHz.

At the product detector stage, use an oscillator frequency of 100 MHz.

The image frequency is  $f'_c = f_c + 2f_{IF}$  and its range is thus  $250 \rightarrow 254$  MHz.

7.1-11

Image frequency  $= f_c' = f_c + 2f_{IF} = 2 + 2 \times 455 = 2.91 \text{ kHz}$ .

For a BPF with center frequency of  $f_0 = f_c \Rightarrow |H(f)| = \frac{1}{\sqrt{1+Q^2(\frac{f}{f_0} - \frac{f_0}{f})^2}}$

and  $Q = f_0/B$ . We use the BPF to reject images and thus we have

$$|H(f)|_{f=2.9 \text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{2.91}{2} - \frac{2}{2.91})^2}} = 0.3123 \Rightarrow 20\log(0.3123) = -10 \text{ dB}.$$

Images are rejected by -10 dB.

7.1-12

(a) With  $f_{LO} = 2.455 \text{ MHz}$  and  $f_{IF} = 455 \text{ KHz}$ , then  $f_c = 2 \text{ MHz}$ , and  $f_c' = 2.910 \text{ MHz}$  (image).

With  $f_{LO} = 2.455 \times 2 = 4.910 \text{ MHz} \Rightarrow$  Input frequencies accepted are:

$f_c'' = 4.910 - 0.455 = 4.455 \text{ MHz}$ , and  $f_c''' = 4.455 + 2 \times 0.455 = 5.365 \text{ MHz}$ .

Given the RCL BPF with  $B = 0.5 \text{ MHz} \Rightarrow Q = 2/0.5 = 4$

$$|H(f)|_{f=2.9 \text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{2.91}{2} - \frac{2}{2.91})^2}} = 0.3123 \Rightarrow 20\log(0.3123) = -10 \text{ dB}$$

We repeat the above calculation for the spurious frequencies of 4.455 and 5.360 MHz.

But because the LO oscillator harmonic is 1/2 that of the fundamental we multiply the result by 1/2. Hence,

$$|H(f)|_{f=4.455 \text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{4.455}{2} - \frac{2}{4.455})^2}} = 0.139 \times 1/2 = 0.070 \Rightarrow 20\log(0.070) = -23.1 \text{ dB}$$

and

$$|H(f)|_{f=5.365 \text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{5.365}{2} - \frac{2}{5.365})^2}} = 0.108 \times 1/2 = 0.054 \Rightarrow 20\log(0.054) = -25.4 \text{ dB}$$

7.1-12 continued

(b) To reduce spurious inputs: (1) use a more selective BPF, (2) Use filter to reject the LO second harmonic, (3) use a higher  $f_{IF}$ .

7.1-13

If  $f_{c_1} = 50 \rightarrow 51$  MHz and  $f_{IF_1} = 7 \rightarrow 8$  MHz.

We could choose a fixed frequency output LO with  $f_{LO} = 43$  MHz.

(a) With  $f_{c_1} = 50$  MHz and  $f_{IF_1} = 7$  MHz, and  $f_{LO} = 43$  MHz, the image frequency is

$$f'_{c_1} = f_{c_1} - 2 \times f_{IF_1} = 50 - 2 \times 7 = 36 \text{ MHz}$$

But, the original 7 MHz receiver also suffers from images, so if the incoming signal is supposed to be 7.0 MHz, it could also be  $7 + 2 \times 0.455 = 7.910$  MHz  $\Rightarrow f'_{IF_1} = 7.910$  MHz.

$\Rightarrow f'_{c_1} = 43 + 7.910 = 50.910$  MHz will also be heard.

(b) Use a more selective BPF at the output of the first mixer and/or at the input of the 7 MHz receiver.

7.1-14

With  $f_c = 50 \rightarrow 54$  MHz, let's use  $f_c = f_0 = 52$  MHz. Assume  $f_{LO} = f_c + f_{IF}$

(a) With  $f_{IF} = 20$  MHz  $\Rightarrow f_{LO} = 72$  MHz and  $\Rightarrow f'_c = 52 + 2 \times 20 = 92$  MHz.

$$Q = f_0 / B = 52 / 4 = 13$$

$$|H(f)|_{f=92 \text{ MHz}} = \frac{1}{\sqrt{1+13^2 \left(\frac{92}{52} - \frac{52}{92}\right)^2}} = 0.064 \Rightarrow 20\log(0.064) = -23.9 \text{ dB}$$

(b) With  $f_{IF} = 100$  MHz  $\Rightarrow f_{LO} = 152$  MHz and  $\Rightarrow f'_c = 52 + 2 \times 100 = 252$  MHz.

$$|H(f)|_{f=152 \text{ MHz}} = \frac{1}{\sqrt{1+13^2 \left(\frac{252}{52} - \frac{52}{252}\right)^2}} = 0.017 \Rightarrow 20\log(0.017) = -35.6 \text{ dB}$$

7.1-15

Given  $f_{c_1} = 850$  MHz and  $f_{c_2} = 1950$  MHz, let's pick a common 500 MHz IF  $\Rightarrow f_{IF} = 500$  MHz.

For  $f_{c_1} = 850$  MHz, select  $f_{LO_1} = f_{c_1} + f_{IF} \Rightarrow f_{LO} = 1350$  MHz

and

for  $f_{c_2} = 1950$  MHz, select  $f_{LO_2} = f_{c_2} - f_{IF} \Rightarrow f_{LO} = 1450$  MHz.

$\Rightarrow f_{LO} = 1350 \rightarrow 1450$  MHz.

Image frequencies:

$f_c = 850$  MHz  $\Rightarrow f'_c = 850 + 2 \times 500 = 1850$  MHz

and

$f_c = 1950$  MHz  $\Rightarrow f'_c = 1950 - 2 \times 500 = 950$  MHz.

7.1-16

$B_{IF_2} = 2W$ ,  $f_{IF_2} \approx 2W / 0.02 = 1$  MHz

From Exercise 7.1-2,  $f_{IF_1} \approx 9.5 f_c = 38$  MHz so  $B_{IF_1} \approx 0.02 \times 38 = 760$  kHz

$f_{LO_1} = f_c + f_{IF_1} = 42$  MHz,  $f_{LO_2} = f_{IF_1} \pm f_{IF_2} = 37$  or 39 MHz

7.1-17

$f_{LO_1} = f_c + f_{IF_1} = 330$  MHz  $\Rightarrow f'_c = 330 + 30 = 360$  MHz

$f_{LO_2} = f_{IF_1} + f_{IF_2} = 33$  MHz, so image frequency at input of 2nd mixer is

$f_{LO_2} + f_{IF_2} = 36$  MHz produced by

$|f'_c - f_{LO_1}| = 36$  MHz  $\Rightarrow f'_c = 294$  and 366 MHz

7.1-18

$f_{LO_1} = f_c - f_{IF_1} = 270$  MHz  $\Rightarrow f'_c = 270 - 30 = 240$  MHz

$f_{LO_2} = f_{IF_1} - f_{IF_2} = 27$  MHz, so image frequency at input of 2nd mixer is

$f_{LO_2} - f_{IF_2} = 24$  MHz produced by

$|f'_c - f_{LO_1}| = 24$  MHz  $\Rightarrow f'_c = 246$  and 294 MHz

7.1-19

$1/T_0 = 20 \text{ Hz}$ , so take  $B < 20 \text{ Hz}$  to resolve lines

$$f_1 = 0, f_2 = 10/T_0 = 200 \text{ Hz}, T \geq \frac{f_2 - f_1}{B^2} > \frac{200 \text{ Hz}}{(20 \text{ Hz})^2} = 0.5 \text{ sec}$$

7.1-20

Take  $B < f_m = 1 \text{ kHz}$  to resolve lines,  $\beta = 5 \Rightarrow 8$  pairs of sideband lines.

$$f_1 = f_c - 8f_m = 92 \text{ kHz}, f_2 = f_c + 8f_m = 108 \text{ kHz}$$

$$T \geq \frac{f_2 - f_1}{B^2} > \frac{16 \text{ kHz}}{(1 \text{ kHz})^2} = 16 \text{ ms}$$

7.1-21

$$h_{bp}(t) = \cos \alpha t^2 \cos \omega_c t - \sin \alpha t^2 \sin \omega_c t \text{ so}$$

$$h_{lp}(t) = \frac{1}{2} (\cos \alpha t^2 + j \sin \alpha t^2) = \frac{1}{2} e^{j\alpha t^2}$$

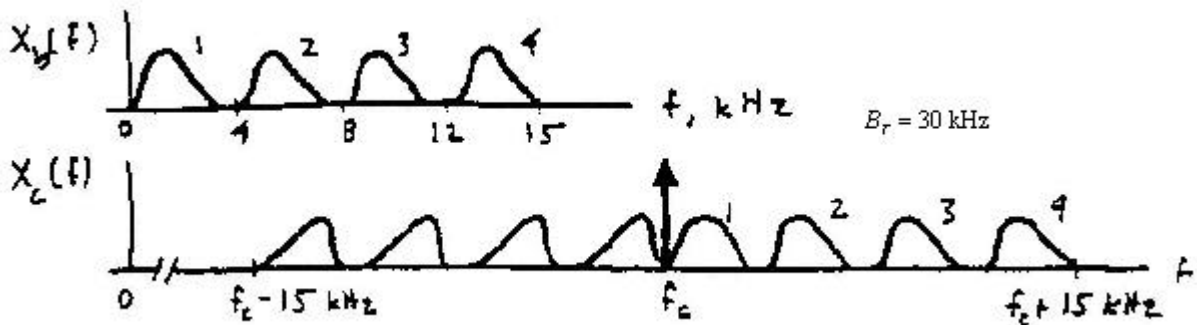
$$x_{bp}(t) = v(t) \cos \alpha t^2 \cos \omega_c t - [-v(t) \sin \alpha t^2] \sin \omega_c t \text{ so}$$

$$x_{lp}(t) = \frac{1}{2} [v(t) \cos \alpha t^2 - j v(t) \sin \alpha t^2] = \frac{1}{2} v(t) e^{-j\alpha t^2}$$

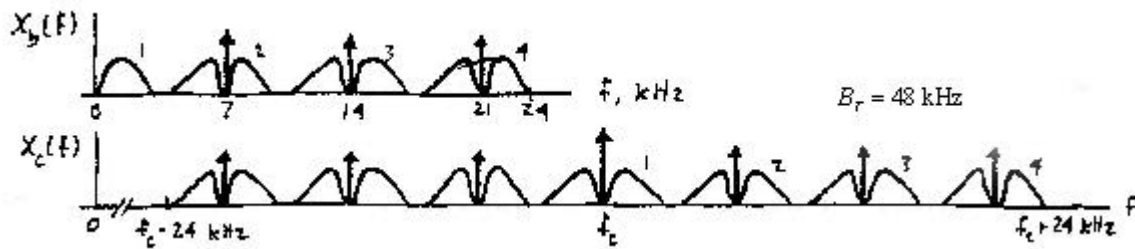
$$y_{lp}(t) = x_{lp} * h_{lp} = \frac{1}{4} \int_{-\infty}^{\infty} v(\lambda) e^{-j\alpha \lambda^2} e^{j\alpha(t-\lambda)^2} d\lambda = \frac{1}{4} e^{j\alpha t^2} \int_{-\infty}^{\infty} v(\lambda) e^{-j2\alpha t \lambda} d\lambda$$

$$A_y(t) = |y_{lp}(t)| = \left| \frac{1}{4} \int_{-\infty}^{\infty} v(\lambda) e^{-j2\pi(\alpha t/\pi)\lambda} d\lambda \right| = \frac{1}{4} |V(f)|_{f=\alpha t/\pi}$$

7.2-1

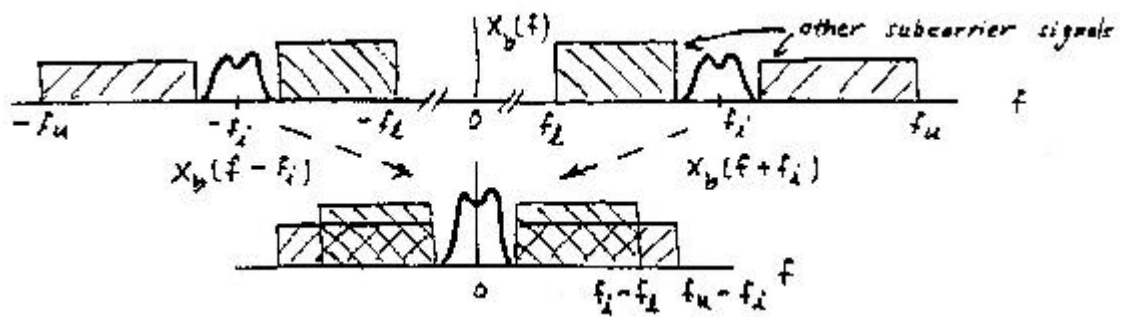


7.2-2

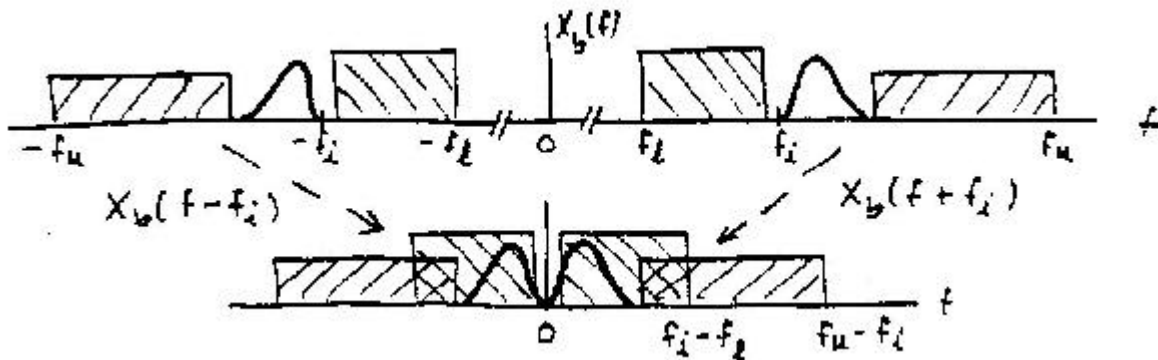


7.2-3

DSB



SSB



7.2-4

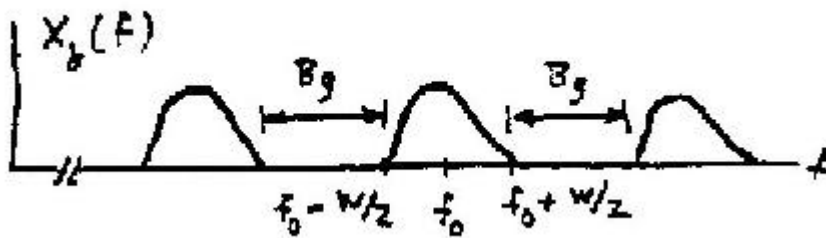
We want  $|H(f)| \leq 0.1$  for  $|f - f_0| \geq \frac{W}{2} + B_g$

so  $\frac{W/2 + B_g}{W} \geq \frac{1}{1.2} \sqrt{\ln(1/0.1)} \approx 1.26 \Rightarrow B_g \geq 0.76W$

Thus,  $B_T = 10W + 9B_g \geq 17W$

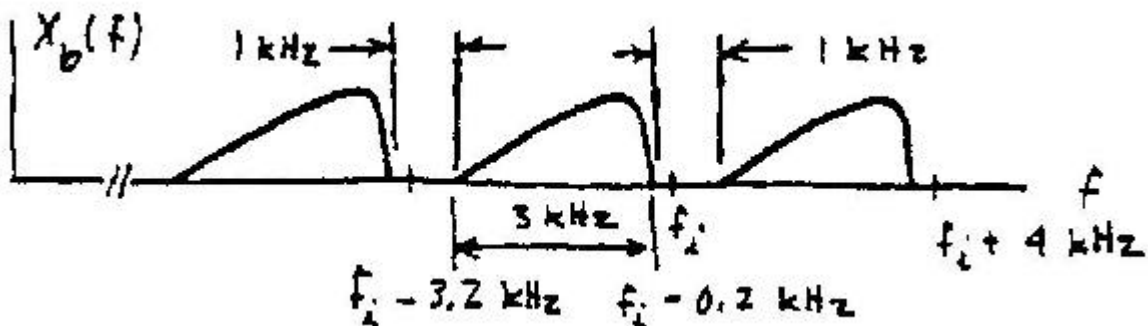


7.2-4 continued



7.2-5

Let  $f_i = i^{th}$  subcarrier, take  $B = 3$  kHz

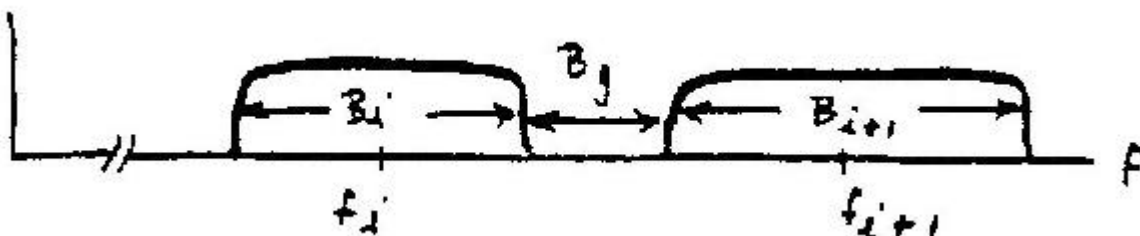


$$f_0 = f_i - 0.2 \text{ kHz} - \frac{B}{2} = f_i - 1.7 \text{ kHz}$$

We want  $|H(f)|^2 \leq 0.01$  for  $|f - f_0| \geq B/2 + 1 \text{ kHz} = 2.5 \text{ kHz}$

$$\text{Thus, } 1 + \left( \frac{2 \times 2.5}{3} \right)^{2n} \geq 100 \Rightarrow n \geq \frac{1}{2} \frac{\ln 99}{\ln(5/3)} \approx 4.5 \Rightarrow n = 5$$

7.2-6



$$(a) B_i = 2M(D)W_i = 2\alpha M(D)f_i. f_i + B_i/2 + B_g = f_{i+1} - B_{i+1}/2$$

$$\text{Thus } f_{i+1} = \frac{[1 + \alpha M(D)]f_i + B_g}{1 - \alpha M(D)}$$

7.2-6 continued

$$(b) \alpha M(D) = \frac{1}{2} B_1 / f_1 = 0.2 \Rightarrow f_{i+1} = (1.2 f_i + 400) / 0.8$$

so  $f_2 = 3.5$  kHz,  $f_3 = 5.75$  kHz,  $f_4 = 9.125$  kHz

7.2-7

$$x_c(t) = x_1(t) \cos \omega_c t + x_2(t) \cos(\omega_c t \pm 90^\circ) \text{ taking } A_c = 1$$

$$2x_c(t) \cos(\omega_c t + \phi) = x_1(t) [\cos \phi + \cos(2\omega_c t + \phi)] \\ + x_2(t) [\cos(\phi \pm 90^\circ) + \cos(2\omega_c t + \phi \pm 90^\circ)]$$

$$2x_c(t) \cos(\omega_c t + \phi \pm 90^\circ) = x_1(t) [\cos(\phi \pm 90^\circ) + \cos(2\omega_c t + \phi \pm 180^\circ)] \\ + x_2(t) [\cos \phi + \cos(2\omega_c t + \phi \pm 180^\circ)]$$

Thus, LPF outputs are

$$y_1(t) = K [x_1(t) \cos \phi + x_2(t) \sin \phi]$$

$$y_2(t) = K [\mp x_1(t) \sin \phi + x_2(t) \cos \phi]$$

7.2-8

$$\text{We want } \left. \begin{aligned} x_0 + x_1 &= 2(L_F + L_R) \\ x_0 - x_1 &= 2(R_F + R_R) \end{aligned} \right\} \Rightarrow x_1(t) = L_F + L_R - (R_F + R_R)$$

Take  $x_2(t) = L_F - L_R - R_F + R_R$  so that

$$x_0 + x_1 + x_3 = 3L_F + L_R + R_F - R_R$$

$$x_0 + x_1 + x_2 + x_3 = 4L_F$$

$$x_0 + x_1 - x_3 = L_F + 3L_R - R_F + R_R$$

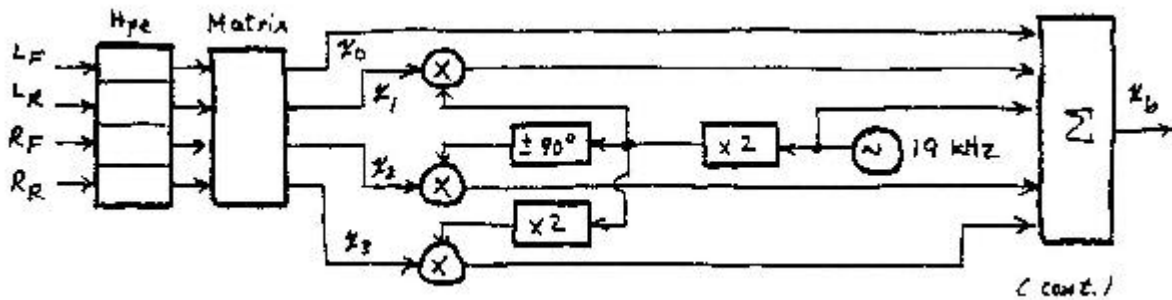
$$x_0 + x_1 - x_2 - x_3 = 4L_R$$

$$x_0 - x_1 + x_3 = L_F - L_R + 3R_F + R_R$$

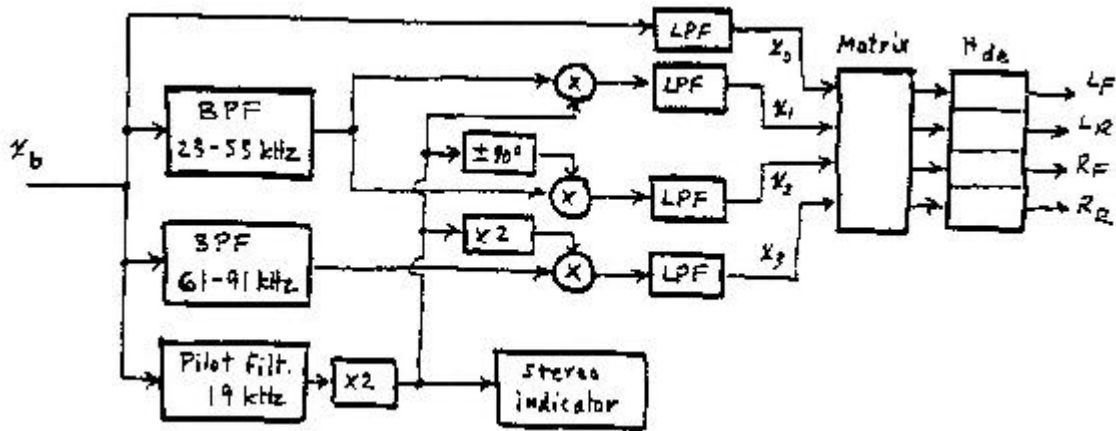
$$x_0 - x_1 - x_2 + x_3 = 4R_F$$

$$x_0 - x_1 - x_3 = -L_F + L_R + R_F + 3R_R$$

$$x_0 - x_1 + x_2 - x_3 = 4R_R$$



7.2-8 continued



7.2-9

$$\Im[x_2(t) \sin \omega_c t] = -\frac{j}{2} [X_2(f - f_c) - X_2(f + f_c)] \text{ so}$$

$$X_c(f) = \frac{A_c}{2} [X_1(f - f_c) + X_1(f + f_c) \mp jX_2(f - f_c) \pm jX_2(f + f_c)]$$

$$Y_c(f) = H_c(f) X_c(f)$$

$$\begin{aligned} \Im[y_c(t) \cos \omega_c t] &= \frac{1}{2} [H_c(f - f_c) X_c(f - f_c) + H_c(f + f_c) X_c(f + f_c)] \\ &= \frac{A_c}{4} \{ H_c(f - f_c) [X_1(f - 2f_c) + X_1(f) \mp jX_2(f - 2f_c) \pm jX_2(f)] \\ &\quad + H_c(f + f_c) [X_1(f) + X_1(f + 2f_c) \mp jX_2(f) \pm jX_2(f + 2f_c)] \} \end{aligned}$$

The output of the lower LPF is

$$Y_1(f) = \frac{A_c}{4} \{ [H_c(f - f_c) + H_c(f + f_c)] X_1(f) \pm j[H_c(f - f_c) - H_c(f + f_c)] X_2(f) \}$$

To remove cross talk from  $X_2(f)$ , we must have  $H_c(f - f_c) - H_c(f + f_c) = 0$  for  $|f| < W$

$$\text{Then, } Y_1(f) = \frac{A_c}{4} [H_c(f - f_c)] X_1(f) \text{ so } Y_1(f) H_{eq}(f) = K X_1(f) e^{-j\omega t_d}$$

$$\text{where the equalizer has } H_{eq}(f) = \frac{4K / A_c}{H_c(f - f_c)} e^{-j\omega t_d}$$

7.2-10

$$r = (24 + 1) \times 8 \text{ kHz} = 200 \text{ kHz}, \tau = 0.5 \frac{T_s}{24 + 1} = 2.5 \text{ } \mu\text{s}, B_r \geq \frac{1}{2\tau} = 200 \text{ kHz}$$

7.2-11

$$r = (24 + 1) \times 6 \text{ kHz} = 150 \text{ kHz}, \quad \tau = 0.3 \frac{T_s}{24 + 1} = 2 \text{ } \mu\text{s}, \quad B_T \geq \frac{1}{2\tau} = 250 \text{ kHz}$$

7.2-12

$$f_s = 2W + B_g = 10 \text{ kHz}$$

$$(a) \quad \tau = 0.25 \frac{T_s}{20} = 1.25 \text{ } \mu\text{s} \Rightarrow B_T \geq 1/\tau = 800 \text{ kHz}$$

$$(b) \quad B_T = B_b = \frac{1}{2} \times 20 f_s = 100 \text{ kHz}$$

7.2-13

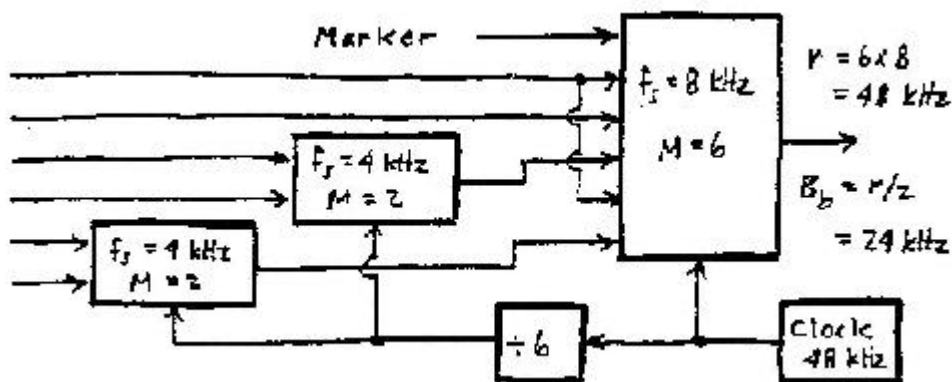
$$f_s = 2W + B_g = 5 \text{ kHz}$$

$$(a) \quad \tau = 0.2 \frac{T_s}{10} = 4 \text{ } \mu\text{s} \Rightarrow B_T \geq 1/\tau = 250 \text{ kHz}$$

$$(b) \quad B_b = \frac{1}{2} \times 10 f_s = 25 \text{ kHz}, \quad D = f_\Delta / B_b = 3, \quad B_T \approx 2(3 + 2)B_b = 250 \text{ kHz}$$

7.2-14

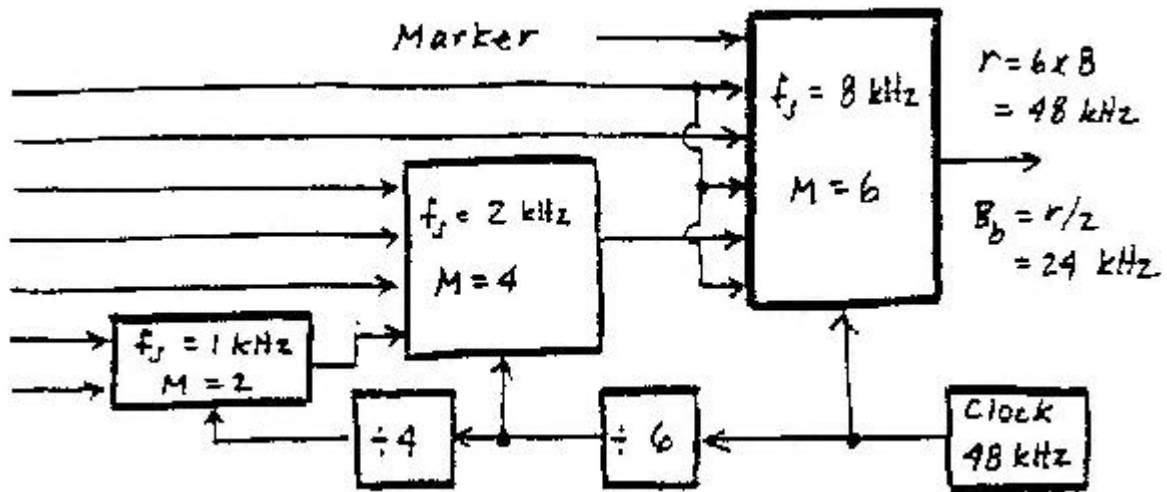
Sampling rate (kHz)	Minimum	Actual
16		2 x 8
7		8
4		4
3.6		4
3		4
2.4		4



$$\text{FDM - SSB: } B_T \geq \sum_i W_i = 18 \text{ kHz}$$

7.2-15

Sampling rate (kHz)	<u>Minimum</u>	<u>Actual</u>
24		3 x 8
8		8
2		2
1.8		2
1.6		2
1.0		1
0.6		1



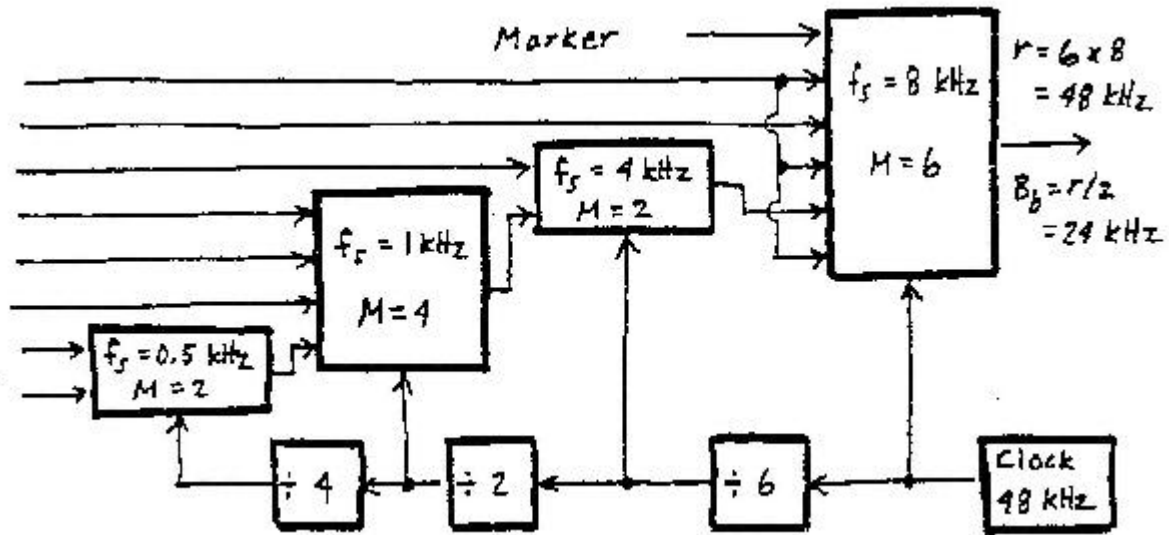
FDM - SSB:  $B_T \geq \sum_i W_i = 19.5 \text{ kHz}$

7.2-16

Sampling rate (kHz)

Sampling rate (kHz)	<u>Minimum</u>	<u>Actual</u>
24		3 x 8
7		8
4		4
1		1
0.8		1
0.6		1
0.4		0.5
0.2		0.5

7.2-16 continued



FDM - SSB:  $B_T \geq \sum_i W_i = 19 \text{ kHz}$

7.2-17

$$-54.5BT_g \leq -40 \Rightarrow T_g \geq 0.734/B$$

$$\frac{T_s}{M} = T_g + 2t_0 + \tau \geq \frac{0.734}{B} + 3t_0, \quad t_0 = \tau = \frac{0.2}{25 \times 8 \text{ kHz}} = 1 \mu\text{s}$$

$$\frac{0.734}{B} \leq \frac{1}{Mf_s} - 3t_0 = 2 \mu\text{s} \Rightarrow B \geq 367 \text{ kHz}$$

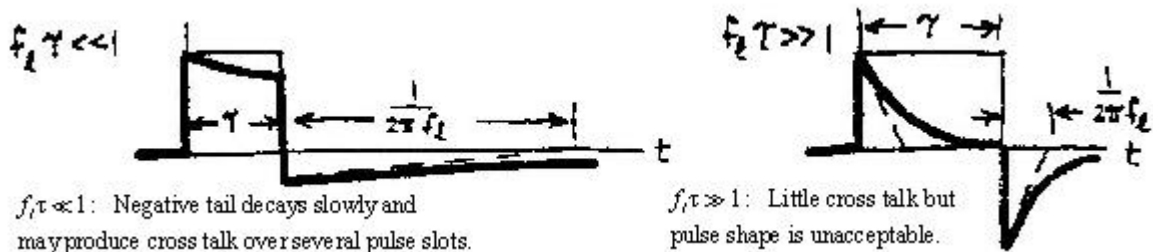
7.2-18

$$-54.5BT_g \leq -30 \Rightarrow T_g \geq 0.55/B$$

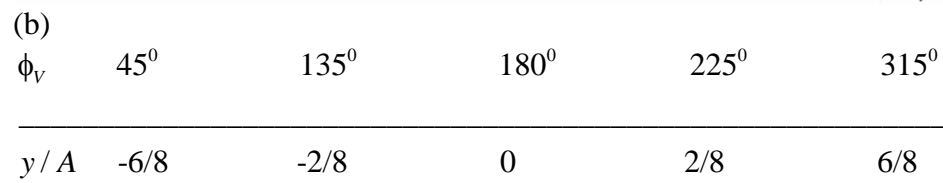
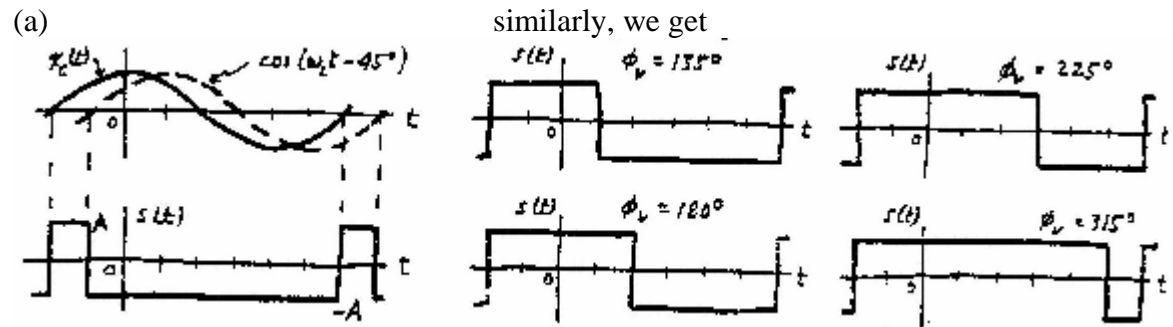
$$\frac{T_s}{M} = T_g + 2t_0 + \tau = T_g + 3 \times \frac{0.25T_s}{M} > \frac{0.55}{B} + \frac{3}{4} \frac{T_s}{M}$$

$$\text{Thus, } \frac{1}{4} \frac{T_s}{M} > \frac{0.55}{B} \Rightarrow M < \frac{1}{4 \times 0.55} \frac{B}{f_s} = 28.4 \Rightarrow M = 28$$

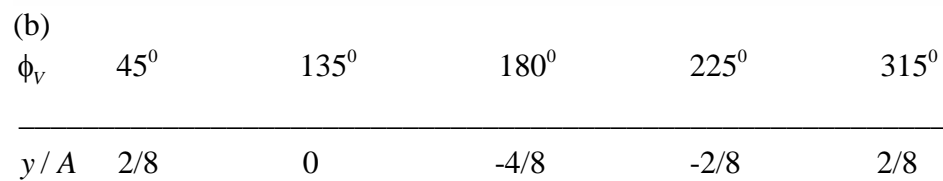
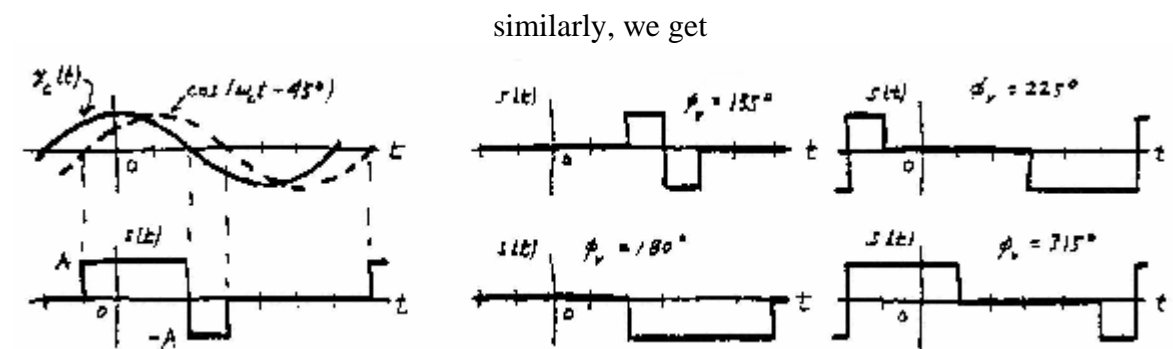
7.2-19



7.3-1



7.3-2



7.3-3

$$t < 0, \quad \varepsilon_{ss} = \Delta f / K$$

$$t > 0, \quad \dot{\phi} = 2\pi f_1 \text{ and } \frac{\Delta f + f_1}{K} \ll 1 \text{ so assume } |\varepsilon| \ll 1 \text{ and } \sin \varepsilon \approx \varepsilon$$

$$\text{Thus, } \dot{\varepsilon} + 2\pi K \varepsilon = 2\pi(\Delta f + f_1) \Rightarrow \text{trial solution } \varepsilon = A + B e^{st}$$

$$\text{Then } B s e^{st} + 2\pi K A + 2\pi K B e^{st} = 2\pi(\Delta f + f_1)$$

$$\text{so } \left. \begin{array}{l} 2\pi K A = 2\pi(\Delta f + f_1) \\ (s + 2\pi K) B e^{st} = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} A = \frac{\Delta f + f_1}{K} \\ s = -2\pi K \end{array} \right.$$

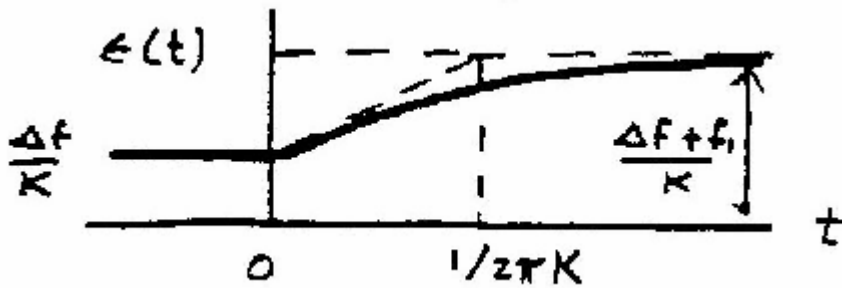
$$\text{and } \varepsilon(t) = \frac{\Delta f + f_1}{K} + B e^{-2\pi K t}, \quad t > 0,$$

Since  $\varepsilon(t)$  can make a step change at  $t = 0$ ,

$$\varepsilon(0^+) = \frac{\Delta f + f_1}{K} + B = \varepsilon(0^-) = \frac{\Delta f}{K} \Rightarrow B = -\frac{f_1}{K}$$

Hence,

$$\varepsilon(t) = \begin{cases} \frac{\Delta f}{K} & t < 0 \\ \frac{\Delta f}{K} + \frac{f_1}{K} (1 - e^{-2\pi K t}) & t > 0 \end{cases}$$



7.3-4

$$x_c(t) = \frac{1}{2} A_c \left[ x(t) \cos \omega_c t - x_q(t) \sin \omega_c t \right] \quad \text{where } x_q(t) = \pm \tilde{x}(t) \text{ for SSB}$$

$$= A(t) \cos[\omega_c t + \phi(t)]$$

$$\text{with } A(t) = \frac{1}{2} A_c \sqrt{x^2(t) + x_q^2(t)}, \quad \phi(t) = \arctan \frac{x_q(t)}{x(t)}$$

If loop locks to  $\phi(t)$  and  $\varepsilon_{ss} \approx 0$ , then the output is proportional to  $A(t)$ .

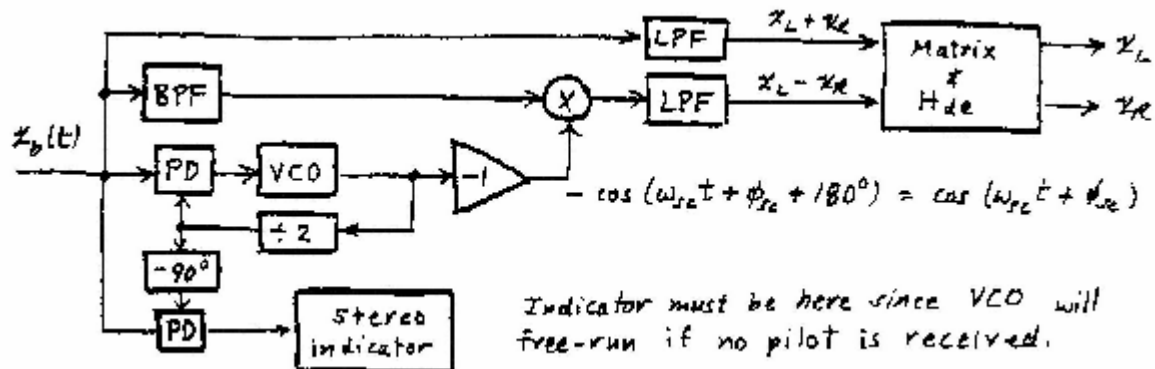
Otherwise,  $\phi(t)$ , may be too rapid for loop to lock.



$$\cos \theta_v(t) \times \cos(\omega_1 t + \phi_1) = \frac{1}{2} \cos[\theta_v(t) - (\omega_1 t + \phi_1)] + \text{high frequency term}$$
$$\text{so } \cos \theta_v(t) = \cos \left[ (\omega_c + \omega_1)t + \phi_0 + \phi_1 + 90^\circ - \varepsilon_{ss} \right]$$
$$\cos[\theta_v(t)/n] = \cos(\omega_c t + \phi_0 + 90^\circ - \varepsilon_{ss})$$
$$\text{so } \cos \theta_v(t) = \cos(n\omega_c t + n\phi_0 + n90^\circ - n\varepsilon_{ss})$$

Let subcarrier be  $\cos(\omega_{sc}t + \phi_{sc})$  so pilot signal is  $\cos[(\omega_{sc}t + \phi_{sc})/2]$

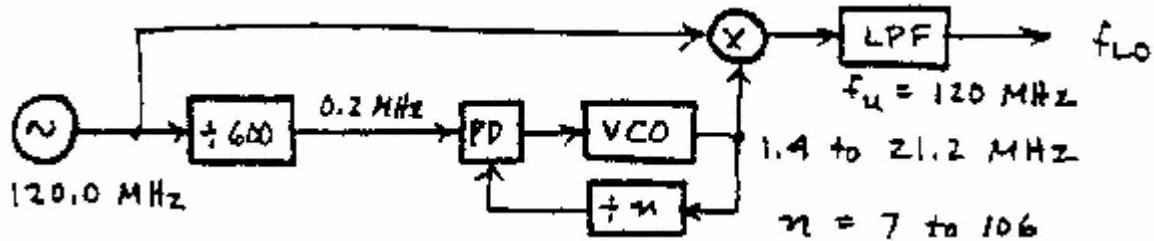
and output of PLL doubler will be  $\cos \phi_v(t) = \cos \left[ 2(\omega_{sc} t + \phi_{sc}) / 2 + 2 \times 90^\circ \right]$



7.3-9

$$f_{LO} = f_c + f_{IF} = 98.8 \text{ to } 118.6 \text{ MHz in steps of } 0.2 \text{ MHz} = 120.0 \text{ MHz} \div 600$$

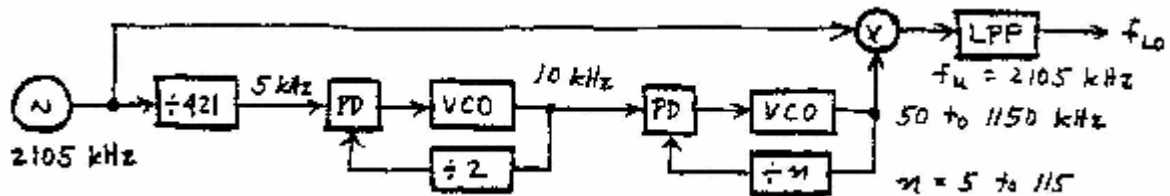
$$120.0 - 98.8 = 106 \times 0.2 \text{ MHz}, \quad 120.0 - 118.6 = 7 \times 0.2 \text{ MHz}$$



7.3-10

$$f_{LO} = f_c + f_{IF} = 955 \text{ to } 2055 \text{ kHz in steps of } 10 \text{ kHz} = 2 \times 2105 \text{ kHz} \div 421$$

$$2105 - 955 = 115 \times 10 \text{ kHz}, \quad 2105 - 2055 = 5 \times 10 \text{ kHz}$$



7.3-11

$$Z(f) = \frac{1}{j2\pi f} Y(f) \quad \text{and} \quad \Phi(f) = \phi_{\Delta} X(f) \quad \text{for PM, so}$$

$$\frac{Z(f)}{X(f)} = \frac{1}{j2\pi f} \frac{1}{K_v} \frac{jfKH(f)}{jf + KH(f)} \phi_{\Delta} = \frac{\phi_{\Delta}}{2\pi K_v} \frac{KH(f)}{jf + KH(f)} = \frac{\phi_{\Delta}}{2\pi K_v} H_L(f)$$

7.4-1

- (a) The frame should have an odd number of lines so that each field has a half-line to fill the small wedge at the top and bottom of the raster.



# 7.4-1 continued

- (b) A linear sweep (sawtooth or triangular) is needed to give the same exposure time to each horizontal element. A triangular sweep would result in excessive retrace time, equal to the line time.

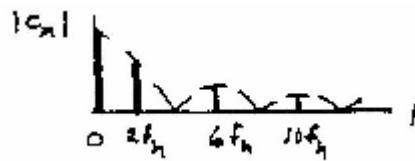
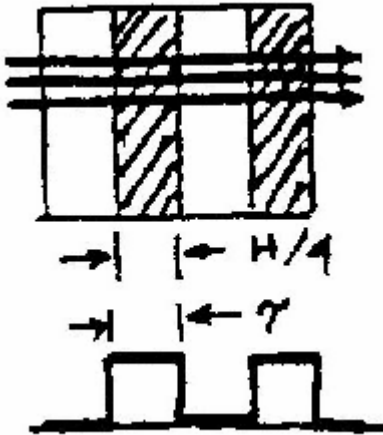
# 7.4-2

- (a) No vertical dependence. Video signal is rectangular pulse train with

$$\tau = (H/4)/s_h = 1/4 f_h \text{ and } T_0 = 2\tau.$$

$$\text{Thus, } f_0 = 2f_h$$

$$c(nf_0) = K \text{sinc } \frac{n}{2}$$



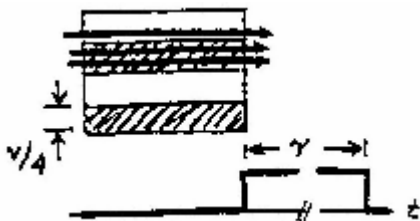
- (b) No horizontal dependence. Video signal is rectangular pulse train with

$$\tau = (V/4)/s_v = 1/4 f_v \text{ and } T_0 = 2\tau.$$

$$\text{Thus, } f_0 = 2f_v$$

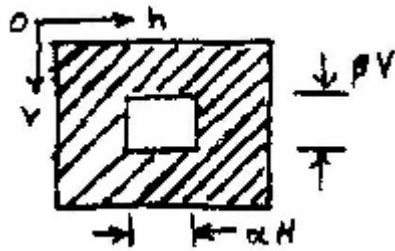
$$c(nf_0) = K \text{sinc } \frac{n}{2}$$

Same spectrum as (a) with  $f_h$  replaced by  $f_v \ll f_h$ , so much smaller bandwidth.



7.4-3

(a)



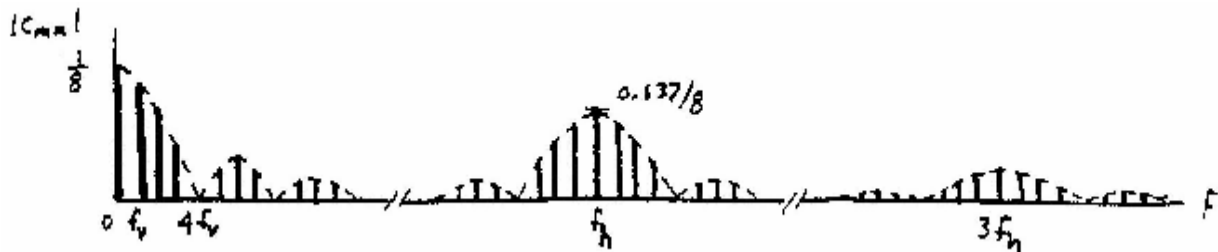
$$I(h, v) = \begin{cases} 1 & \frac{H}{2} - \frac{\alpha H}{2} < h < \frac{H}{2} + \frac{\alpha H}{2}, \quad \frac{V}{2} - \frac{\beta V}{2} < v < \frac{V}{2} + \frac{\beta V}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$c_{mn} = \frac{1}{HV} \int_{(1-\alpha)H/2}^{(1+\alpha)H/2} e^{-j2\pi mh/H} dh \int_{(1-\beta)V/2}^{(1+\beta)V/2} e^{-j2\pi nv/V} dv$$

$$= \frac{1}{HV} \left( \frac{e^{-j\pi m\alpha} - e^{j\pi m\alpha}}{-j2\pi m/H} e^{-j\pi m} \right) \left( \frac{e^{-j\pi n\beta} - e^{j\pi n\beta}}{-j2\pi n/V} e^{-j\pi n} \right)$$

$$\text{Thus, } |c_{mn}| = \left| \frac{\sin \pi m\alpha}{\pi m} \right| \left| \frac{\sin \pi n\beta}{\pi n} \right| = \alpha\beta |\text{sinc } m\alpha \text{ sinc } n\beta|$$

$$(b) \quad |c_{mn}| = \frac{1}{8} \left| \text{sinc } \frac{m}{2} \text{ sinc } \frac{n}{4} \right|, \quad f_{mn} = mf_h + nf_v = \left( m + \frac{n}{100} \right) f_h$$



7.4-4

$$n_v = 0.7 \times 230, \quad n_p = 1 \times n_v^2 = 25,921$$

$$B = 0.35 \times 1 \times 230/100 \mu s = 805 \text{ kHz}$$

7.4-5

$$n_v = 0.7 (1125 - N_{vr}) \approx 787, \quad n_p = 5/3 \times 787^2 = 1.03 \times 10^6$$

$$T_{line} = \frac{(2/60) \text{ sec}}{1125} = 29.6 \text{ } \mu\text{s}, \quad B = 0.35 \times \frac{5}{3} \times \frac{1125}{(1 - 0.2)29.6 \text{ } \mu\text{s}} = 27.7 \text{ MHz}$$

7.4-6

$$n_v = 0.7 (625 - 48) = 404, \quad n_p = 4/3 \times 404^2 = 2.18 \times 10^5$$

$$T_{line} = \frac{1}{15.625 \text{ kHz}} = 64 \text{ } \mu\text{s}, \quad B = 0.35 \times \frac{4}{3} \times \frac{625 - 48}{(64 - 10) \text{ } \mu\text{s}} = 4.99 \text{ MHz}$$

7.4-7

- (a) Since  $\tilde{x}(t)$  is proportional to  $x(t)$  averaged over the previous  $\tau$  seconds, the picture will be smeared in the horizontal direction and five vertical lines will be lost.

$$(b) \quad \tilde{x}(t) = \int_{-\infty}^t x(\lambda) d\lambda - \int_{-\infty}^{t-\tau} x(\lambda) d\lambda = \int_{-\infty}^t x(\lambda) d\lambda - \int_{-\infty}^t x(\lambda - \tau) d\lambda$$

$$\text{so } \tilde{X}(f) = \frac{1}{j2\pi f} X(f) - \frac{1}{j2\pi f} X(f) e^{-j2\pi f \tau} = \frac{1 - e^{-j2\pi f \tau}}{j2\pi f} X(f)$$

$$Y(f) = H_{eq}(f) \tilde{X}(f) = KX(f) e^{-j\omega t_d}$$

$$\text{Thus, } H_{eq}(f) = \frac{j2\pi f K e^{-j\omega t_d}}{1 - e^{-j2\pi f \tau}} = \frac{K}{\tau \text{ sinc } f \tau} e^{-j\omega(t_d - \tau/2)}$$

which can only hold for  $|f| < 1/\tau$  since  $H_{eq}(f) \rightarrow \infty$  at  $f = 1/\tau, 2/\tau, \dots$

7.4-8

- (a) If gain of the chrominance amp is too high, then  $|x_c|$  will be too large and all colors will be saturated and pastel colors will be too bright. If the gain of the chrominance amp is too low, then  $|x_c|$  will be too small and all colors will be unsaturated and appear as "washed-out" pastels.

- (b) If  $+90^\circ$  error, then red  $\rightarrow$  blue, blue  $\rightarrow$  green, green  $\rightarrow$  red.

If  $-90^\circ$  error, then red  $\rightarrow$  green, blue  $\rightarrow$  red, green  $\rightarrow$  blue.

If  $180^\circ$  error, then red  $\rightarrow$  blue-green, blue  $\rightarrow$  yellow (red-green),  
green  $\rightarrow$  purple (red-blue).

7.4-9

Let  $x'_b(t)$  be the BPF output in Fig. 7.4-11 so, from Eq. (15),

$$x'_b(t) = x_{YH}(t) + x_Q(t) \sin \omega_{cc} t + x_I(t) \cos \omega_{cc} t + \hat{x}_{IH}(t) \sin \omega_{cc} t$$

where  $x_{YH}(t)$  is the high-frequency portion of  $x_Y(t)$ .

7.4-9 continued

Thus,

$$\begin{aligned} v_I(t) &= x'_b(t) \times 2 \cos \omega_{cc} t = 2x_{YH} \cos \omega_{cc} t + x_Q(t) \sin 2\omega_{cc} t + x_I(t)(1 + \cos 2\omega_{cc} t) + \hat{x}_{IH}(t) \sin 2\omega_{cc} t \\ &= x_I(t) + 2x_{YH} \cos \omega_{cc} t + x_I(t) \cos 2\omega_{cc} t + [x_Q(t) + \hat{x}_{IH}(t)] \sin 2\omega_{cc} t \end{aligned}$$

$$\begin{aligned} v_Q(t) &= x'_b(t) \times 2 \sin \omega_{cc} t = 2x_{YH} \sin \omega_{cc} t + x_Q(t)(1 - \cos 2\omega_{cc} t) + x_I \sin 2\omega_{cc} t + \hat{x}_{IH}(1 - \cos 2\omega_{cc} t) \\ &= x_Q(t) + \hat{x}_{IH}(t) + 2x_{YH}(t) \sin \omega_{cc} t + x_I(t) \sin 2\omega_{cc} t - [x_Q(t) + \hat{x}_{IH}(t)] \cos 2\omega_{cc} t \end{aligned}$$

7.4-10

To modify Eq. (15) to account for asymmetric sidebands in Q channel, let

$x_{QH}(t)$  be the high-frequency portion of  $x_Q(t)$ . Then

$$\begin{aligned} x_b(t) &= x_Y(t) + [x_I(t) \cos \omega_{cc} t + \hat{x}_{IH}(t) \sin \omega_{cc} t] + [x_Q(t) \underbrace{\cos(\omega_{cc} t - 90^\circ)}_{\sin \omega_{cc} t} \\ &\quad + \hat{x}_{QH}(t) \underbrace{\sin(\omega_{cc} t - 90^\circ)}_{-\cos \omega_{cc} t}] \end{aligned}$$

Let  $x'_b(t)$  be the BPF output at the receiver, so

$$x'_b(t) = x_{YH} + [x_I(t) \cos \omega_{cc} t + \hat{x}_{IH}(t) \sin \omega_{cc} t] + [x_Q \sin \omega_{cc} t - \hat{x}_{QH} \cos \omega_{cc} t]$$

Thus

$$v_I(t) = x_b' \times 2 \cos \omega_{cc} t = x_I(t) - \hat{x}_{QH}(t) + 2x_{YH}(t) \cos \omega_{cc} t + [x_I(t) - \hat{x}_{QH}(t)] \cos 2\omega_{cc} t \\ + [x_Q(t) + \hat{x}_{IH}(t)] \sin 2\omega_{cc} t$$

$$v_Q(t) = x_b' \times 2 \sin \omega_{cc} t = x_Q(t) + \hat{x}_{IH}(t) + 2x_{YH}(t) \sin \omega_{cc} t + [x_I(t) - \hat{x}_{QH}(t)] \sin 2\omega_{cc} t \\ - [x_Q(t) + \hat{x}_{IH}(t)] \cos 2\omega_{cc} t$$

and lowpass filtering with  $B = 1.5$  MHz yields

$$v_I'(t) = x_I(t) - \hat{x}_{QH}(t) + 2x_{YH}(t) \cos \omega_{cc} t$$

$$v_Q'(t) = x_Q(t) + \hat{x}_{IH}(t) + 2x_{YH}(t) \sin \omega_{cc} t$$

Now we have cross talk between I and Q channels since both  $\hat{x}_{QH}(t)$  and  $\hat{x}_{IH}(t)$  have components in  $0.5 \text{ MHz} < f < 1.5 \text{ MHz}$ . This quadrature color cross talk is eliminated by reducing the bandwidth of  $x_Q(t)$  to 0.5 MHz so  $\hat{x}_{QH}(t) = 0$  and the Q-channel LPF removes  $\hat{x}_{IH}(t)$ .

## Chapter 8

8.1-1

$M = 12$  equally likely outcomes

$$P(A) = 6/12, P(B) = 4/12, P(C) = 3/12$$

$$P(AB) = 2/12, P(AC) = 0, P(BC) = 1/12$$

$$P(A^cB) = 2/12$$

A	1	2	3	4	C
	5	6	7	8	
B	9	10	11	12	

8.1-2

$M = 16$  equally likely outcomes

$$P(A) = 4/16, P(B) = 6/16, P(C) = 6/16$$

$$P(AB) = 0, P(AC) = 2/16, P(BC) = 2/16$$

$$P(A^cB) = 6/16$$

A	1,1				
	1,2		2,1	B	
	1,3		2,2		3,1
	1,4		2,3		3,2
		2,4		3,3	4,2
			3,4		4,3
C			4,4		

8.1-3

$$P(AB^c) = N_{AB^c} / N = (N_A - N_{AB}) / N = P(A) - P(AB)$$

8.1-4

$$N_A = N_{AB} + N_{AB^c} \quad N_B = N_{AB} + N_{A^cB}$$

$$P(A+B) = \frac{N_{AB} + N_{AB^c} + N_{A^cB}}{N} = \frac{N_A + N_B - N_{AB}}{N} = P(A) + P(B) - P(AB)$$

8.1-5

$$N_A = N_{AB} + N_{AB^c} \quad N_B = N_{AB} + N_{A^cB}$$

$$P(C) = \frac{N_{AB^c} + N_{A^cB}}{N} = \frac{N_A + N_B - 2N_{AB}}{N} = P(A) + P(B) - 2P(AB)$$



8.1-6

$$P(\text{match}) = P(HH + TT) = P(HH) + P(TT) = P(H)P(H) + P(T)P(T)$$

$$P(T) = P(H^c) = 1 - \frac{1+\epsilon}{2} = \frac{1-\epsilon}{2} \quad P(\text{match}) = \left(\frac{1+\epsilon}{2}\right)^2 + \left(\frac{1-\epsilon}{2}\right)^2 = \frac{1+\epsilon^2}{2} > \frac{1}{2}$$

8.1-7

Let  $A$  = “A fails,”  $B$  = “B fails,”  $C$  = “computer inoperable”

$$P(A) = 0.01, P(B) = 0.005, P(B|A) = 4 \times 0.005 = 0.02$$

$$P(C) = P(AB) = P(B|A)P(A) = 0.0002, P(A|B) = P(AB)/P(B) = 0.04$$

8.1-8

Let  $M$  = “match,”  $H_1$  = “heads on first toss,” etc.

$$(a) \quad P(H_1) = 1/2, P(MH_1) = P(H_1H_2) = (1/2)^2, P(M|H_1) = P(MH_1)/P(H_1) = 1/2$$

$$(b) \quad \text{Let } A = “H_1 \text{ or } H_2,” P(A) = P(H_1T_2 + T_1H_2 + H_1H_2) = 3/4$$

$$P(MA) = 1/4, P(M|A) = P(MA)/P(A) = 1/3$$

$$(c) \quad P(M) = P(H_1H_2 + T_1T_2) = 1/2, P(A|M) = P(A)P(M|A)/P(M) = 1/2$$

8.1-9

Let  $M$  = “match,”  $H_1$  = “heads on first toss,” etc.

$$(a) \quad P(H_1) = 1/4, P(MH_1) = P(H_1H_2) = (1/4)^2, P(M|H_1) = P(MH_1)/P(H_1) = 1/4$$

$$(b) \quad \text{Let } A = “H_1 \text{ or } H_2,” P(A) = P(H_1T_2 + T_1H_2 + H_1H_2) = 2 \times 1/4 \times 3/4 + (1/4)^2 = 7/16$$

$$P(MA) = P(H_1H_2) = (1/4)^2, P(M|A) = P(MA)/P(A) = 1/7$$

$$(c) \quad P(M) = P(H_1H_2) + P(T_1T_2) = (1/4)^2 + (3/4)^2 = 10/16, P(A|M) = P(A)P(M|A)/P(M) = 1/10$$

8.1-10

Since  $P(AB) = P(A|B)P(B) = P(B|A)P(A)$ ,  $P(XYZ) = P(X)P(YZ|X)$  where

$$P(YZ|X) = \frac{P(XYZ)}{P(X)} = \frac{P(XY)P(YZ|XY)}{P(X)} = P(Y|X)P(Z|XY) \text{ so } P(XYZ) = P(X)P(Y|X)P(Z|XY)$$

8.1-11

Let  $F$  = “fair coin,”  $L$  = “loaded coin,”  $A$  = “all tails,”  $P(F) = 1/3$ ,  $P(L) = 2/3$ ,  $P(A|F) = (1/2)^2$ ,  $P(A|L) =$

$$(3/4)^2 \quad (a) \quad P(A) = P(A|F)P(F) + P(A|L)P(L) = 11/24$$

$$(b) \quad P(L|A) = P(L)P(A|L)/P(A) = 9/11$$

8.1-12

Let  $F$  = “fair coin,”  $L$  = “loaded coin,”  $A$  = “all tails;”  $P(F) = 1/3$ ,  $P(L) = 2/3$ ,  $P(A|F) = (1/2)^2$ ,  $P(A|L) =$

$$(3/4)^2 \quad (a) \quad P(A) = P(A|F)P(F) + P(A|L)P(L) = 31/96$$

$$(b) \quad P(L|A) = P(L)P(A|L)/P(A) = 27/31$$

8.1-13

Let  $R_1$  = “first marble is red,” etc.,  $M$  = “match;”  $P(R_1) = 5/10$ ,  $P(W_1) = 3/10$ ,  $P(G_1) = 2/10$ ,

$$P(M|R_1) = P(R_2|R_1) = (5 - 1)/(10 - 1) = 4/9, P(M|W_1) = 2/9, P(M|G_1) = 1/9$$

$$(a) \quad P(M) = P(M|R_1) \times P(R_1) + P(M|W_1) \times P(W_1) + P(M|G_1) \times P(G_1)$$

$$= \frac{4}{9} \times \frac{5}{10} + \frac{2}{9} \times \frac{3}{10} + \frac{1}{9} \times \frac{2}{10} = \frac{14}{45}$$

$$(b) \quad P(W_1|M) = P(W_1)P(M|W_1)/P(M) = 3/14$$

8.1-14

Let  $R_1$  = “first marble is red,” etc.,  $M$  = “match;”  $P(R_1) = 5/10$ ,  $P(W_1) = 3/10$ ,

$$P(G_1) = 2/10, P(M|R_1) = P(R_3R_2|R_1) = P(R_3|R_2R_1)P(R_2|R_1) = \frac{5-2}{10-2} \times \frac{5-1}{10-1} = \frac{3}{8} \times \frac{4}{9}, P(M|W_1) = \frac{1}{8} \times \frac{2}{9},$$

$$P(M|G_1) = \frac{0}{8} \times \frac{1}{9}$$

$$(a) \quad P(M) = P(M|R_1) \times P(R_1) + P(M|W_1) \times P(W_1) + P(M|G_1) \times P(G_1)$$

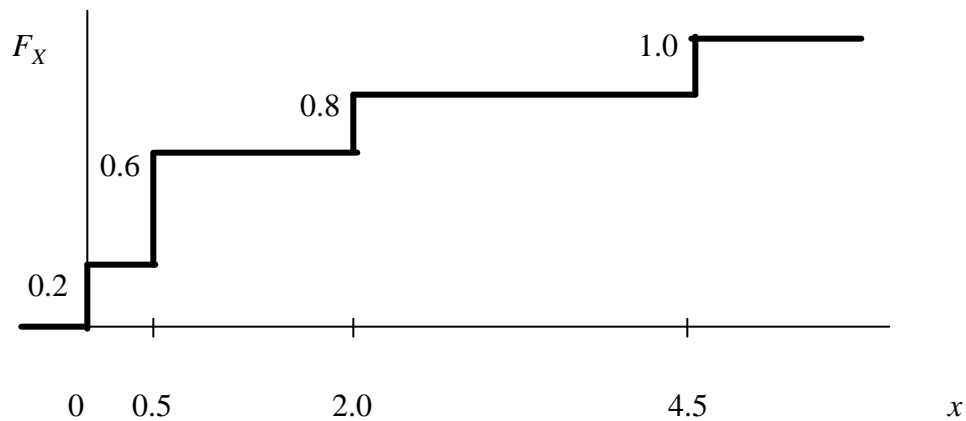
$$= \frac{12}{72} \times \frac{5}{10} + \frac{2}{72} \times \frac{3}{10} + 0 \times \frac{2}{10} = \frac{11}{120}$$

$$(b) \quad P(W_1|M) = P(W_1)P(M|W_1)/P(M) = 1/11$$

8.2-1

$P(N_i) = 1/5$

$N$	$x_i$	$P_X(x_i)$	$F_X(x_i)$
0	0	0.2	0.2
-1,1	0.5	0.4	0.6
2	2	0.2	0.8
3	4.5	0.2	1.0



(cont.)

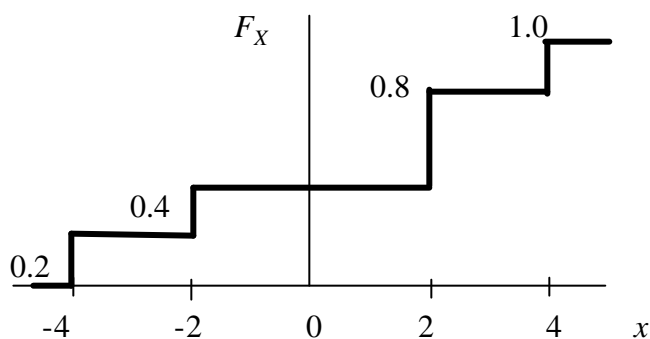
$$P(X \leq 0) = F_X(0) = 0.2, P(2 < X \leq 3) = F_X(3) - F_X(2) = 0, P(X < 2) = F_X(2 - \epsilon) = 0.6,$$

$$P(X > 2) = 1 - 0.6 = 0.4$$

8.2-2

$P(N_i) = 1/5$

$N$	$x_i$	$P_X(x_i)$	$F_X(x_i)$
3	-4	0.2	0.2
2	-2	0.2	0.4
-1,1	2	0.4	0.8
0	4	0.2	1.0



$$P(X \leq 0) = F_X(0) = 0.4, P(2 < X \leq 3) = F_X(3) - F_X(2) = 0, P(X < 2) = F_X(2 - \epsilon) = 0.4,$$

$$P(X \geq 2) = 1 - 0.4 = 0.6$$

8.2-3

$$F_X(x) = \int_{-\infty}^x p_X(\lambda) d\lambda = \begin{cases} 0 & x \leq 0 \\ \int_0^x \lambda e^{-\lambda} d\lambda = 1 - (x+1)e^{-x} & x > 0 \end{cases}$$

$$P(X \leq 1) = F_X(1) = 0.264, P(X > 2) = 1 - F_X(2) = 0.406, P(1 < X \leq 2) = F_X(2) - F_X(1) = 0.330$$

8.2-4

$$F_X(x) = \int_{-\infty}^x p_X(\lambda) d\lambda = \begin{cases} \int_{-\infty}^x \frac{1}{2} e^{\lambda} d\lambda = \frac{1}{2} e^x & x \leq 0 \\ \frac{1}{2} + \int_0^x \frac{1}{2} e^{-\lambda} d\lambda = 1 - \frac{1}{2} e^{-x} & x > 0 \end{cases}$$

$$P(X \leq 0) = F_X(0) = 1/2, P(X > 1) = 1 - F_X(1) = 0.184, P(0 < X \leq 1) = F_X(1) - F_X(0) = 0.316$$

8.2-5

$$F_X(\infty) = 100K = 1 \Rightarrow K = 0.01 \text{ so } p_X(x) = dF_X(x)/dx = 0.2x[u(x) - u(x - 10)]$$

$$P(X \leq 5) = F_X(5) = K \times 5^2 = 0.25, P(5 < X \leq 7) = F_X(7) - F_X(5) = 0.49 - 0.25 = 0.24$$

8.2-6

$$F_X(\infty) = K/\sqrt{2} = 1 \Rightarrow K = \sqrt{2} \text{ so } p_X(x) = dF_X(x)/dx = \frac{\sqrt{2}\pi}{40} \cos \frac{\pi x}{40} [u(x) - u(x - 10)]$$

$$P(X \leq 5) = F_X(5) = K \sin \frac{\pi}{8} = 0.541, P(5 < X \leq 7) = F_X(7) - F_X(5) = K \sin \frac{7\pi}{40} - 0.541 = 0.198$$

8.2-7

$$P(Z < 0) = 0, P(Z \leq 0) = P(X \leq 0) = 1/2, P(Z \leq z) = P(X \leq z) \text{ for } z > 0$$

$$F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{1}{2} + \frac{1}{\pi} \arctan z & z \geq 0 \end{cases} \quad p_Z(z) = \frac{d}{dz} F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{1}{2} \delta(z) + \frac{1}{\pi(1+z^2)} & z \geq 0 \end{cases}$$

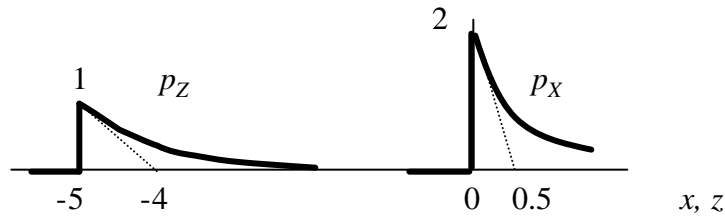
8.2-8

$P(Z < -1) = 0$ ,  $P(Z \leq -1) = P(X \leq 0) = 1/2$ ,  $P(Z \leq z) = P(X \leq z)$  for  $z > 0$

$$F_Z(z) = \begin{cases} 0 & z < -1 \\ 1/2 & -1 \leq z \leq 0 \\ \frac{1}{2} + \frac{1}{\pi} \arctan z & z \geq 0 \end{cases} \quad p_Z(z) = \frac{d}{dz} F_Z(z) = \begin{cases} \frac{1}{2} \delta(z+1) & z \leq 0 \\ \frac{1}{\pi(1+z^2)} & z > 0 \end{cases}$$

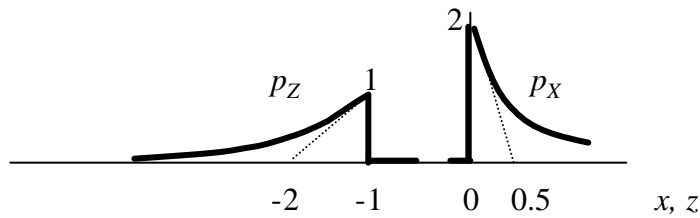
8.2-9

$$p_Z(z) = \frac{1}{|2|} 2e^{-2(z+5)/2} u\left(\frac{z+5}{2}\right) = e^{-(z+5)} u(z+5)$$



8.2-10

$$p_Z(z) = \frac{1}{|-2|} 2e^{-2(z-1)/(-2)} u\left(\frac{z-1}{-2}\right) = e^{(z-1)} u[-(z-1)]$$



8.2-11

Monotonic transformation with  $g^{-1}(z) = z^2 - 1$ ,  $dg^{-1}/dz = 2z$ ,  $p_X(x) = 1/4$  for  $-1 \leq x \leq 3$ , so

$$p_Z(z) = \frac{1}{4} |2z| [u(z) - u(z-2)] = \frac{z}{2} [u(z) - u(z-2)]$$

8.2-12

$$g_1(x) = -x[u(x+1) - u(x)], g_1^{-1}(z) = -z[u(z) - u(z-1)], dg_1^{-1}/dz = -1$$

$$g_2(x) = x[u(x) - u(x-3)], g_2^{-1}(z) = z[u(z) - u(z-3)], dg_2^{-1}/dz = 1, p_X(x) = 1/4 \text{ for } -1 \leq x \leq 3, \text{ so}$$

$$p_Z(z) = \begin{cases} \frac{1}{4}|-1| + \frac{1}{4}|1| = \frac{1}{2} & 0 \leq z \leq 1 \\ \frac{1}{4}|1| = \frac{1}{4} & 1 < z \leq 3 \end{cases}$$

8.2-13

$$g_1(x) = \sqrt{-x}[u(x+1) - u(x)], g_1^{-1}(z) = -z^2[u(z) - u(z-1)], dg_1^{-1}/dz = -2z$$

$$g_2(x) = \sqrt{x}[u(x) - u(x-3)], g_2^{-1}(z) = z^2[u(z) - u(z-\sqrt{3})], dg_2^{-1}/dz = 2z, p_X(x) = 1/4 \text{ for } -1 \leq x \leq 3, \text{ so}$$

$$p_Z(z) = \begin{cases} \frac{1}{4}|-2z| + \frac{1}{4}|2z| = z & 0 \leq z \leq 1 \\ \frac{1}{4}|2z| = \frac{z}{2} & 1 < z \leq \sqrt{3} \end{cases}$$

8.2-14

$$g_1(x) = x^2 u(-x), g_1^{-1}(z) = -\sqrt{z} u(z), dg_1^{-1}/dz = -1/2 \sqrt{z}, g_2(x) = x^2 u(x),$$

$$g_2^{-1}(z) = +\sqrt{z} u(z), dg_2^{-1}/dz = +1/2 \sqrt{z} \quad (\text{cont.})$$

$$p_Z(z) = 0 \text{ for } z < 0, p_Z(z) = p_X(\sqrt{z}) \left| \frac{1}{2\sqrt{z}} \right| + p_X(-\sqrt{z}) \left| -\frac{1}{2\sqrt{z}} \right| \text{ for } z > 0, \text{ so}$$

$$p_Z(z) = \frac{1}{2\sqrt{z}} [p_X(\sqrt{z}) + p_X(-\sqrt{z})] u(z)$$

8.2-15

$$p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x, y) dx = y e^{-y} u(y) \int_0^{\infty} e^{-yx} dx = e^{-y} u(y),$$

$$p_{XY}(x, y) = y e^{-yx} u(x) e^{-y} u(y) \neq p_X(x) p_Y(y), p_X(x|y) = p_{XY}(x, y) / p_Y(y) = y e^{-yx} u(x)$$

8.2-16

$$p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x, y) dx = \frac{1}{40} \Pi\left(\frac{y}{6}\right) \int_{-1}^1 (x^2 + 2xy + y^2) dx = \frac{1}{60} (1 + 3y^2) \Pi\left(\frac{y}{6}\right),$$

$$p_{XY}(x, y) = \frac{3}{2} \frac{(x+y)^2}{(1+3y^2)} \Pi\left(\frac{x}{2}\right) \frac{1}{60} (1+3y^2) \Pi\left(\frac{y}{6}\right) \neq p_X(x) p_Y(y)$$

$$p_X(x|y) = p_{XY}(x, y) / p_Y(y) = \frac{3(x+y)^2}{2(1+3y^2)} \Pi\left(\frac{x}{2}\right)$$

8.2-17

$$\int_{-\infty}^{\infty} p_X(x|y) dx = \int_{-\infty}^{\infty} \frac{p_{XY}(x, y)}{p_Y(y)} dx = \frac{1}{p_Y(y)} \int_{-\infty}^{\infty} p_{XY}(x, y) dx = 1$$

For any given  $Y = y$ ,  $X$  must be somewhere in the range  $-\infty < x < \infty$ .

8.2-18

$$p_{XY}(x, y) = p_X(x|y)p_Y(y), \quad p_X(x) = \int_{-\infty}^{\infty} p_{XY}(x, y) dy = \int_{-\infty}^{\infty} p_X(x|y)p_Y(y) dy$$

$$\text{Thus, } p_Y(y|x) = p_{XY}(x, y) / p_X(x) = p_X(x|y)p_Y(y) / \int_{-\infty}^{\infty} p_X(x|y)p_Y(y) dy$$

8.3-1

$$m_X = a \int_0^{\infty} x e^{-ax} dx = 1/a, \quad \overline{X^2} = a \int_0^{\infty} x^2 e^{-ax} dx = 2/a^2, \quad \text{so } \sigma_X = \sqrt{\frac{2}{a^2} - \left(\frac{1}{a}\right)^2} = \frac{1}{a}$$

8.3-2

$$m_X = a^2 \int_0^{\infty} x^2 e^{-ax} dx = 2/a, \quad \overline{X^2} = a^2 \int_0^{\infty} x^3 e^{-ax} dx = 6/a^2, \quad \text{so } \sigma_X = \sqrt{\frac{6}{a^2} - \left(\frac{2}{a}\right)^2} = \frac{\sqrt{2}}{a}$$

8.3-3

$$m_X = \frac{\sqrt{2}}{\pi} \int_{-\infty}^{\infty} \frac{x}{1+(x-a)^4} dx = \frac{\sqrt{2}}{\pi} \left[ \int_{-\infty}^{\infty} \frac{\lambda}{1+\lambda^4} d\lambda + \int_{-\infty}^{\infty} \frac{a}{1+\lambda^4} d\lambda \right] = a,$$

$$\overline{X^2} = \frac{\sqrt{2}}{\pi} \int_{-\infty}^{\infty} \frac{x^2}{1+(x-a)^4} dx = \frac{\sqrt{2}}{\pi} \left[ \int_{-\infty}^{\infty} \frac{\lambda^2}{1+\lambda^4} d\lambda + \int_{-\infty}^{\infty} \frac{2\lambda}{1+\lambda^4} d\lambda + \int_{-\infty}^{\infty} \frac{a^2}{1+\lambda^4} d\lambda \right] = 1 + a^2, \quad \text{so}$$

$$\sigma_X = \sqrt{(1+a^2) - a^2} = 1$$

8.3-4

$$P(X=b)=1-p, m_X=ap+b(1-p), \overline{X^2}=a^2p+b^2(1-p),$$

$$\sigma_X^2=a^2p+b^2(1-p)-[ap+b(1-p)]^2=(a-b)^2p(1-p), \sigma_X=|a-b|\sqrt{p(1-p)}$$

8.3-5

$$m_X=\sum_{i=0}^{K-1} ai\frac{1}{K}=\frac{a}{K}\frac{(K-1)K}{2}=\frac{K-1}{2}a,$$

$$\overline{X^2}=\sum_{i=0}^{K-1}(ai)^2\frac{1}{K}=\frac{a^2}{K}\frac{(K-1)K[2(K-1)+1]}{6}=\frac{(K-1)(2K-1)}{6}a^2$$

$$\sigma_X^2=\overline{X^2}-m_X^2=\frac{(K-1)a^2}{2}\frac{2(2K-1)-3(K-1)}{6}=\frac{K^2-1}{12}a^2, \quad \sigma_X=\sqrt{\frac{K^2-1}{3}}\frac{a}{2}$$

8.3-6

$$m_Y=\int_{-\infty}^{\infty} a \cos x p_X(x) dx = \frac{a}{2\pi} \int_{\theta}^{\theta+2\pi} \cos x dx = 0,$$

$$\overline{Y^2}=\int_{-\infty}^{\infty} a^2 \cos^2 x p_X(x) dx = \frac{a^2}{2\pi} \int_{\theta}^{\theta+2\pi} \cos^2 x dx = \frac{a^2}{2} \quad \sigma_Y=\sqrt{a^2/2-0}=a/\sqrt{2}$$

8.3-7

$$m_Y=\int_{-\infty}^{\infty} a \cos x p_X(x) dx = \frac{a}{\pi} \int_{\theta}^{\theta+\pi} \cos x dx = -\frac{2a}{\pi} \sin \theta,$$

$$\overline{Y^2}=\int_{-\infty}^{\infty} a^2 \cos^2 x p_X(x) dx = \frac{a^2}{\pi} \int_{\theta}^{\theta+\pi} \cos^2 x dx = \frac{a^2}{2}$$

$$\sigma_Y=\sqrt{\frac{a^2}{2}-\left(-\frac{2a}{\pi}\sin\theta\right)^2}=a\sqrt{\frac{1}{2}-\frac{4}{\pi^2}\sin^2\theta}$$

8.3-8

$$m_Y=\alpha m_X+\beta, \quad \overline{Y^2}=E[(\alpha X+\beta)^2]=E[\alpha^2 X^2+2\alpha\beta X+\beta^2]=\alpha^2 \overline{X^2}+2\alpha\beta m_X+\beta^2$$

$$\sigma_Y^2=\overline{Y^2}-m_Y^2=\alpha^2(\overline{X^2}-m_X^2)=\alpha^2\sigma_X^2, \quad \sigma_Y=|\alpha|\sigma_X$$



8.3-9

$$\overline{Y^2} = E[(X + \beta)^2] = E[X^2 + 2\beta X + \beta^2] = \overline{X^2} + 2\beta m_X + \beta^2$$

$$\frac{d}{d\beta} \overline{Y^2} = 2m_X + 2\beta = 0 \Rightarrow \beta = -m_X$$

8.3-10

$$P(X \geq a) = \int_a^\infty p_X(x) dx \quad \text{and} \quad p_X(x) = 0 \text{ for } x < 0$$

$$E[X] = \int_0^\infty p_X(x) dx \geq \int_a^\infty x p_X(x) dx \geq a \int_a^\infty p_X(x) dx = aP(X \geq a), \text{ so } P(X \geq a) \leq m_X / a$$

8.3-11

$$E[(X \pm Y)^2] = E[X^2 \pm 2XY + Y^2] = \overline{X^2} \pm 2\overline{XY} + \overline{Y^2} \geq 0$$

$$\text{so } 2\overline{XY} \geq -(\overline{X^2} + \overline{Y^2}) \text{ and } 2\overline{XY} \leq (\overline{X^2} + \overline{Y^2}), \Rightarrow -\frac{\overline{X^2} + \overline{Y^2}}{2} \leq \overline{XY} \leq \frac{\overline{X^2} + \overline{Y^2}}{2}$$

8.3-12

$$C_{XY} = E[XY - m_X Y - m_Y X + m_X m_Y] = \overline{XY} - m_X m_Y$$

$$(a) \overline{XY} = \overline{X} \overline{Y} = m_X m_Y \Rightarrow C_{XY} = 0$$

$$(b) \overline{XY} = E[X(\alpha X + \beta)] = \alpha \overline{X^2} + \beta m_X \text{ and } m_Y = \alpha m_X + \beta \text{ so } C_{XY} = \alpha(\overline{X^2} - m_X^2) = \alpha \sigma_X^2$$

8.3-13

$$\epsilon^2 = E[Y^2 - 2(\alpha X + \beta)Y + (\alpha X + \beta)^2] = \overline{Y^2} - 2\alpha \overline{XY} - 2\beta \overline{Y} + \alpha^2 \overline{X^2} + 2\alpha \beta \overline{X} + \beta^2$$

$$\partial \epsilon^2 / \partial \alpha = -2\overline{XY} + 2\alpha \overline{X^2} + 2\beta \overline{Y} = 0 \text{ and } \partial \epsilon^2 / \partial \beta = -2\overline{Y} - 2\alpha \overline{X} + 2\beta = 0 \text{ so}$$

$$\alpha = (\overline{XY} - \overline{X} \overline{Y}) / \sigma_X^2 \text{ and } \beta = \overline{Y} - \alpha \overline{X}$$

8.3-14

$$\frac{d^n}{dv^n} \Phi_X(v) = \frac{d^n}{dv^n} E[e^{jvX}] = E[(jX)^n e^{jvX}] = j^n E[X^n e^{jvX}], \text{ so}$$

$$\frac{d^n}{dv^n} \Phi_X(0) = j^n E[X^n] \Rightarrow E[X^n] = j^{-n} \frac{d^n}{dv^n} \Phi_X(v) \Big|_{v=0}$$

8.3-15

$$F[ae^{-at}u(t)] = \frac{a}{a + j2\pi f} \Rightarrow F^{-1}[ae^{-af}u(f)] = \frac{a}{a + j2\pi(-t)} \text{ so}$$

$$\Phi_X(2\pi t) = \frac{a}{a - j2\pi t} \quad \text{and} \quad \Phi_X(v) = \frac{a}{a - jv} = \left(1 - j\frac{v}{a}\right)^{-1}$$

$$\frac{d\Phi_X}{dv} = -\left(1 - j\frac{v}{a}\right)^{-2} \left(-\frac{j}{a}\right) \Rightarrow \overline{X} = j^{-1} \left(\frac{j}{a}\right) = \frac{1}{a}$$

$$\frac{d^2\Phi_X}{dv^2} = 2\left(1 - j\frac{v}{a}\right)^{-3} \left(-\frac{j}{a}\right)^2 \Rightarrow \overline{X^2} = j^{-2} 2 \left(\frac{j}{a}\right)^2 = \frac{2}{a^2}$$

$$\frac{d^3\Phi_X}{dv^3} = -6\left(1 - j\frac{v}{a}\right)^{-4} \left(-\frac{j}{a}\right)^3 \Rightarrow \overline{X^3} = j^{-3} (-6) \left(\frac{j}{a}\right)^3 = \frac{6}{a^3}$$

8.3-16

$$\Phi_Y(v) = E[e^{jvx^2}] = \int_0^\infty e^{jvx^2} 2axe^{-ax^2} dx = \int_0^\infty e^{jv\lambda} ae^{-a\lambda} d\lambda \Rightarrow p_Y(y) = ae^{-ay}u(y)$$

8.3-17

$$\Phi_Y(v) = E[e^{jv\sin x}] = \int_{-\pi/2}^{\pi/2} e^{jv\sin x} \frac{1}{\pi} dx$$

Let  $\lambda = \sin x$ ,  $d\lambda = \cos x dx$  where  $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \lambda^2}$ , so

$$\Phi_Y(v) = \int_{\sin(-\pi/2)}^{\sin \pi/2} e^{jv\lambda} \frac{1}{\pi} \frac{d\lambda}{\sqrt{1 - \lambda^2}} = \int_{-1}^1 e^{jv\lambda} \frac{1}{\pi\sqrt{1 - \lambda^2}} d\lambda \Rightarrow p_Y(y) = \frac{1}{\pi\sqrt{1 - y^2}} \Pi\left(\frac{y}{2}\right)$$

8.4-1

Binomial distribution with  $\alpha = (1 - \alpha) = 1/2$ , so  $m = 10 \times 1/2 = 5$ ,  $\sigma^2 = 5 \times 1/2 = 2.5$ ,  $m \pm 2\sigma \approx 2$  to  $8$

$$P(i < 3) = F_I(2) = \left[ \binom{10}{0} + \binom{10}{1} + \binom{10}{2} \right] \left(\frac{1}{2}\right)^{10} = \frac{1 + 10 + 45}{1024} = 0.0547$$

8.4-2

Binomial distribution with  $\alpha = 3/5$  and  $(1 - \alpha) = 2/5$ , so  $m = 10 \times 3/5 = 6$ ,  $\sigma^2 = 5 \times 2/5 = 2.4$ ,

$m \pm 2\sigma \approx 3$  to  $9$

$$P(i < 3) = F_I(2) = \left[ \binom{10}{0} 3^0 2^{10} + \binom{10}{1} 3^1 2^9 + \binom{10}{2} 3^2 2^8 \right] \frac{2^8}{5^{10}} = \frac{(1 \times 4 + 10 \times 6 + 45 \times 9) 256}{9.87 \times 10^6} = 0.0122$$

8.4-3

Let  $I$  = number of forward steps, binomial distribution with  $m_I = 100 \times \frac{3}{4} = 75$ ,  $\sigma_I^2 = 75 \times \frac{1}{4}$ ,

$$\overline{I^2} = \frac{75}{4} + 75^2, X = Il - (100 - I)l = (2I - 100)l \text{ so } m_X = (2m_I - 100)l = 50l \text{ and}$$

$$\overline{X^2} = \overline{(2I - 100)^2 l^2} = (4\overline{I^2} - 400m_I + 10^4)l^2 = 2575l^2, \quad \sigma_X = \sqrt{2575l^2 - (50l)^2} = \sqrt{75}l$$

8.4-4

Binomial distribution with  $1 - \alpha = 0.99$  so

$$P(I > 1) = 1 - P_I(0) - P_I(1) = 1 - \binom{10}{0} 0.01^0 0.99^{10} - \binom{10}{1} 0.01^1 0.99^9 = 0.0042$$

Poisson approximation with  $m = 10 \times 0.01 = 0.1$

$$P(I > 1) \approx 1 - e^{-0.1} \frac{(0.1)^0}{0!} - e^{-0.1} \frac{(0.1)^1}{1!} = 0.0047$$

8.4-5

$\mu = 0.5$  particles/sec,  $T = 2$  sec,  $\mu T = 1$ , so

$$(a) \quad P_I(1) = e^{-1} \frac{1^1}{1!} = 0.368 \quad (b) \quad P(I > 1) = 1 - P_I(0) - P_I(1) = 1 - e^{-1} \frac{1^0}{0!} - e^{-1} \frac{1^1}{1!} = 0.264$$

8.4-6

$$E[I] = \sum_{i=0}^{\infty} i e^{-m} \frac{m^i}{i!} = e^{-m} \sum_{i=0}^{\infty} i \frac{m^i}{i!}, \quad E[I^2] = e^{-m} \sum_{i=0}^{\infty} i^2 \frac{m^i}{i!}, \text{ where}$$

$$e^m = 1 + m + \frac{1}{2!} m^2 + \dots = \sum_{i=0}^{\infty} \frac{m^i}{i!} \text{ so } \frac{d}{dm} e^m = \frac{d}{dm} \sum_{i=0}^{\infty} \frac{m^i}{i!} = \sum_{i=0}^{\infty} i \frac{m^{i-1}}{i!} = \frac{1}{m} \sum_{i=0}^{\infty} i \frac{m^i}{i!} \text{ and}$$

$$\frac{d^2}{dm^2} e^m = \frac{d}{dm} \sum_{i=0}^{\infty} i \frac{m^{i-1}}{i!} = \sum_{i=0}^{\infty} i(i-1) \frac{m^{i-2}}{i!} = \frac{1}{m^2} \sum_{i=0}^{\infty} (i^2 - i) \frac{m^i}{i!}$$

(cont.)

$$\text{But } \frac{d}{dm} e^m = \frac{d^2}{dm^2} e^m = e^m \text{ so } \sum_{i=0}^{\infty} i \frac{m^i}{i!} = m e^m \text{ and } \sum_{i=0}^{\infty} (i^2 - i) \frac{m^i}{i!} = \sum_{i=0}^{\infty} i^2 \frac{m^i}{i!} - m e^m = m^2 e^m$$

$$\text{Thus, } E[I] = e^{-m}(m e^m) = m \text{ and } E[I^2] = e^{-m}(m^2 e^m + m e^m) = m^2 + m$$

8.4-7

$$\overline{X} = m = 100, \quad \sigma^2 = \overline{X^2} - \overline{X}^2 \Rightarrow \overline{X^2} = \sigma^2 + m^2 = 10,004$$

$$P(X < m - \sigma \text{ or } X > m + \sigma) = P(X \leq m - \sigma) + P(X > m + \sigma) = 2Q(1) \approx 0.32$$

8.4-8

$$m = \overline{X} = 2, \quad \sigma = \sqrt{\overline{X^2} - \overline{X}^2} = 3$$

$$P(X > 5) = P(X > m + \sigma) = Q(1) \approx 0.16,$$

$$P(2 < X \leq 5) = P(X > m) - P(X > m + \sigma) = \frac{1}{2} - Q(1) \approx 0.34$$

8.4-9

$$m = 10, \quad \sigma = \sqrt{500 - 100} = 20, \quad P(X > 20) = P(X > m + \sigma/2) = Q(0.5) \approx 0.31$$

$$P(10 < X \leq 20) = P(X > m) - P(X > m + \sigma/2) = 1/2 - Q(0.5) \approx 0.19$$

$$P(0 < X \leq 20) = P(|X - m| < \sigma/2) = 1 - 2Q(0.5) \approx 0.38$$

$$P(X > 0) = 1 - P(X \leq m - \sigma/2) = 1 - Q(0.5) \approx 0.69$$

8.4-10

$$m = 100 \times \frac{1}{2} = 50, \quad \sigma^2 = 50 \times \frac{1}{2} = 25, \quad \sigma = 5$$

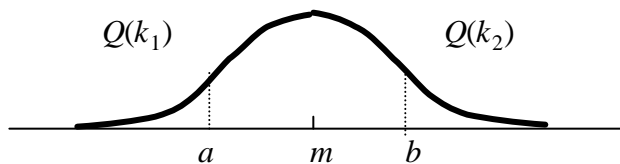
$$(a) \quad P(X > 70) = P(X > m + 4\sigma) = Q(4) \approx 3.5 \times 10^{-5}$$

$$(b) \quad P(40 < X \leq 60) = P(|X - m| \leq 2\sigma) = 1 - 2Q(2) = 0.95$$

8.4-11

Let  $a = m - k_1\sigma$  and  $b = m + k_2\sigma$  so

$$P(a < X \leq b) = 1 - Q(k_1) - Q(k_2) = 1 - Q\left(\frac{m-a}{\sigma}\right) - Q\left(\frac{m-b}{\sigma}\right)$$



8.4-12

$$m = 0, \sigma = 3, P(|X| \leq c) = P\left(|X - m| \leq \frac{c}{\sigma}\right) = 1 - 2Q\left(\frac{c}{\sigma}\right)$$

$$(a) \quad 1 - 2Q(c/3) = 0.9 \Rightarrow Q(c/3) = 0.05, c \approx 3 \times 1.65 = 4.95$$

$$(b) \quad 1 - 2Q(c/3) = 0.99 \Rightarrow Q(c/3) = 0.005, c \approx 3 \times 2.57 = 7.71$$

8.4-13

$$\begin{aligned} Q(k) &= \frac{1}{\sqrt{2\pi}} \int_k^\infty e^{-\lambda^2/2} d\lambda = \frac{1}{\sqrt{2\pi}} \int_k^\infty \left(-\frac{1}{\lambda}\right) d\left(e^{-\lambda^2/2}\right) = \frac{1}{\sqrt{2\pi}} \left[ -\frac{1}{\lambda} e^{-\lambda^2/2} \Big|_k^\infty - \int_k^\infty e^{-\lambda^2/2} d\left(-\frac{1}{\lambda}\right) \right] \\ &= \frac{1}{\sqrt{2\pi}k^2} e^{-k^2/2} - \frac{1}{\sqrt{2\pi}} \int_k^\infty \frac{1}{\lambda^2} e^{-\lambda^2/2} d\lambda \end{aligned}$$

$$\text{so } Q(k) < \frac{1}{\sqrt{2\pi}k^2} e^{-k^2/2} \text{ and}$$

$$\frac{1}{\sqrt{2\pi}} \int_k^\infty \frac{1}{\lambda^2} e^{-\lambda^2/2} d\lambda < \frac{1}{\sqrt{2\pi}} \frac{1}{k^2} \int_k^\infty e^{-\lambda^2/2} d\lambda = \frac{1}{k^2} Q(k) < Q(k) \text{ if } k > 1$$

$$\text{Thus, } Q(k) \approx \frac{1}{\sqrt{2\pi}k^2} e^{-k^2/2} \text{ for } k > 1$$

8.4-14

$$E[(X - m)^n] = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^\infty (x - m)^n e^{-(x-m)^2/2\sigma^2} dx = \frac{(2\sigma^2)^{n/2}}{\sqrt{\pi}} \int_{-\infty}^\infty \lambda^n e^{-\lambda^2} d\lambda$$

$$(a) \quad E[(X - m)^n] = 0 \text{ for odd } n \text{ since } \lambda^n e^{-\lambda^2} \text{ has odd symmetry}$$

$$(b) \quad E[(X - m)^n] = K_n 2 \int_0^\infty \lambda^n e^{-\lambda^2} d\lambda \text{ for even } n, \text{ where } K_n = \frac{(2\sigma^2)^{n/2}}{\sqrt{\pi}} = \frac{2^{n/2} \sigma^n}{\sqrt{\pi}}$$

$$\text{But } e^{-\lambda^2} d\lambda = -\frac{1}{2\lambda} d(e^{-\lambda^2}) \text{ so}$$

$$\begin{aligned} E[(X - m)^n] &= -K_n \int_0^\infty \lambda^{n-1} d(e^{-\lambda^2}) = -K_n \left[ \lambda^{n-1} e^{-\lambda^2} \Big|_0^\infty - \int_0^\infty e^{-\lambda^2} d(\lambda^{n-1}) \right] = K_n (n-1) \int_0^\infty \lambda^{n-2} e^{-\lambda^2} d\lambda \\ &= (n-1) \frac{K_n}{2K_{n-2}} 2 \int_0^\infty \lambda^{n-2} e^{-\lambda^2} d\lambda = (n-1) \sigma^2 E[(X - m)^{n-2}] \end{aligned}$$

(cont.)

Thus,  $E[(X-m)^4] = (4-1)\sigma^2 E[(X-m)^2] = 3\sigma^4$ ,  $E[(X-m)^6] = (6-1)\sigma^2 (3\sigma^4) = 3 \cdot 5\sigma^6$ , and  $E[(X-m)^n] = 1 \cdot 3 \cdot 5 \cdots (n-1)\sigma^n$ ,  $n = 2, 4, 6, \dots$

8.4-15

$$p_X(f) = \frac{1}{b} e^{-\pi[(f-m)/b]^2} \quad \text{where} \quad b = \sqrt{2\pi\sigma^2}$$

If  $m = 0$ ,  $\Phi_X(2\pi t) = e^{-\pi(bt)^2} = e^{-\sigma^2(2\pi t)^2/2}$ , so  $\Phi_X(v) = e^{-\sigma^2 v^2/2}$

For  $m \neq 0$ , use frequency-translation theorem with  $\omega_c = 2\pi m$ , so

$$\Phi_X(2\pi t) = e^{-\pi(bt)^2} e^{j\omega_c t} = e^{-\sigma^2(2\pi t)^2/2} e^{jm(2\pi t)} \quad \text{and} \quad \Phi_X(v) = e^{-\sigma^2 v^2/2} e^{jm v}$$

8.4-16

$$\Phi_Z(v) = \Phi_X(v)\Phi_Y(v) = e^{-\sigma_X^2 v^2/2} e^{jm_X v} e^{-\sigma_Y^2 v^2/2} e^{jm_Y v} = e^{-\sigma_Z^2 v^2/2} e^{jm_Z v}$$

$$\text{where } \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2, m_Z = m_X + m_Y. \text{ Hence, } p_Z(z) = \frac{1}{\sqrt{2\pi(\sigma_X^2 + \sigma_Y^2)}} e^{-[z-(m_X+m_Y)]^2/2(\sigma_X^2 + \sigma_Y^2)}$$

If  $Z = \frac{1}{n} \sum_{i=1}^n X_i = Y_1 + Y_2 + \cdots + Y_n$  where  $Y_i = \frac{X_i}{n}$  is gaussian with

$$\overline{Y_i} = \frac{\overline{X_i}}{n}, \overline{Y_i^2} = \frac{\overline{X_i^2}}{n^2}, \sigma_{Y_i}^2 = \frac{\overline{X_i^2} - \overline{X_i}^2}{n^2} = \frac{\sigma_{X_i}^2}{n^2}$$

Then  $\Phi_Z(v) = \Phi_{Y_1}(v)\Phi_{Y_2}(v)\cdots\Phi_{Y_n}(v) = e^{-\sigma_Z^2 v^2/2} e^{jm_Z v}$  where

$$\sigma_Z^2 = \sum \sigma_{Y_i}^2 = \frac{1}{n^2} \sum_{i=1}^n \sigma_{X_i}^2, \quad m_Z = \sum m_{Y_i} = \frac{1}{n} \sum_{i=1}^n m_{X_i}$$

8.4-17

$$X = \ln Y \Rightarrow Y = e^X$$

$$E[Y] = E[e^X] = E[e^{jvX}] \Big|_{jv=1} = \Phi_X(-j) = e^{\sigma_X^2/2} e^{m_X}$$

$$E[Y^2] = E[e^{2X}] = E[e^{jvX}] \Big|_{jv=2} = \Phi_X(-j2) = e^{2\sigma_X^2} e^{2m_X}$$

8.4-18

$$(a) \Phi_Z(v) = \int_{-\infty}^{\infty} e^{jvx^2} p_X(x) dx = \frac{2}{\sqrt{2\pi\sigma^2}} \int_0^{\infty} e^{-\left(\frac{1}{2\sigma^2} - jv\right)x^2} dx = (1 - j2\sigma^2 v)^{-1/2}$$

$$(b) \frac{d\Phi_Z}{dv} = j\sigma^2 (1 - j2\sigma^2 v)^{-3/2} \Rightarrow E[Z] = j^{-1}(j\sigma^2) = \sigma^2$$

$$\frac{d^2\Phi_Z}{dv^2} = -3\sigma^4 (1 - j2\sigma^2 v)^{-5/2} \Rightarrow E[Z^2] = j^{-2}(-3\sigma^4) = 3\sigma^4$$

$$\frac{d^3\Phi_Z}{dv^3} = -j15\sigma^6 (1 - j2\sigma^2 v)^{-7/2} \Rightarrow E[Z^3] = j^{-3}(-j15\sigma^6) = 15\sigma^6$$

$$\text{Thus, } E[X^2] = E[Z] = \sigma^2, E[X^4] = E[Z^2] = 3\sigma^4, E[X^6] = E[Z^3] = 15\sigma^6$$

8.4-19

$$\overline{R^2} = 2\sigma^2 = 32 \Rightarrow \sigma^2 = 16 \text{ so } p_R(r) = \frac{r}{16} e^{-r^2/32} u(r) \text{ and } P(R \leq r) = F_R(r) = (1 - e^{-r^2/32}) u(r)$$

$$\text{Thus, } P(R > 6) = 1 - P(R \leq 6) = e^{-6^2/32} = 0.325 \text{ and}$$

$$P(4.5 < R < 5.5) = P(R \leq 5.5) - P(R \leq 4.5) = e^{-4.5^2/32} - e^{-5.5^2/32} = 0.143$$

8.4-20

$$\overline{X^2} = 2\sigma^2 = 18 \Rightarrow \sigma^2 = 9 \text{ so } p_X(x) = \frac{x}{9} e^{-x^2/18} u(x) \text{ and } P(X \leq x) = (1 - e^{-x^2/18}) u(x)$$

$$\text{Thus, } P(X < 3) = P(X \leq 3) = 1 - e^{-3^2/18} = 0.393, P(X > 4) = 1 - P(X \leq 4) = e^{-4^2/18} = 0.411, \text{ and}$$

$$P(3 < X \leq 4) = P(X > 3) - P(X > 4) = [1 - P(X \leq 3)] - [1 - P(X \leq 4)] = e^{-3^2/18} - e^{-4^2/18} = 0.195$$

8.4-21

Since  $Z \succcurlyeq 0$  and  $X \succcurlyeq 0$ , monotonic transformation with  $g(x) = x^2$ ,  $g^{-1}(z) = +\sqrt{z}$ ,  $dg^{-1}/dz = 1/(2\sqrt{z})$

$$p_X(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} u(x), m = \overline{Z} = E[X^2] = 2\sigma^2. \text{ Thus}$$

$$p_Z(z) = \frac{\sqrt{z}}{\sigma^2} e^{-(\sqrt{z})^2/2\sigma^2} u(+\sqrt{z}) \frac{1}{2\sqrt{z}} = \frac{1}{m} e^{-z/m} u(z)$$

$$P(Z \leq km) = \int_0^{km} \frac{1}{m} e^{-z/m} dz = 1 - e^{-k} = \begin{cases} 0.632 & k=1 \\ 0.095 & k=0.1 \end{cases}$$

8.4-22

(a)  $A = R_1^2 = X^2 + Y^2$  where  $X$  and  $Y$  are gaussian with  $\bar{X} = \bar{Y} = 0$ ,  $\sigma_X^2 = \sigma_Y^2 = \sigma^2$

$$\Phi_A(v) = \Phi_{X^2}(v) \Phi_{Y^2}(v) = \left[ (1 - j2\sigma^2 v)^{-1/2} \right]^2 = (1 - j2\sigma^2 v)^{-1}$$

$$\Phi_A(2\pi t) = \frac{1}{1 + j2\sigma^2(-2\pi t)} = \frac{b}{b + j2\pi(-t)}, b = \frac{1}{2\sigma^2} \text{ so } p_A(a) = b e^{-ba} u(a) = \frac{1}{2\sigma^2} e^{-a/2\sigma^2} u(a)$$

(b)  $p_W(w) = p_{R_1^2}(w) * p_{R_2^2}(w)$ ,  $p_{R_1^2} = p_{R_2^2} = p_A$

$$= \begin{cases} 0 & w < 0 \\ \int_0^w \frac{1}{2\sigma^2} e^{-\lambda/2\sigma^2} \frac{1}{2\sigma^2} e^{-(w-\lambda)/2\sigma^2} d\lambda = \left( \frac{1}{2\sigma^2} \right)^2 e^{-w/2\sigma^2} \int_0^w d\lambda & w > 0 \end{cases}$$

$$\text{so } p_W(w) = \frac{w}{4\sigma^4} e^{-w/2\sigma^2} u(w)$$

8.4-23

$$\text{Let } a^2 = \frac{1}{2\sigma^2(1-\rho^2)} \text{ so } p_{XY}(x, y) = \frac{a}{\sqrt{2\pi^2\sigma^2}} e^{-a^2(x^2 - 2\rho xy + y^2)}$$

$$p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x, y) dx = \frac{a}{\sqrt{2\pi^2\sigma^2}} e^{-a^2 y^2} \int_{-\infty}^{\infty} e^{-a^2(x^2 - 2\rho xy)} dx = \frac{e^{-a^2 y^2}}{\sqrt{2\pi^2\sigma^2}} e^{a^2 \rho^2 y^2} 2 \int_0^{\infty} e^{-\lambda^2} d\lambda = \frac{e^{-y^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$$

$$p_X(x|y) = \frac{p_{XY}(x, y)}{p_Y(y)} = \frac{1}{\sqrt{2\pi\sigma^2(1-\rho^2)}} e^{-(x-\rho y)^2/2\sigma^2(1-\rho^2)}$$

8.4-24

Since  $Z$  is a linear combination of gaussian RVs,  $p_Z(z)$  is a gaussian PDF with

$$m_Z = E[X + 3Y] = m_X + 3m_Y = 0$$

$$\sigma_Z^2 = E[X^2 + 6XY + 9Y^2] = (\sigma_X^2 + m_X^2) + 6E[XY] + 9(\sigma_Y^2 + m_Y^2) = 100$$

8.4-25

$$E[X^n] = \int_{-\infty}^{\infty} x^n p_X(x) dx = 0 \text{ for } n \text{ odd, } E[Y] = E[X^2] = \sigma_X^2$$

$$E[(X - m_X)(Y - m_Y)] = E[X(X^2 - \sigma_X^2)] = E[X^3] - \sigma_X^2 E[X] = 0 \Rightarrow \rho = 0$$



## Chapter 9

9.1-1

$$E[e^{Xt}] = \frac{1}{2} \int_0^2 e^{xt} dx = \frac{1}{2t} (e^{2t} - 1), \quad \overline{v(t)} = E[6e^{Xt}] = \frac{3}{t} (e^{2t} - 1)$$

$$R_v(t_1, t_2) = E[36e^{X(t_1+t_2)}] = \frac{18}{t_1+t_2} [e^{2(t_1+t_2)} - 1], \quad \overline{v^2(t)} = \frac{9}{t} (e^{4t} - 1)$$

9.1-2

$$E[\cos Xt] = \frac{1}{2} \int_0^2 \cos xt dx = \frac{1}{2t} \sin 2t, \quad \overline{v(t)} = E[6 \cos Xt] = \frac{3}{t} \sin 2t$$

$$\begin{aligned} R_v(t_1, t_2) &= E[36 \cos X t_1 \cos X t_2] = 18 E[\cos X(t_1 - t_2) + \cos X(t_1 + t_2)] \\ &= 18 \left[ \frac{\sin 2(t_1 - t_2)}{2(t_1 - t_2)} + \frac{\sin 2(t_1 + t_2)}{2(t_1 + t_2)} \right] \end{aligned}$$

$$\overline{v^2(t)} = 18 \left( 1 + \frac{\sin 4t}{4t} \right)$$

9.1-3

$$\overline{X} = 0, \quad \overline{X^2} = 1/3, \quad \overline{v(t)} = E[Y + 3Xt]t = \overline{Y}t + 3\overline{X}t^2 = 2t$$

$$\begin{aligned} R_v(t_1, t_2) &= E[Y^2 + 3YX(t_1 + t_2) + 9X^2 t_1 t_2]_{t_1 t_2} = [\overline{Y^2} + 3\overline{X}\overline{Y}(t_1 + t_2) + 9\overline{X^2} t_1 t_2]_{t_1 t_2} = 6t_1 t_2 + 3(t_1 t_2)^2 \\ \overline{v^2(t)} &= 6t^2 + 3t^4 \end{aligned}$$

9.1-4

$$E[e^{Xt}] = \frac{1}{2} \int_{-1}^1 e^{xt} dx = \frac{1}{2t} (e^t - e^{-t}), \quad \overline{v(t)} = E[Y e^{Xt}] = \overline{Y} E[e^{Xt}] = \frac{1}{t} (e^t - e^{-t})$$

$$R_v(t_1, t_2) = E[Y^2 e^{X(t_1+t_2)}] = \overline{Y^2} E[e^{X(t_1+t_2)}] = \frac{3}{t_1+t_2} [e^{(t_1+t_2)} - e^{-(t_1+t_2)}], \quad \overline{v^2(t)} = \frac{3}{2t} (e^{2t} - e^{-2t})$$

9.1-5

$$E[\cos Xt] = \frac{1}{2} \int_{-1}^1 \cos xt dx = \frac{\sin t}{t}, \quad \overline{v(t)} = E[Y \cos Xt] = \overline{Y} E[\cos Xt] = 2 \frac{\sin t}{t}$$

(cont.)

$$R_v(t_1, t_2) = E[Y^2 \cos X t_1 \cos X t_2] = \frac{1}{2} \overline{Y^2} E[\cos X(t_1 - t_2) + \cos X(t_1 + t_2)]$$

$$= 3 \left[ \frac{\sin(t_1 - t_2)}{t_1 - t_2} + \frac{\sin(t_1 + t_2)}{t_1 + t_2} \right]$$

$$\overline{v^2(t)} = 3 \left( 1 + \frac{\sin 2t}{2t} \right)$$

9.1-6

$$p_F \Phi(f, \varphi) = \frac{1}{2\pi} p_F(f), \quad 0 \leq \varphi \leq 2\pi, \quad R_v(t_1, t_2) = A^2 \int_{-\infty}^{\infty} g(f) p_F(f) df \text{ where}$$

$$\begin{aligned} g(f) &= \frac{1}{2\pi} \int_0^{2\pi} \cos(2\pi f t_1 + \varphi) \cos(2\pi f t_2 + \varphi) d\varphi \\ &= \frac{1}{4\pi} \int_0^{2\pi} \cos 2\pi f(t_1 - t_2) d\varphi + \frac{1}{4\pi} \int_0^{2\pi} \cos[2\pi f(t_1 + t_2) + 2\varphi] d\varphi = \frac{1}{2} \cos 2\pi f(t_1 - t_2) \end{aligned}$$

$$\text{Thus, with } f = \lambda, \quad R_v(t_1, t_2) = \frac{A^2}{2} \int_{-\infty}^{\infty} \cos 2\pi \lambda(t_1 - t_2) p_F(\lambda) d\lambda$$

$$\overline{v(t)} = A \int_{-\infty}^{\infty} \left[ \frac{1}{2\pi} \int_0^{2\pi} \cos(2\pi f t + \varphi) d\varphi \right] p_F(f) df = 0, \quad \overline{v^2(t)} = R_v(t, t) = \frac{A^2}{2} \int_{-\infty}^{\infty} p_F(f) df = \frac{A^2}{2}$$

9.1-7

$$\begin{aligned} v(t_1)w(t_2) &= XY(\cos \omega_0 t_1 \cos \omega_0 t_2 - \sin \omega_0 t_1 \sin \omega_0 t_2) - X^2 \cos \omega_0 t_1 \sin \omega_0 t_2 \\ &\quad + Y^2 \sin \omega_0 t_1 \cos \omega_0 t_2 \end{aligned}$$

$$E[XY] = \overline{XY} = 0, \quad E[X^2] = E[Y^2] = \sigma^2, \text{ so}$$

$$R_{vw}(t_1, t_2) = E[v(t_1)w(t_2)] = \sigma^2 (\sin \omega_0 t_1 \cos \omega_0 t_2 - \cos \omega_0 t_1 \sin \omega_0 t_2) = \sigma^2 \sin \omega_0(t_1 - t_2)$$

9.1-8

$$(a) \quad \overline{v(t)} = E[X \cos \omega_0 t + Y \sin \omega_0 t] = \overline{X} \cos \omega_0 t + \overline{Y} \sin \omega_0 t = 0$$

$$\begin{aligned} R_v(t_1, t_2) &= E[X^2 \cos \omega_0 t_1 \cos \omega_0 t_2 + XY(\cos \omega_0 t_1 \sin \omega_0 t_2 + \sin \omega_0 t_1 \cos \omega_0 t_2) + Y^2 \sin \omega_0 t_1 \sin \omega_0 t_2] \\ &= \overline{X^2} \cos \omega_0 t_1 \cos \omega_0 t_2 + \overline{Y^2} \sin \omega_0 t_1 \sin \omega_0 t_2 = \sigma^2 \cos \omega_0(t_1 - t_2) \end{aligned}$$

$$\text{so } R_v(\tau) = \sigma^2 \cos \omega_0 \tau$$

$$(b) \quad \overline{v^2(t)} = R_v(0) = \sigma^2 \quad \text{(cont.)}$$

$$\begin{aligned}
\langle v_i^2(t) \rangle &= \langle (X_i \cos \omega_0 t + Y_i \sin \omega_0 t)^2 \rangle = X_i^2 \langle \cos^2 \omega_0 t \rangle + 2X_i Y_i \langle \cos \omega_0 t \sin \omega_0 t \rangle \\
&\quad + Y_i^2 \langle \sin^2 \omega_0 t \rangle \\
&= \frac{1}{2} X_i^2 + \frac{1}{2} Y_i^2 \neq \sigma^2
\end{aligned}$$

9.1-9

$$(a) \quad \overline{v(t)} = \int_{-\infty}^{\infty} a p_A(a) da \int_0^{2\pi} \cos(\omega_0 t + \varphi) \frac{d\varphi}{2\pi} = \overline{A} \times 0 = 0$$

$$R_v(t_1, t_2) = \int_{-\infty}^{\infty} a^2 p_A(a) da \int_0^{2\pi} \cos(\omega_0 t_1 + \varphi) \cos(\omega_0 t_2 + \varphi) \frac{d\varphi}{2\pi} = \overline{A^2} \times \frac{1}{2} \cos \omega_0(t_1 - t_2)$$

$$\text{so } R_v(\tau) = \frac{\overline{A^2}}{2} \cos \omega_0 \tau$$

$$(b) \quad \overline{v^2(t)} = \overline{A^2} / 2, \quad \langle v_i^2(t) \rangle = \langle A_i^2 \cos^2(\omega_0 t + \Phi_i) \rangle = A_i^2 \langle \cos^2(\omega_0 t + \Phi_i) \rangle = A_i^2 / 2 \neq \overline{A^2} / 2$$

9.1-10

$$m_z = \overline{z(t)} = E[v(t) - v(t+T)] = \overline{v(t)} - \overline{v(t+T)} = 0$$

$$\sigma_z^2 = \overline{z^2(t)} = \overline{v^2(t)} - 2\overline{v(t)v(t+T)} + \overline{v^2(t+T)} = R_v(0) - 2R_v(T) + R_v(0) = 2[R_v(0) - R_v(T)]$$

9.1-11

$$m_z = \overline{z(t)} = E[v(t) + v(t-T)] = \overline{v(t)} + \overline{v(t-T)} = 2\sqrt{R_v(\pm\infty)}$$

$$\overline{z^2(t)} = \overline{v^2(t)} + 2\overline{v(t)v(t-T)} + \overline{v^2(t-T)} = R_v(0) + 2R_v(T) + R_v(0),$$

$$\sigma_z^2 = 2[R_v(0) + R_v(T) - 2R_v(\pm\infty)]$$

9.2-1

$$F \left[ e^{-(\sqrt{\pi} b t)^2} \right] = \frac{1}{b} e^{-(\sqrt{\pi} f / b)^2} \quad \text{so } G_v(f) = 2\sqrt{\pi} e^{-(\pi f / 8)^2} + 9\delta(f)$$

$$\langle v(t) \rangle = \sqrt{R_v(\pm\infty)} = \pm 3, \quad \langle v^2(t) \rangle = R_v(0) = 25, \quad v_{\text{rms}} = \sqrt{25 - 9} = 4$$

9.2-2

$$G_v(f) = \frac{32}{8} \Lambda \left( \frac{f}{8} \right) + \frac{4}{2} [\delta(f-8) + \delta(f+8)]$$

$$\langle v(t) \rangle = m_v = 0, \quad \langle v^2(t) \rangle = R_v(0) = 36, \quad v_{\text{rms}} = \sqrt{36 - 0} = 6$$

9.2-3

$$R_v(\tau) = \frac{A^2}{2} \int_{-\infty}^{\infty} \cos 2\pi\lambda\tau p_F(\lambda) d\lambda,$$

$$\begin{aligned} G_v(f) &= F_{\tau}[R_v(\tau)] = \frac{A^2}{2} \int_{-\infty}^{\infty} F_{\tau}[\cos 2\pi\lambda\tau] p_F(\lambda) d\lambda = \frac{A^2}{2} \int_{-\infty}^{\infty} \frac{1}{2} [\delta(f-\lambda) + \delta(f+\lambda)] p_F(\lambda) d\lambda \\ &= \frac{A^2}{4} [p_F(f) + p_F(-f)] \end{aligned}$$

$$p_F(f) = \delta(f-f_0) \Rightarrow G_v(f) = \frac{A^2}{4} [\delta(f-f_0) + \delta(f+f_0)] \quad \text{since } \delta(-f-f_0) = \delta(f+f_0)$$

9.2-4

$$\begin{aligned} (a) \quad E[\tilde{G}_v(f)] &= \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) R_v(\tau) e^{-j\omega\tau} d\tau = \int_{-\infty}^{\infty} \Lambda\left(\frac{\tau}{T}\right) R_v(\tau) e^{-j\omega\tau} d\tau \\ &= F_{\tau}\left[\Lambda\left(\frac{\tau}{T}\right) R_v(\tau)\right] = (T \operatorname{sinc}^2 fT) * G_v(f) \end{aligned}$$

$$(b) \quad \lim_{T \rightarrow \infty} \Lambda\left(\frac{\tau}{T}\right) = 1 \quad \text{so} \quad \lim_{T \rightarrow \infty} T \operatorname{sinc}^2 fT = F_{\tau}[1] = \delta(f) \quad \text{and}$$

$$\lim_{T \rightarrow \infty} E[\tilde{G}_v(f)] = \delta(f) * G_v(f) = G_v(f)$$

9.2-5

$$v_T(t) = A \cos(\omega_0 t + \Phi) \Pi\left(\frac{t}{T}\right), \quad V_T(f, s) = \frac{AT}{2} [\operatorname{sinc}(f-f_0)T e^{j\Phi} + \operatorname{sinc}(f+f_0)T e^{-j\Phi}]$$

$$E[|V_T(f, s)|^2] = \frac{A^2 T^2}{4} \{ \operatorname{sinc}^2(f-f_0)T + \operatorname{sinc}^2(f+f_0)T + E[e^{j2\Phi} + e^{-j2\Phi}] \operatorname{sinc}(f-f_0)T \operatorname{sinc}(f+f_0)T \}$$

$$\text{But } E[e^{j2\Phi} + e^{-j2\Phi}] = E[2\cos 2\Phi] = 0 \quad \text{and} \quad \lim_{T \rightarrow \infty} T \operatorname{sinc}^2 fT = \delta(f), \text{ so}$$

$$G_v(f) = \lim_{T \rightarrow \infty} \frac{A^2}{4} [T \operatorname{sinc}^2(f-f_0)T + T \operatorname{sinc}^2(f+f_0)T] = \frac{A^2}{4} [\delta(f-f_0) + \delta(f+f_0)]$$

9.2-6

$$R_{vw}(t_1, t_2) = E[v(t_1)w(t_2)] = m_v m_w \quad \text{so} \quad R_{vw}(\tau) = R_{wv}(\tau) = m_v m_w$$

$$R_z(\tau) = R_v(\tau) + R_w(\tau) \pm 2m_v m_w, \quad G_z(f) = G_v(f) + G_w(f) \pm 2m_v m_w \delta(f)$$

(cont.)

$$R_z(\pm\infty) = R_v(\pm\infty) + R_w(\pm\infty) \pm 2m_v m_w = m_v^2 + m_w^2 \pm 2m_v m_w = (m_v \pm m_w)^2$$

$$\overline{z^2} = R_z(0) = R_v(0) + R_w(0) \pm 2m_v m_w = \overline{v^2} + \overline{w^2} \pm 2m_v m_w$$

$$= \sigma_v^2 + m_v^2 + \sigma_w^2 + m_w^2 \pm 2m_v m_w = \sigma_v^2 + \sigma_w^2 + (m_v \pm m_w)^2 > 0$$

9.2-7

$$R_{vw}(t_1, t_2) = E[w(t_1)v(t_2)] = E[v(t_2)w(t_1)] = R_{vw}(t_2 - t_1) \quad \text{so} \quad R_{vw}(\tau) = R_{vw}(-\tau)$$

$$G_{vw}(f) = F_\tau[R_{vw}(-\tau)] = \frac{1}{|-1|} G_{vw}(-f) = G_{vw}(-f)$$

9.2-8

$$\begin{aligned} R_z(t_1, t_2) &= E[v(t_1)v(t_2) + v(t_1 + T)v(t_2 + T) - v(t_1 + T)v(t_2) - v(t_1)v(t_2 + T)] \\ &= R_v(t_1 - t_2) + R_v(t_1 + T - t_2 - T) - R_v(t_1 + T - t_2) - R_v(t_1 - t_2 - T) \quad \text{so} \end{aligned}$$

$$R_z(\tau) = 2R_v(\tau) - R_v(\tau + T) - R_v(\tau - T) \quad \text{and}$$

$$G_z(f) = 2G_v(f) - G_v(f)(e^{j\omega T} + e^{-j\omega T}) = 2G_v(f)(1 - \cos 2\pi f T)$$

9.2-9

$$\begin{aligned} R_z(t_1, t_2) &= E[v(t_1)v(t_2) + v(t_1 - T)v(t_2 - T) + v(t_1)v(t_2 - T) + v(t_1 - T)v(t_2)] \\ &= R_v(t_1 - t_2) + R_v(t_1 - T - t_2 + T) + R_v(t_1 - t_2 + T) + R_v(t_1 - T - t_2) \quad \text{so} \end{aligned}$$

$$R_z(\tau) = 2R_v(\tau) + R_v(\tau + T) + R_v(\tau - T) \quad \text{and}$$

$$G_z(f) = 2G_v(f) + G_v(f)(e^{j\omega T} + e^{-j\omega T}) = 2G_v(f)(1 + \cos 2\pi f T)$$

9.2-10

$$z(t) = v(t) \cos(2\pi f_2 t + \Phi_2) \quad \text{with} \quad v(t) = A \cos(2\pi f_1 t + \Phi_1) \quad \text{so}$$

$$G_v(f) = (A^2/2)[\delta(f - f_1) + \delta(f + f_1)]$$

$$\text{Thus, } G_z(f) = \frac{A^2}{16}[\delta(f - f_1 - f_2) + \delta(f + f_1 - f_2) + \delta(f - f_1 + f_2) + \delta(f + f_1 + f_2)]$$

$$\text{For } f_1 = f_2, \quad G_z(f) = \frac{A^2}{16}[2\delta(f) + \delta(f - 2f_2) + \delta(f + 2f_2)]$$

9.2-11

$$R_y(t_1, t_2) = E[y(t_1)y(t_2)], \quad y(t_2) = \int_{-\infty}^{\infty} h(\lambda)x(t_2 - \lambda) d\lambda \text{ so}$$

$$R_y(t_1, t_2) = \int_{-\infty}^{\infty} h(\lambda)E[y(t_1)x(t_2 - \lambda)] d\lambda \quad (\text{cont.})$$

$$\text{But } E[y(t_1)x(t_2 - \lambda)] = R_{yx}(t_1, t_2 - \lambda) = R_{yx}(t_1 - t_2 + \lambda) = R_{yx}(\tau + \lambda) \text{ so}$$

$$R_y(\tau) = \int_{-\infty}^{\infty} h(\lambda)R_{yx}(\tau + \lambda) d\lambda = \int_{-\infty}^{\infty} h(-\mu)R_{yx}(\tau - \mu) d\mu = h(-\tau) * R_{yx}(\tau)$$

9.2-12

$$R_y(\tau) = F_{\tau}^{-1}[(2\pi f)^2 G_x(f)] = -F_{\tau}^{-1}[(j2\pi f)^2 G_x(f)] = -d^2 R_x(\tau) / d\tau^2$$

$$G_{yx}(f) = F_{\tau}[h(\tau) * R_x(\tau)] = H(f)G_x(f) \text{ where } H(f) = j2\pi f,$$

$$\text{so } R_{yx}(\tau) = F_{\tau}^{-1}[(j2\pi f)G_x(f)] = dR_x(\tau) / d\tau$$

9.2-13

$$\text{If } x(t) \text{ is deterministic, then } Y(f) = X(f) - \alpha X(f)e^{-j\omega T} \Rightarrow H(f) = 1 - \alpha e^{-j\omega T}$$

$$|H(f)|^2 = 1 + \alpha^2 - \alpha(e^{j\omega T} + e^{-j\omega T}) = 1 + \alpha^2 - 2\alpha \cos \omega T \text{ so}$$

$$G_y(f) = (1 + \alpha^2 - 2\alpha \cos \omega T)G_x(f), \quad R_y(\tau) = (1 + \alpha^2)R_x(\tau) - \alpha[R_x(\tau + T) + R_x(\tau - T)]$$

9.2-14

$$R_{\hat{x}}(\tau) = h_Q(\tau) * R_x(\tau) = \int_{-\infty}^{\infty} \frac{1}{\pi(\tau - \lambda)} R_x(\lambda) d\lambda$$

$$\begin{aligned} R_{\hat{x}}(\tau) &= R_{\hat{x}}(-\tau) = \int_{-\infty}^{\infty} \frac{1}{\pi(-\tau - \lambda)} R_x(\lambda) d\lambda = -\int_{-\infty}^{\infty} \frac{1}{\pi\mu} R_x(\mu - \tau) d\mu = -\int_{-\infty}^{\infty} \frac{1}{\pi\mu} R_x(\tau - \mu) d\mu \\ &= -[h_Q(\tau) * R_x(\tau)] = -\hat{R}_x(\tau) \end{aligned}$$

9.2-15

$$\text{Let } x(t) = \delta(t) \text{ so } y(t) = h(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \delta(\lambda) d\lambda = \begin{cases} 1/T & t-T/2 < 0 < t+T/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Thus, } h(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & |t| > T/2 \end{cases} \Rightarrow H(f) = \text{sinc } fT, \text{ and}$$

$$R_y(\tau) = F_{\tau}^{-1} [\text{sinc}^2 fT G_x(f)] = \frac{1}{T} \Lambda\left(\frac{\tau}{T}\right) * R_x(\tau) = \frac{1}{T} \int_{-T}^T \left(1 - \frac{|\lambda|}{T}\right) R_x(\tau - \lambda) d\lambda$$

9.3-1

$$\text{Let } x = h|f|/kT, \quad e^x - 1 = x + \frac{1}{2}x^2 + \dots \approx x \left(1 + \frac{1}{2}x\right) \text{ for } |x| \ll 1. \text{ Then}$$

$$(e^x - 1)^{-1} \approx \frac{1}{x} \left(1 + \frac{1}{2}x\right)^{-1} = \frac{1}{x} \left(1 - \frac{1}{2}x + \frac{1}{2}x^2 + \dots\right) \approx \frac{1}{x} \left(1 - \frac{1}{2}x\right), \text{ so}$$

$$G_y(f) \approx \frac{2Rh|f|}{h|f|/kT} \left(1 - \frac{1}{2} \frac{h|f|}{kT}\right) = 2RkT \left(1 - \frac{h|f|}{2kT}\right)$$

9.3-2

$$R_{yx}(\tau) = h(\tau) * \frac{N_0}{2} \delta(\tau) = \frac{N_0}{2} h(\tau)$$

$$R_y(\tau) = h(-\tau) * \frac{N_0}{2} h(\tau) = \frac{N_0}{2} \int_{-\infty}^{\infty} h(-\lambda) h(\tau - \lambda) d\lambda = \frac{N_0}{2} \int_{-\infty}^{\infty} h(t) h(t + \tau) dt$$

9.3-3

$$G_y(f) = \frac{N_0 T^2}{2} \text{sinc}^2 fT, \quad R_y(\tau) = \frac{N_0 T}{2} \Lambda\left(\frac{\tau}{T}\right), \quad \overline{y^2} = R_y(0) = \frac{N_0 T}{2}$$

9.3-4

$$G_y(f) = \frac{N_0 K^2}{2} e^{-2(af)^2} = \frac{N_0 K^2}{2} e^{-\pi(f/\sqrt{\pi/2a^2})^2},$$

$$R_y(\tau) = \frac{N_0 K^2}{a} \sqrt{\frac{\pi}{8}} e^{-\pi\tau^2/\sqrt{\pi/2a^2}}, \quad \overline{y^2} = R_y(0) = \frac{N_0 K^2}{a} \sqrt{\frac{\pi}{8}}$$

9.3-5

$$G_y(f) = \frac{N_0 K^2}{2} \left[ \Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right) \right],$$

$$R_y(\tau) = N_0 K^2 B \text{sinc}^2 B \tau \cos 2\pi f_0 \tau, \quad \overline{y^2} = R_y(0) = N_0 K^2 B$$

9.3-6

$$G_y(f) = \frac{N_0 K^2}{2} \left[ 1 - \Pi\left(\frac{f}{2f_0}\right) \right], \quad R_y(\tau) = \frac{N_0 K^2}{2} [\delta(\tau) - 2f_0 \text{sinc} 2f_0 \tau], \quad \overline{y^2} = R_y(0) = \infty$$

9.3-7

$$H(f) = \frac{R}{R + j\omega L} = \frac{1}{1 + j(f/B)}, \quad B = \frac{R}{2\pi L}, \quad \text{so } G_y(f) = |H(f)|^2 G_x(f) = \frac{N_{0v}/2}{1 + (f/B)^2}$$

$$R_y(\tau) = \frac{N_{0v}}{2} \pi B e^{-2\pi B |\tau|} = \frac{N_{0v} R}{4L} e^{-R|\tau|/L}, \quad \overline{y^2} = R_y(0) = \frac{N_{0v} R}{4L}$$

9.3-8

$$H(f) = \frac{j\omega L}{R + j\omega L} = \frac{j2\pi f}{b + j2\pi f}, \quad b = \frac{R}{L}, \quad \text{so}$$

$$G_y(f) = |H(f)|^2 G_x(f) = \frac{N_{0v}}{2} \frac{(2\pi f)^2}{b^2 + (2\pi f)^2} = -\frac{N_{0v}}{4b} (j2\pi f)^2 \frac{2b}{b^2 + (2\pi f)^2}.$$

$$R_y(\tau) = -\frac{N_{0v}}{4b} \frac{d^2}{d\tau^2} [e^{-b|\tau|}] = -\frac{N_{0v}}{4b} \frac{d}{d\tau} [-be^{-b\tau} u(\tau) + be^{b\tau} u(-\tau)]$$

$$= -\frac{N_{0v}}{4b} [b^2 e^{-b\tau} u(\tau) - b\delta(\tau) + b^2 e^{b\tau} u(-\tau) - b\delta(-\tau)]$$

$$= \frac{N_{0v}}{4} [2\delta(\tau) - be^{-b|\tau|}], \quad \overline{y^2} = R_y(0) = \infty$$

9.3-9

$$\overline{i^2} = \int_{-\infty}^{\infty} G_i(f) df = N_{0v} \int_0^{\infty} \frac{df}{R^2 + (2\pi fL)^2} = \frac{N_{0v}}{4LR}. \quad \text{Thus, } \frac{1}{2} L \frac{N_{0v}}{4LR} = \frac{1}{2} kT \Rightarrow N_{0v} = 4RkT$$



9.3-10

$y$  is gaussian with  $\bar{y} = 0$ ,  $\overline{y^2} = \sigma^2 = 4RkT_0B = 4 \times 10^{-10}$

$$\bar{z} = \int_{-\infty}^{\infty} |y| p_Y(y) dy = 2 \int_0^{\infty} \frac{y}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} dy = \sqrt{\frac{2}{\pi}} \sigma \int_0^{\infty} e^{-\lambda^2} d(\lambda^2) = \sqrt{\frac{2}{\pi}} \sigma \approx 16 \mu V$$

$$z^2 = y^2 \text{ so } \bar{z^2} = \overline{y^2} = \sigma^2, \quad \sigma_z = \sqrt{\sigma^2 - \left(\sqrt{\frac{2}{\pi}} \sigma\right)^2} \approx 12 \mu V$$

9.3-11

$y$  is gaussian with  $\bar{y} = 0$ ,  $\overline{y^2} = \sigma^2 = 4RkT_0B = 4 \times 10^{-10}$  and  $z = y u(y)$

(cont.)

$$\bar{z} = \int_0^{\infty} y p_Y(y) dy = \int_0^{\infty} \frac{y}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} dy = \frac{\sigma}{\sqrt{2\pi}} \int_0^{\infty} e^{-\lambda^2} d(\lambda^2) = \frac{\sigma}{\sqrt{2\pi}} \approx 8 \mu V$$

$$\bar{z^2} = \int_0^{\infty} \frac{y^2}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} dy = \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} \lambda^2 e^{-\lambda^2} d\lambda = \frac{\sigma^2}{2}, \quad \sigma_z = \sqrt{\frac{\sigma^2}{2} - \left(\frac{\sigma}{\sqrt{2\pi}}\right)^2} \approx 12 \mu V$$

9.3-12

$$\bar{y} = \bar{z} = 0, \quad \overline{y^2} = \overline{z^2} = \sigma^2 = 4RkT_0B = 4 \times 10^{-10}$$

$$E[(Y - m_Y)(Z - m_Z)] = E[y(t)y(t-T)] = R_y(T) = \sigma^2 \text{sinc } 2BT = 0 \text{ since } 2BT = 5$$

$$\text{Thus, } \rho = 0 \text{ and } p_{YZ}(y, z) = \frac{1}{2\pi\sigma^2} e^{(y^2+z^2)/2\sigma^2}$$

9.3-13

$$\bar{y} = \bar{z} = 0, \quad \overline{y^2} = \overline{z^2} = \sigma^2 = 4RkT_0B = 4 \times 10^{-10}$$

$$E[(Y - m_Y)(Z - m_Z)] = E[y(t)y(t-T)] = R_y(T) = \sigma^2 \text{sinc } 2BT \text{ so } \rho = \frac{\sigma^2}{\sigma\sigma} \text{sinc } 0.5 = 0.637$$

$$\text{Thus, } p_{YZ}(y, z) = \frac{1}{2\pi 0.77\sigma^2} e^{-(y^2 + z^2 - 1.274 yz)/1.19\sigma^2}$$

9.3-14

$$g = |H(0)|^2 = K^2, \quad B_N = \int_0^\infty e^{-2a^2 f^2} df = \frac{\sqrt{\pi}}{2\sqrt{2}a}$$

$$\text{At } f = B, |H(B)|^2 = K^2 e^{-2a^2 B^2} = \frac{K^2}{2} \Rightarrow B = \sqrt{\frac{\ln 2}{2}} \frac{1}{a} \text{ so } \frac{B_N}{B} = \frac{\sqrt{\pi}}{2\sqrt{\ln 2}} = 1.06$$

9.3-15

$$X_T(f, s) = \int_{-T/2}^{T/2} \left[ \sum_{k=-K_1}^{K_2} A_k \delta(t - T_k) \right] e^{-j\omega t} dt = \sum_{k=-K_1}^{K_2} A_k e^{-j\omega T_k} \quad \text{where } T_{-K_1} > -\frac{T}{2} \text{ and } T_{K_2} < \frac{T}{2}$$

$$|X_T(f, s)|^2 = \sum_k \sum_m A_k A_m e^{-j\omega(T_k - T_m)}, \quad E[|X_T(f, s)|^2] = \sum_k \sum_m E[A_k A_m] E[e^{-j\omega(T_k - T_m)}]$$

$$\text{where } E[A_k A_m] = \begin{cases} \sigma^2 & m = k \\ 0 & m \neq k \end{cases}. \text{ So } E[|X_T(f, s)|^2] = \sum_k \sigma^2 E[e^{-j\omega(T_k - T_k)}] = \sigma^2 (K_1 + K_2)$$

with  $K_1 + K_2 = \text{expected number of impulses in } T \text{ seconds} = \mu T$  (cont.)

$$\text{Thus, } G_x(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \sigma^2 \mu T = \mu \sigma^2$$

9.4-1

$$10 \log_{10} \left( \frac{T_N}{T_0} W \right) = 10 \log_{10} (4 \times 10^6) \approx 66 \text{ dB}, \quad S_R = 2 \times 10^{-5} \text{ mW} \approx -47 \text{ dBm}$$

$$(S/N)_{\text{dB}} \approx -47 + 174 - 66 = 61 \text{ dB}$$

9.4-2

$$10 \log_{10} \left( \frac{T_N}{T_0} W \right) = 10 \log_{10} (5 \times 2 \times 10^6) = 70 \text{ dB}, \quad S_R = 4 \times 10^{-6} \text{ mW} \approx -54 \text{ dBm}$$

$$(S/N)_{\text{dB}} \approx -54 + 174 - 70 = 50 \text{ dB}$$

9.4-3

$$S_T/LN_0W = 46 \text{ dB} = 4 \times 10^4 \Rightarrow LN_0 = 5 \times 10^{-10}$$

$$(a) \quad W = 20 \text{ kHz}, (S/N)_D = 55\text{-}65 \text{ dB}, S_{T_{\text{dBm}}} = (S/N)_D - 10\log_{10}(5 \times 10^{-10} \times 20 \times 10^3) + 30 =$$

$$35 \text{ to } 45 \text{ dBm}, S_T = 3.2\text{-}32 \text{ W}$$

$$(b) \quad W = 3.2 \text{ kHz}, (S/N)_D = 25\text{-}35 \text{ dB}, S_{T_{\text{dBm}}} = (S/N)_D - 10\log_{10}(5 \times 10^{-10} \times 3.2 \times 10^3) + 30 =$$

$$-3 \text{ to } +7 \text{ dBm}, S_T = 0.5\text{-}5 \text{ mW}$$

9.4-4

$$(S/N)_D = S_R/N_0B_N = (W/B_N)(S_R/N_0W)$$

$$(a) \quad B_N = \frac{\pi}{2} B = 23.6 \text{ kHz} = 2.36W \Rightarrow (S/N)_D = 0.424(S_R/N_0W)$$

$$(b) \quad B_N = \frac{\pi B}{4 \sin \pi/4} = 13.4 \text{ kHz} = 1.34W \Rightarrow (S/N)_D = 0.746(S_R/N_0W)$$

9.4-5

$$S_D = \int_{-\infty}^{\infty} |H_C(f)|^2 |H_R(f)|^2 G_x(f) df = K^2 \int_{-\infty}^{\infty} G_x(f) df = K^2 S_T$$

$$N_D = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_R(f)|^2 df = N_0 K^2 L \int_0^W \left[ 1 + \left( \frac{f}{W} \right)^2 \right] df = N_0 K^2 L \frac{3W}{2} \text{ so } \left( \frac{S}{N} \right)_D = \frac{2}{3} \frac{S_T}{LN_0W}$$

9.4-6

$$S_D = \int_{-\infty}^{\infty} |H_C(f)|^2 |H_R(f)|^2 G_x(f) df = K^2 \int_{-\infty}^{\infty} G_x(f) df = K^2 S_T$$

$$N_D = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_R(f)|^2 df = N_0 K^2 L \int_0^W \left[ 1 + \left( \frac{2f}{W} \right)^4 \right] df = N_0 K^2 L \frac{21W}{5} \text{ so } \left( \frac{S}{N} \right)_D = \frac{5}{21} \frac{S_T}{LN_0W}$$

9.4-7

$$(a) \quad S_{T_{\text{dBm}}} - L + 174 - 10\log_{10}(10 \times 5 \times 10^3) = 60 \text{ dB},$$

$$L = 3 \times 40 = 120 \text{ dB}, S_T = 53 \text{ dBm} = 200 \text{ W}$$

$$(b) \quad L_1 = 60 \text{ dB} = 10^6, L = 120 \text{ dB} = 10^{12}, S_T = \frac{2L_1}{L} \times 200 \text{ W} = 0.4 \text{ mW}$$

9.4-8

$$L_1 = \frac{240}{6} = 40 \text{ dB} = 10^4, \left( \frac{S}{N} \right)_D = \frac{1}{6 \times 10^4} \frac{S_T}{N_0 W} = 10^3 \Rightarrow \frac{S_T}{N_0 W} = 6 \times 10^7$$

$$(a) L_1 = 20 \text{ dB} = 100, \left( \frac{S}{N} \right)_D = \frac{1}{12 \times 100} \times 6 \times 10^7 = 5 \times 10^4 = 47 \text{ dB}$$

$$(b) L_1 = 60 \text{ dB} = 10^6, \left( \frac{S}{N} \right)_D = \frac{1}{4 \times 10^6} \times 6 \times 10^7 = 15 \approx 12 \text{ dB}$$

9.4-9

$$L = 0.5 \times 400 = 200 \text{ dB}, L_1 = 200/m \text{ dB},$$

$$\left( \frac{S}{N} \right)_D = 10 \log_{10} \left( \frac{S_T}{m L_1 N_0 W} \right) = 80 - 10 \log_{10} m - \frac{200}{m} \geq 30 \text{ dB so } \log_{10} m + \frac{20}{m} \leq 5$$

$$\frac{m}{10} \log m + 20/m$$

$$10 \quad 1.0 + 2 = 3$$

$$5 \quad 0.7 + 4 = 4.7 \Rightarrow m_{\min} = 5$$

$$4 \quad 0.6 + 5 = 5.6$$

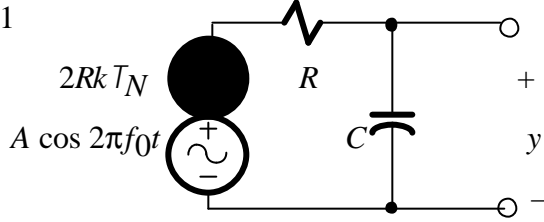
9.4-10

$$L_{1\text{dB}} = \frac{L_{\text{dB}}}{m} \Rightarrow L_1 = L^{1/m}, \text{ so } \left( \frac{S}{N} \right)_D = K m^{-1} L^{-m^{-1}} \text{ where } K = \frac{S_T}{N_0 W} \quad (\text{cont.})$$

$$\frac{d}{dm} \left( \frac{S}{N} \right)_D = K \left[ (-m)^{-2} L^{-m^{-1}} + m^{-1} L^{-m^{-1}} (\ln L)(m^{-2}) \right] = 0 \text{ so}$$

$$m = \ln L = \frac{\ln 10}{10} (10 \log_{10} L) = 0.23 L_{\text{dB}}$$

9.4-11



$$N = kT_N/C \text{ (from Example 9.3-1)}$$

$$S = (A^2/2)[1 + (2\pi f_0 RC)^2]^{-1}$$

$$\frac{S}{N} = \frac{A^2}{2kT_N} \frac{C}{1 + (2\pi f_0 RC)^2} \quad \text{so} \quad \frac{d}{dC} \left( \frac{S}{N} \right) = \frac{A^2}{2kT_N} \frac{1 + (2\pi f_0 RC)^2 - (2\pi f_0 RC)^2 2C \times C}{[1 + (2\pi f_0 RC)^2]^2} = 0$$

$$\text{and } C = \frac{1}{2\pi f_0 R}$$

9.5-1

$$\left( \frac{\sigma_A}{A} \right)^2 = \frac{N_0 B_N}{A^2} = \frac{kT_N B_N \tau}{E_p} = \frac{4 \times 10^{-21} \times 1}{10^{-20}} = 0.4$$

9.5-2

$$\sigma_i^2 = \frac{t_r^2 N_0 B_N}{A^2} = \frac{t_r^2 N_0 B_N \tau}{A^2 \tau} \Rightarrow \left| \frac{\sigma_i}{t_r} \right| = \sqrt{\frac{N_0 B_N \tau}{E_p}}$$

$$\sigma_A^2 = N_0 B_N \Rightarrow \left| \frac{\sigma_A}{A} \right| = \sqrt{\frac{N_0 B_N}{A^2}} = \sqrt{\frac{N_0 B_N \tau}{E_p}}$$

9.5-3

$$\text{Take } B_N \approx 1/2\tau = 100 \text{ kHz} \ll B_T, \text{ so } \sigma_A^2 \approx \frac{N_0}{2E_p} A^2 \leq \left( \frac{A}{100} \right)^2 \Rightarrow E_p \geq \frac{10^4 N_0}{2} = 5 \times 10^{-9}$$

$$\text{Then } \sigma_i / \tau \approx \sqrt{N_0 / 4B_N E_p} = 0.01$$

9.5-4

$$\text{Take } B_N \approx B_T = 1 \text{ MHz, so } \sigma_i^2 \approx \frac{N_0 \tau}{4B_T E_p} \leq \left( \frac{\tau}{100} \right)^2 \Rightarrow E_p \geq \frac{10^4 N_0}{4B_T \tau} = 5 \times 10^{-11}$$

$$\text{Then } \sigma_A / A = \sqrt{N_0 B_N \tau / E_p} = 1$$

9.5-5

$$\sigma_A^2 = N_0 B_N \leq \left( \frac{A}{100} \right)^2 = \frac{E_p / \tau}{10^4} \Rightarrow B_N \leq \frac{E_p}{10^4 N_0 \tau} = 10^{11} E_p$$

$$\sigma_i^2 = \frac{N_0 \tau}{4 B_N E_p} \leq \left( \frac{\tau}{1000} \right)^2 \Rightarrow B_N \geq \frac{10^6 N_0}{4 \tau E_p} = \frac{1}{4 \times 10^3 E_p}. \text{ Thus,}$$

$$10^{11} E_p \geq \frac{1}{4 \times 10^3 E_p} \Rightarrow E_p \geq 5 \times 10^{-8}$$

$$\text{and } B_N = 10^{11} E_{p_{\min}} = 5 \text{ kHz so } \frac{1}{2\tau} < B_N < B_T$$

9.5-6

$$B \gg 1/\tau \Rightarrow y(t) \approx xR(t), E_p = A_p^2 \tau, \text{ so } A^2 \approx A_p^2 = E_p / \tau$$

$$B_N = \pi B / 2 \Rightarrow \sigma^2 = N_0 B_N = N_0 \pi B / 2, \text{ so } \left( \frac{A}{\sigma} \right)^2 = \frac{A_p^2}{N_0 \pi B / 2} = \frac{2 E_p}{\pi N_0 B \tau} \square \frac{2 E_p}{N_0}$$

9.5-7

$$\text{Assuming pulse arrives at } t = 0, y(t) = A_p (1 - e^{-2\pi B t}), 0 < t < \tau, \text{ so } A = y(\tau) = A_p (1 - e^{-2\pi B \tau})$$

$$\sigma^2 = N_0 B_N = N_0 \pi B / 2, \text{ so } \left( \frac{A}{\sigma} \right)^2 = \frac{A_p^2 (1 - e^{-2\pi B \tau})^2}{N_0 \pi B / 2} = \frac{(1 - e^{-2\pi B \tau})^2}{\pi B \tau} \frac{2 E_p}{N_0}$$

9.5-8

$$P(f) = \tau \text{sinc}^2 f \tau \Rightarrow H_{\text{opt}}(f) = \frac{2K\tau}{N_0} \text{sinc}^2 f \tau e^{-j\omega t_d} \text{ and } h_{\text{opt}}(t) = \frac{2K\tau}{N_0} \Lambda \left( \frac{t - t_d}{\tau} \right)$$

Want  $h_{\text{opt}}(t) = 0$  for  $t < 0$  for realizability, so  $t_d \rightarrow \tau$ .

9.5-9

$$P(f) = \frac{1}{b + j2\pi f} \Rightarrow H_{\text{opt}}(f) = \frac{2K}{N_0} \frac{1}{b - j2\pi f} e^{-j\omega t_d} \text{ and } h_{\text{opt}}(t) = \frac{2K}{N_0} e^{-b(t_d - t)} u(t_d - t)$$

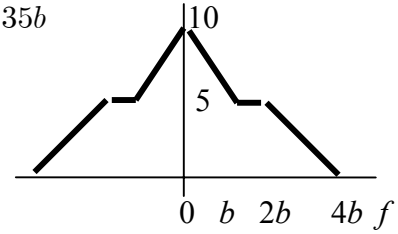
Want  $h_{\text{opt}}(t) \approx 0$  for  $t < 0$  for approximate realizability, so take  $t_d \rightarrow 5/b$  which yields

$h_{\text{opt}}(t) \ll 2K/N_0$  for  $t < 0$ .

## Chapter 10

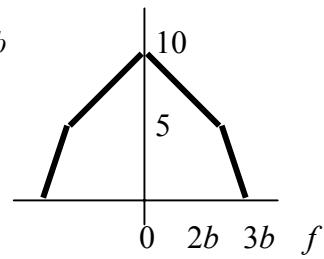
10.1-1

$$\overline{n^2} = 2 \times 5 \left[ \frac{b}{2} + 2b + \frac{2b}{2} \right] = 35b, \quad \overline{n_i^2} = 2 \times 5 \left[ 2b + \frac{b}{2} + \frac{2b}{2} \right] = 35b$$



10.1-2

$$\overline{n^2} = 2 \times 5 \left[ \frac{b}{2} + 2b + \frac{2b}{2} \right] = 35b, \quad \overline{n_i^2} = 2 \times 5 \left[ 2b + \frac{b}{2} + \frac{2b}{2} \right] = 35b$$



10.1-3

$$G_n(f) = \frac{N_0/2}{1 + \left[ \frac{4}{3} \left( \frac{f}{f_c} - \frac{f_c}{f} \right) \right]^2}$$

$ff_c$	0	$\pm 0.5$	$\pm 1$	$\pm 1.5$	$\pm 2$
$G_n/N_0$	0	0.1	0.5	0.22	0.1

10.1-4

$$(a) \quad G_{lp}(f) = \frac{N_0}{2} \left| H_R(f + f_c) u(f + f_c) \right|^2 = 0 \quad \text{for } f < -f_c$$

$$\text{For } f > 0, \quad G_n(f) = \frac{N_0}{2} \left| H_R(f) u(f) \right|^2 = G_{lp}(f - f_c)$$

$$\text{For } f < 0, \quad G_n(f) = G_n(-f) = \frac{N_0}{2} \left| H_{lp}(-f - f_c) \right|^2 = \frac{N_0}{2} \left| H_{lp}(f + f_c) \right|^2 = G_{lp}(f + f_c)$$

But  $G_{lp}(f - f_c) = 0$  for  $f < 0$  and  $G_{lp}(f + f_c) = 0$  for  $f > 0$ , so

(cont.)

$$G_n(f) = G_{lp}(f - f_c) + G_{lp}(f + f_c) = \begin{cases} G_{lp}(f - f_c) & f > 0 \\ G_{lp}(f + f_c) & f < 0 \end{cases}$$

$$(b) G_n(f)u(f) = G_{lp}(f - f_c) \Rightarrow G_n(f + f_c)u(f + f_c) = G_{lp}(f - f_c + f_c) = G_{lp}(f)$$

$$G_n(f)[1 - u(f)] = G_{lp}(f + f_c) \Rightarrow G_n(f - f_c)[1 - u(f - f_c)] = G_{lp}(f - f_c + f_c) = G_{lp}(f)$$

$$\text{Thus, } G_{ni}(f) = G_{lp}(f) + G_{lp}(f) = 2G_{lp}(f)$$

10.1-5

$$(a) H_{lp}(f) = H_R(f + f_c)u(f + f_c) \approx 1/(1 + j2f/B_T) \text{ for } f > -f_c. \text{ Thus,}$$

$$G_{lp}(f) = \frac{N_0}{2} \left| \frac{1}{1 + j2f/B_T} \right|^2 = \frac{N_0/2}{1 + (2f/B_T)^2} \text{ for } f > -f_c, \text{ which looks like the output of a}$$

1<sup>st</sup>-order LPF with  $B = B_T/2$ .

$$(b) G_{ni}(f) = 2G_{lp}(f)$$

$$\overline{n_i^2} = \int_{-\infty}^{\infty} 2G_{lp}(f) df \approx N_0 \int_{-f_c}^{\infty} \frac{df}{1 + (2f/B_T)^2} = \frac{N_0 B_T}{2} \left[ \frac{\pi}{2} + \arctan \frac{2f_c}{B_T} \right]$$

$$\approx N_0 \pi B_T / 2 \text{ since } 2f_c/B_T = 2Q \gg 1$$

10.1-6

$$\begin{aligned} y(t) &= 2[n_i(t) \cos \omega_c t - n_q(t) \sin \omega_c t] \cos(\omega_c t + \theta) \\ &= \underbrace{n_i(t) \cos \theta + n_q(t) \sin \theta}_{y_{lp}(t)} + \underbrace{n_i(t) \cos(2\omega_c t + \theta) - n_q(t) \sin(2\omega_c t + \theta)}_{y_{bp}(t)} \end{aligned}$$

Since  $n_i$  and  $n_q$  are independent and  $\overline{n_i} = \overline{n_q} = 0$

$$\overline{y_{lp}} = 0, \overline{y_{lp}^2} = \overline{n_i^2} \cos^2 \theta + \overline{n_q^2} \sin^2 \theta = \overline{n^2}$$

$$\begin{aligned} \overline{y_{bp}} &= 0, \overline{y_{bp}^2} = \overline{n_i^2} \cos^2(2\omega_c t + \theta) + \overline{n_q^2} \sin^2(2\omega_c t + \theta) = \overline{n^2} [\cos^2(2\omega_c t + \theta) + \sin^2(2\omega_c t + \theta)] \\ &= \overline{n^2} \end{aligned}$$

10.1-7

$$y(t) = A_n(t) - \overline{A_n} \text{ with } \overline{A_n^2} = 2\sigma_n^2 = 8 \text{ and } \overline{A_n} = \sqrt{\pi\sigma_n^2/2} = \sqrt{2\pi}$$

$$p_Y(y) = p_{A_n}(y + \overline{A_n}) = \frac{1}{4}(y + \sqrt{2\pi})e^{-(y + \sqrt{2\pi})^2/8}u(y + \sqrt{2\pi}) \quad (\text{cont.})$$



$$\bar{y} = 0, \sigma_y^2 = \overline{y^2} = \overline{(A_n - \bar{A}_n)^2} = \overline{A_n^2} - \bar{A}_n^2 = 8 - 2\pi \Rightarrow \sigma_y = \sqrt{8 - 2\pi} = 1.3$$

10.1-8

$$\bar{y} = \overline{A_n^2} = 2\sigma_n^2$$

$$\overline{y^2} = \overline{A_n^4} = \int_0^\infty A_n^4 \frac{A_n}{\sigma_n^2} e^{-A_n^2/2\sigma_n^2} dA_n = 4\sigma_n^4 \int_0^\infty \lambda^2 e^{-\lambda} d\lambda = 8\sigma_n^4$$

Since  $A_n \geq 0$  and  $y \geq 0$ , transformation with  $g^{-1}(A_n) = y^{1/2}$  yields

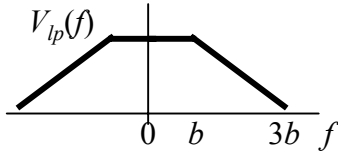
$$p_Y(y) = p_{A_n}(y^{1/2}) \left| \frac{dy^{1/2}}{dy} \right| = \frac{y^{1/2}}{\sigma_n^2} e^{-y/2\sigma_n^2} u(y^{1/2})^{\frac{1}{2}} y^{-1/2} = \frac{1}{2\sigma_n^2} e^{-y/2\sigma_n^2} u(y)$$

10.1-9

$$\mathcal{F}[j\hat{v}(t)] = j(-j \operatorname{sgn} f)V(f) = (\operatorname{sgn} f)V(f) \text{ so}$$

$$V_{lp}(f) = \frac{1}{2}V(f + f_c) + \frac{1}{2}\operatorname{sgn}(f + f_c)V(f + f_c) = \frac{1}{2}[1 + \operatorname{sgn}(f + f_c)]V(f + f_c)$$

$$= u(f + f_c)V(f + f_c) \text{ since } 1 + \operatorname{sgn}(f + f_c) = \begin{cases} 0 & f < -f_c \\ 2 & f > -f_c \end{cases}$$



10.1-10

$$G_n(f) = \frac{N_0}{2} \left[ \Pi \left( \frac{f - f_c}{B_T} \right) + \Pi \left( \frac{f + f_c}{B_T} \right) \right],$$

$$R_n(\tau) = \frac{N_0}{2} B_T \operatorname{sinc} B_T \tau (e^{j\omega_c \tau} + e^{-j\omega_c \tau}) = N_0 B_T \operatorname{sinc} B_T \tau \cos \omega_c \tau$$

$$(-j \operatorname{sgn} f)G_n(f) = \frac{-jN_0}{2} \left[ \Pi \left( \frac{f - f_c}{B_T} \right) - \Pi \left( \frac{f + f_c}{B_T} \right) \right],$$

$$\hat{R}_n(\tau) = \frac{-jN_0}{2} B_T \operatorname{sinc} B_T \tau (e^{j\omega_c \tau} - e^{-j\omega_c \tau}) = N_0 B_T \operatorname{sinc} B_T \tau \sin \omega_c \tau$$

(cont.)

$$\begin{aligned}
R_{n_i}(\tau) &= (N_0 B_T \text{sinc } B_T \tau \cos \omega_c \tau) \cos \omega_c \tau + (N_0 B_T \text{sinc } B_T \tau \sin \omega_c \tau) \sin \omega_c \tau \\
&= N_0 B_T \text{sinc } B_T \tau (\cos^2 \omega_c \tau + \sin^2 \omega_c \tau) = N_0 B_T \text{sinc } B_T \tau
\end{aligned}$$

$$G_{n_i}(f) = \frac{N_0}{2} \Pi\left(\frac{f}{B_T}\right) + \frac{N_0}{2} \Pi\left(\frac{f}{B_T}\right) = N_0 \Pi\left(\frac{f}{B_T}\right) \text{ so } R_{n_i}(\tau) = \mathcal{F}_\tau^{-1}[G_{n_i}(f)]$$

10.1-11

Let  $f_0 = f_c + B_T/2$  so  $\omega_0 = \omega_c + \pi B_T$ . Then  $G_n(f) = \frac{N_0}{2} \left[ \Pi\left(\frac{f-f_0}{B_T}\right) + \Pi\left(\frac{f+f_0}{B_T}\right) \right]$  and

$$R_n(\tau) = \frac{N_0}{2} B_T \text{sinc } B_T \tau (e^{j\omega_0 \tau} + e^{-j\omega_0 \tau}) = N_0 B_T \text{sinc } B_T \tau \cos \omega_0 \tau$$

$$(-j \text{sgn } f) G_n(f) = \frac{-jN_0}{2} \left[ \Pi\left(\frac{f-f_0}{B_T}\right) - \Pi\left(\frac{f+f_0}{B_T}\right) \right]$$

$$\hat{R}_n(\tau) = \frac{-jN_0}{2} B_T \text{sinc } B_T \tau (e^{j\omega_0 \tau} - e^{-j\omega_0 \tau}) = N_0 B_T \text{sinc } B_T \tau \sin \omega_0 \tau$$

$$\begin{aligned}
R_{n_i}(\tau) &= [N_0 B_T \text{sinc } B_T \tau \cos(\omega_c + \pi B_T) \tau] \cos \omega_c \tau + [N_0 B_T \text{sinc } B_T \tau \sin(\omega_c + \pi B_T) \tau] \sin \omega_c \tau \\
&= N_0 B_T \text{sinc } B_T \tau \cos \pi B_T \tau = \frac{N_0}{\pi \tau} \sin \pi B_T \tau \cos \pi B_T \tau = \frac{N_0}{2\pi \tau} \sin 2\pi B_T \tau = N_0 B_T \text{sinc } 2B_T \tau
\end{aligned}$$

$$G_{n_i}(f) = \frac{N_0}{2} \left[ \Pi\left(\frac{f-B_T/2}{B_T}\right) + \Pi\left(\frac{f+B_T/2}{B_T}\right) \right] = \frac{N_0}{2} \Pi\left(\frac{f}{2B_T}\right) \text{ so } R_{n_i}(\tau) = \mathcal{F}_\tau^{-1}[G_{n_i}(f)]$$

10.1-12

$$R_{n_i}(\tau) = \mathcal{F}_\tau^{-1}[2G_{lp}(f)] = 2 \int_{-\infty}^{\infty} G_{lp}(f) e^{j\omega \tau} df$$

$$\begin{aligned}
R_n(\tau) &= \mathcal{F}_\tau^{-1}[G_{lp}(f-f_c) + G_{lp}(f+f_c)] = \int_{-\infty}^{\infty} G_{lp}(f-f_c) e^{j\omega \tau} df + \int_{-\infty}^{\infty} G_{lp}(f+f_c) e^{j\omega \tau} df \\
&= \int_{-\infty}^{\infty} G_{lp}(\lambda_1) e^{j2\pi(\lambda_1+f_c)\tau} d\lambda_1 + \int_{-\infty}^{\infty} G_{lp}(\lambda_2) e^{j2\pi(\lambda_2-f_c)\tau} d\lambda_2 \\
&= 2 \int_{-\infty}^{\infty} G_{lp}(\lambda) e^{j2\pi\lambda\tau} d\lambda \frac{1}{2} (e^{j\omega_c \tau} + e^{-j\omega_c \tau}) = R_{n_i}(\tau) \cos \omega_c \tau
\end{aligned}$$

$$\hat{R}_n(\tau) \sin \omega_c \tau = R_{n_i}(\tau) - [R_{n_i}(\tau) \cos \omega_c \tau] \cos \omega_c \tau = R_{n_i}(\tau) \sin^2 \omega_c \tau \text{ so } \hat{R}_n(\tau) = R_{n_i}(\tau) \sin \omega_c \tau$$

## 10.1-13

$E[n_q(t)n_q(t-\tau)] = E_1 - E_2 - E_3 + E_4$  where

$$E_1 = E[\hat{n}(t) \cos \omega_c t \times \hat{n}(t-\tau) \cos \omega_c (t-\tau)] = \frac{1}{2} R_{\hat{n}}(\tau) [\cos \omega_c \tau + \cos \omega_c (2t-\tau)]$$

$$E_2 = E[\hat{n}(t) \cos \omega_c t \times n(t-\tau) \sin \omega_c (t-\tau)] = \frac{1}{2} R_{\hat{n}n}(\tau) [-\sin \omega_c \tau + \sin \omega_c (2t-\tau)]$$

$$E_3 = E[n(t) \sin \omega_c t \times \hat{n}(t-\tau) \cos \omega_c (t-\tau)] = \frac{1}{2} R_{n\hat{n}}(\tau) [\sin \omega_c \tau + \sin \omega_c (2t-\tau)]$$

$$E_4 = E[n(t) \sin \omega_c t \times n(t-\tau) \sin \omega_c (t-\tau)] = \frac{1}{2} R_n(\tau) [\cos \omega_c \tau - \cos \omega_c (2t-\tau)]$$

$$\begin{aligned} \text{Thus, } R_{n_q}(t, t-\tau) &= \frac{1}{2} [R_{\hat{n}}(\tau) + R_n(\tau)] \cos \omega_c \tau + \frac{1}{2} [R_{\hat{n}n}(\tau) - R_{n\hat{n}}(\tau)] \sin \omega_c \tau \\ &\quad + \frac{1}{2} [R_{\hat{n}}(\tau) - R_n(\tau)] \cos \omega_c (2t-\tau) - \frac{1}{2} [R_{\hat{n}n}(\tau) + R_{n\hat{n}}(\tau)] \sin \omega_c (2t-\tau) \end{aligned}$$

But  $R_{\hat{n}} = R_n$  so  $R_{\hat{n}} + R_n = 2R_n$  and  $R_{\hat{n}} - R_n = 0$  and

$$-R_{n\hat{n}} = R_{\hat{n}n} = \hat{R}_n \text{ so } R_{\hat{n}n} - R_{n\hat{n}} = 2\hat{R}_n \text{ and } R_{\hat{n}n} + R_{n\hat{n}} = 0. \text{ Hence,}$$

$$R_{n_q}(t, t-\tau) = R_{n_q}(\tau) = R_n(\tau) \cos \omega_c \tau + \hat{R}_n(\tau) \sin \omega_c \tau$$

## 10.1-14

$E[n_i(t)n_q(t-\tau)] = E_1 - E_2 + E_3 - E_4$  where

$$E_1 = E[n(t) \cos \omega_c t \times \hat{n}(t-\tau) \cos \omega_c (t-\tau)] = \frac{1}{2} R_{n\hat{n}}(\tau) [\cos \omega_c \tau + \cos \omega_c (2t-\tau)]$$

$$E_2 = E[n(t) \cos \omega_c t \times n(t-\tau) \sin \omega_c (t-\tau)] = \frac{1}{2} R_n(\tau) [-\sin \omega_c \tau + \sin \omega_c (2t-\tau)]$$

$$E_3 = E[\hat{n}(t) \sin \omega_c t \times \hat{n}(t-\tau) \cos \omega_c (t-\tau)] = \frac{1}{2} R_{\hat{n}}(\tau) [\sin \omega_c \tau + \sin \omega_c (2t-\tau)]$$

$$E_4 = E[\hat{n}(t) \sin \omega_c t \times n(t-\tau) \sin \omega_c (t-\tau)] = \frac{1}{2} R_{\hat{n}n}(\tau) [\cos \omega_c \tau - \cos \omega_c (2t-\tau)]$$

$$\begin{aligned} \text{Thus, } R_{n_i n_q}(t, t-\tau) &= \frac{1}{2} [R_{n\hat{n}}(\tau) + R_{\hat{n}n}(\tau)] \cos \omega_c \tau + \frac{1}{2} [R_n(\tau) - R_{\hat{n}}(\tau)] \sin \omega_c \tau \\ &\quad + \frac{1}{2} [R_{n\hat{n}}(\tau) - R_{\hat{n}n}(\tau)] \cos \omega_c (2t-\tau) - \frac{1}{2} [R_n(\tau) + R_{\hat{n}}(\tau)] \sin \omega_c (2t-\tau) \end{aligned}$$

But  $R_{\hat{n}} = R_n$  so  $R_{\hat{n}} + R_n = 2R_n$  and  $R_{\hat{n}} - R_n = 0$  and

$$R_{n\hat{n}} = -R_{\hat{n}n} = -\hat{R}_n \text{ so } R_{n\hat{n}} - R_{\hat{n}n} = -2\hat{R}_n \text{ and } R_{n\hat{n}} + R_{\hat{n}n} = 0. \text{ Hence,}$$

$$R_{n_i n_q}(t, t-\tau) = R_{n_i n_q}(\tau) = R_n(\tau) \sin \omega_c \tau + \hat{R}_n(\tau) \cos \omega_c \tau$$

## 10.1-15

$$(a) R_{n_i n_q}(\tau) = [R_{n_i}(\tau) \cos \omega_c \tau] \sin \omega_c \tau - [R_{n_i}(\tau) \sin \omega_c \tau] \cos \omega_c \tau = 0 \quad (\text{cont.})$$

$$(b) \quad G_n(f)u(f) = G_{lp}(f - f_c) \Rightarrow G_n(f + f_c)u(f + f_c) = G_{lp}(f + f_c - f_c) = G_{lp}(f)$$

$$G_n(f)[1 - u(f)] = G_{lp}(f + f_c) \Rightarrow G_n(f - f_c)[1 - u(f - f_c)] = G_{lp}(f - f_c + f_c) = G_{lp}(f)$$

$$\text{Thus, } R_{n_i n_q}(\tau) = \mathcal{F}_\tau^{-1}\{j[G_{lp}(f) - G_{lp}(f)]\} = 0$$

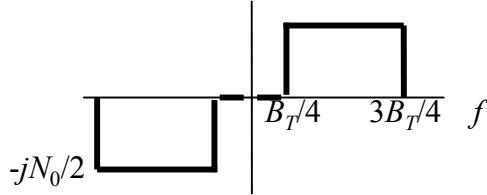
10.1-16

$$G_n(f + f_c)u(f + f_c) = \frac{N_0}{2} \Pi\left(\frac{f - B_T/2}{B_T}\right), \quad G_n(f - f_c)[1 - u(f - f_c)] = \frac{N_0}{2} \Pi\left(\frac{f + B_T/2}{B_T}\right)$$

$$R_{n_i n_q}(\tau) = \mathcal{F}_\tau^{-1}\left\{j \frac{N_0}{2} \left[ \Pi\left(\frac{f - B_T/2}{B_T}\right) - \Pi\left(\frac{f + B_T/2}{B_T}\right) \right]\right\} = j \frac{N_0}{2} B_T \text{sinc} B_T \tau (e^{j\pi B_T \tau} - e^{-j\pi B_T \tau})$$

$$= -N_0 B_T \text{sinc} B_T \tau \sin \pi B_T \tau = -\pi N_0 B_T^2 \tau \text{sinc}^2 B_T \tau$$

10.1-17



$$G_n(f + f_c)u(f + f_c) - G_n(f - f_c)[1 - u(f - f_c)] = \frac{N_0}{2} \left[ \Pi\left(\frac{f - B_T/2}{B_T/2}\right) - \Pi\left(\frac{f + B_T/2}{B_T/2}\right) \right]$$

$$R_{n_i n_q}(\tau) = \mathcal{F}_\tau^{-1}\left\{j \frac{N_0}{2} \left[ \Pi\left(\frac{f - B_T/2}{B_T/2}\right) - \Pi\left(\frac{f + B_T/2}{B_T/2}\right) \right]\right\} = j \frac{N_0}{2} \frac{B_T}{2} \text{sinc} \frac{B_T \tau}{2} (e^{j\pi B_T \tau} - e^{-j\pi B_T \tau})$$

$$= -\frac{N_0 B_T}{2} \text{sinc} \frac{B_T \tau}{2} \sin \pi B_T \tau = -\frac{\pi N_0 B_T^2 \tau}{2} \text{sinc} \frac{B_T \tau}{2} \text{sinc} B_T \tau$$

10.2-1

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N / \mathcal{T}_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}$$

$$(S/N)_D = S_R / N_0 W = 20 \times 10^{-9} / (4 \times 10^{-20} \times 5 \times 10^6) = 10^5 = 50 \text{ dB}$$

10.2-2

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N / \mathcal{T}_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}$$

$$\left(\frac{S}{N}\right)_D = \frac{0.4}{1 + 0.4} \frac{20 \times 10^{-9}}{4 \times 10^{-20} \times 5 \times 10^6} = 2.86 \times 10^4 = 44.6 \text{ dB}$$

### 10.2-3

$$\begin{aligned}
 y(t) &= \left\{ [A_c x(t) + n_i(t)] \cos \omega_c t - n_q(t) \sin \omega_c t \right\} 2 \cos(\omega_c t + \phi') \\
 &= A_c x(t) \cos \phi' + n_i(t) \cos \phi' + n_q(t) \sin \phi' + \text{high-frequency terms} \\
 y_D(t) &= A_c x(t) \cos \phi' + n_i(t) \cos \phi' + n_q(t) \sin \phi' \text{ so } S_D = A_c^2 \overline{x^2} \cos^2 \phi' \\
 N_D &= E[n_i^2 \cos^2 \phi' + 2n_i n_q \cos \phi' \sin \phi' + n_q^2 \sin^2 \phi'] \\
 &= \overline{n_i^2} \cos^2 \phi' + \overline{n_q^2} \sin^2 \phi' = \overline{n^2} (\cos^2 \phi' + \sin^2 \phi') = \overline{n^2} = 2N_0 W
 \end{aligned}$$

$$\text{Thus, } (S/N)_D = A_c^2 \overline{x^2} \cos^2 \phi' / 2N_0 W = S_R / N_0 W \times \cos^2 \phi' = \gamma \cos^2 \phi'$$

### 10.2-4

$$\begin{aligned}
 \text{DSB: } S_p &= x_c^2|_{\max} = A_c^2 \Rightarrow S_D = A_c^2 \overline{x^2} = S_p S_x \text{ so } (S/N)_D = S_x S_p / 2N_0 W = \frac{1}{2} S_x \gamma_p \\
 \text{AM: } S_p &= A_c^2 (1 + \overline{x^2})|_{\max} = 4A_c^2 \Rightarrow S_D = A_c^2 \overline{x^2} = \frac{1}{4} S_p S_x \text{ so} \\
 (S/N)_D &= \frac{1}{4} S_x S_p / 2N_0 W = \frac{1}{8} S_x \gamma_p
 \end{aligned}$$

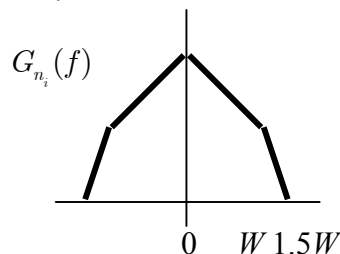
### 10.2-5

$$\begin{aligned}
 v(t) &= [A_c x_1(t) + n_i(t)] \cos \omega_c t \pm [A_c x_2(t) \mp n_q(t)] \sin \omega_c t \text{ so } y_{D_1}(t) = A_c x_1(t) + n_i(t) \text{ and} \\
 y_{D_2}(t) &= A_c x_2(t) \mp n_q(t) \text{ where } \overline{x_1^2} = \overline{x_2^2} = S_x, S_R = \frac{1}{2} A_c^2 \overline{x_1^2} + \frac{1}{2} A_c^2 \overline{x_2^2} = A_c^2 S_x, \text{ and} \\
 \overline{n_i^2} &= \overline{(\mp n_q)^2} = \overline{n^2} = 2N_0 W. \text{ Thus, both outputs have } \left( \frac{S}{N} \right)_D = \frac{A_c^2 S_x}{2N_0 W} = \frac{S_R}{2N_0 W} = \frac{1}{2} \gamma
 \end{aligned}$$

### 10.2-6

For USSB, any noise component in  $f_c - W < |f| < f_c$  will be translated to  $|f| < W$  and cannot be removed by LPF; similarly, for LSSB, noise components in  $f_c < |f| < f_c + W$  cannot be removed by LPF. For DSB, noise components outside  $f_c - W < |f| < f_c + W$  are translated to  $|f| > W$  and can be removed by LPF.

### 10.2-7



(cont.)

With ideal LPF at output,  $N_D = \int_{-W}^W G_{n_i}(f) df = \frac{3}{2} N_0 W$  so  $\left(\frac{S}{N}\right)_D = \frac{S_R}{(\frac{3}{2} N_0 W)} = \frac{2}{3} \gamma$

10.2-8

USSB:  $y_D(t) = \frac{1}{2} A_c x(t) + n_i(t)$ ,  $S_D = S_R$

$$N_D = 2 \int_0^W \frac{N_0 f_c}{2(f + f_c)} df = N_0 f_c [\ln(W + f_c) - \ln f_c] = N_0 f_c \ln(1 + W / f_c)$$

$$\left(\frac{S}{N}\right)_D = \frac{S_R}{N_0 f_c \ln(1 + W / f_c)} = \frac{W / f_c}{\ln(1 + W / f_c)} \gamma = \begin{cases} 1.10\gamma & W / f_c = 1/5 \\ 1.01\gamma & W / f_c = 1/50 \end{cases}$$

DSB:  $y_D(t) = A_c x(t) + n_i(t)$ ,  $S_D = 2S_R$ ,  $G_{n_i}(f) = \frac{N_0 f_c}{2} \left( \frac{1}{f + f_c} - \frac{1}{f - f_c} \right) = \frac{N_0 f_c^2}{f^2 - f_c^2}$

$$N_D = 2 \int_0^W \frac{N_0 f_c^2}{f^2 - f_c^2} df = 2N_0 f_c \frac{1}{2f_c} \ln \left( \frac{f_c + W}{f_c - W} \right) = N_0 f_c \ln \left( \frac{1 + W / f_c}{1 - W / f_c} \right)$$

$$\left(\frac{S}{N}\right)_D = \frac{2S_R}{N_0 f_c \ln \left( \frac{1 + W / f_c}{1 - W / f_c} \right)} = \frac{2W / f_c}{\ln \left( \frac{1 + W / f_c}{1 - W / f_c} \right)} \gamma = \begin{cases} 0.99\gamma & W / f_c = 1/5 \\ 1.00\gamma & W / f_c = 1/50 \end{cases}$$

Note: no significant difference between LSSB and DSB when  $W/f_c \ll 1$ .

10.2-9

$$v(t) = \left[ \frac{1}{2} A_c x(t) + n_i(t) \right] \cos \omega_c t - \left[ \frac{1}{2} A_c \hat{x}(t) + n_q(t) \right] \sin \omega_c t$$

$$y(t) = v(t) 2 \cos [\omega_c t + \phi(t)] = \left[ \frac{1}{2} A_c x(t) + n_i(t) \right] \cos \phi(t) + \left[ \frac{1}{2} A_c \hat{x}(t) + n_q(t) \right] \sin \phi(t) +$$

high-frequency terms. Since  $\phi(t)$  has slow variations compared to  $x(t)$ ,

$$y_D(t) \approx \left[ \frac{1}{2} A_c x(t) + n_i(t) \right] \cos \phi(t) + \left[ \frac{1}{2} A_c \hat{x}(t) + n_q(t) \right] \sin \phi(t) = \frac{1}{2} A_c x(t) \text{ when } \phi = n_i = n_q = 0$$

which implies that  $K = 2 / A_c$ . Then

$$\begin{aligned} E \left\{ [x(t) - K y_D(t)]^2 \right\} &= E \left\{ x^2 + \left( x + \frac{2}{A_c} n_i \right)^2 \cos^2 \phi + \left( \hat{x} + \frac{2}{A_c} n_q \right)^2 \sin^2 \phi - 2x \left( x + \frac{2}{A_c} n_i \right) \cos \phi \right. \\ &\quad \left. - 2x \left( \hat{x} + \frac{2}{A_c} n_q \right) \sin \phi + 2 \left( x + \frac{2}{A_c} n_i \right) \left( \hat{x} + \frac{2}{A_c} n_q \right) \cos \phi \sin \phi \right\} \end{aligned}$$

(cont.)

where  $\overline{x^2} = \overline{\hat{x}^2} = S_x$ ,  $\overline{x\hat{x}} = 0$ ,  $\frac{4}{A_c^2} = \frac{S_x}{S_R}$ ,  $\overline{n_i^2} = \overline{n_q^2} = N_0 W$ ,  $\overline{n_i} = \overline{n_q} = 0$ ,  $\overline{n_i n_q} = 0$ . Thus,

$$\begin{aligned}\epsilon^2 &= \left[ S_x + (S_x + S_x N_0 W / S_R) \overline{\cos^2 \phi} + (S_x + S_x N_0 W / S_R) \overline{\sin^2 \phi} - 2 S_x \overline{\cos \phi} \right] / S_x \\ &= 1 + \overline{\cos^2 \phi} + \overline{\sin^2 \phi} - 2 \overline{\cos \phi} + \frac{1}{S_R / N_0 W} (\overline{\cos^2 \phi} + \overline{\sin^2 \phi})\end{aligned}$$

But  $\overline{\cos^2 \phi} + \overline{\sin^2 \phi} = \overline{\cos^2 \phi + \sin^2 \phi} = 1$  so  $\epsilon^2 = 2(1 - \overline{\cos \phi}) + 1/\gamma$

### 10.2-10

Envelope detection without mutilation requires  $S_R \gg N_R = N_0 B_N$ , where  $B_N$  is the noise equivalent bandwidth of  $H_R(f)$ , so  $B_N$  should be as small as possible, namely  $B_N = B_T = 2W$  for an ideal BPF.

With synchronous detection, there is no mutilation and noise components outside

$f_c - W < |f| < f_c + W$  are translated to  $|f| > W$  and can be removed by the LPF.

### 10.2-11

With  $S_x = \frac{1}{2}$ ,  $\left(\frac{S}{N}\right)_D = \frac{\frac{1}{2}}{1 + \frac{1}{2}} \gamma = 10^4 \Rightarrow \gamma = 3 \times 10^4$ , whereas  $\gamma_{th} \approx 20$ .

Thus,  $g_T = \gamma / \gamma_{th} \approx 1500 = 32 \text{ dB}$

### 10.2-12

$$\left(\frac{S}{N}\right)_D = \frac{\frac{1}{2}}{1 + \frac{1}{2}} \gamma = 10^3 \Rightarrow \gamma = \frac{S_R}{N_0 W} = 3 \times 10^3 \Rightarrow \frac{S_R}{N_0} = \gamma W = 24 \times 10^6$$

$$\gamma_{th} \approx 20 = \left(\frac{S_R}{N_0 W}\right)_{\min} \Rightarrow W_{\max} = \frac{1}{20} \frac{S_R}{N_0} = 1.2 \text{ MHz}$$

### 10.2-13

$$\begin{aligned}y_D(t) &= y(t) = A_n(t) + A_c x(t) \cos \phi_n(t) - \overline{A_n} \\ &= A_c x(t) \text{ if } n(t) = 0, \text{ so } K = 1/A_c\end{aligned}$$

$$\epsilon^2 = E \left\{ \left[ x - \frac{1}{A_c} A_n - x \cos \phi_n + \frac{1}{A_c} \overline{A_n} \right]^2 \right\} / S_x, \quad S_x = 1$$

(cont.)

$$\begin{aligned} \epsilon^2 = E \left\{ \overline{x^2} + \frac{A_n^2}{A_c^2} + x^2 \cos^2 \phi_n + \frac{\overline{A_n^2}}{A_c^2} - \frac{2}{A_c} A_n x - 2x^2 \cos \phi_n + \frac{2}{A_c} \overline{A_n} x \right. \\ \left. - \frac{2}{A_c} x A_n \cos \phi_n - \frac{2}{A_c^2} \overline{A_n} A_n - \frac{2}{A_c} \overline{A_n} x \cos \phi_n \right\} \end{aligned}$$

where  $\overline{x^2} = 1$ ,  $\overline{x} = 0$ ,  $\overline{A_n^2} = 2\overline{n^2} = 4N_0W$ ,  $\overline{A_n^2} = \sqrt{\pi N_0W}$ ,  $\overline{A_n \cos \phi_n} = \overline{n_i} = 0$ ,  $\overline{\cos \phi_n} = 0$ .

$$\overline{\cos^2 \phi_n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos^2 \phi_n d\phi_n = \frac{1}{2}, A_c^2 = 2S_R / (1 + S_x) = S_R. \text{ Thus,}$$

$$\epsilon^2 = 1 + \frac{4N_0W}{S_R} + \frac{1}{2} + \frac{\pi N_0W}{S_R} - \frac{2}{S_R} \pi N_0W = \frac{3}{2} + \frac{4 - \pi}{\gamma} \text{ with } \gamma < \gamma_{th} \approx 20$$

10.3-1

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N / \mathcal{T}_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}$$

$$\text{PM: } N_D = N_0W / S_R = 4 \times 10^{-20} \times 500 \times 10^3 / 10 \times 10^{-9} = 2 \times 10^{-6}$$

$$\text{FM: } N_D = N_0W^3 / 3S_R = 4 \times 10^{-20} (500 \times 10^3)^3 / 3 \times 10 \times 10^{-9} = 1.67 \times 10^5$$

$$\text{Deemphasized FM: } N_D \approx N_0 B_{de}^2 W / S_R = 4 \times 10^{-20} (5 \times 10^3)^2 500 \times 10^3 / 10 \times 10^{-9} = 50$$

10.3-2

$$\begin{aligned} N_D = \int_{-\infty}^{\infty} |H_D(f)|^2 G_{\xi}(f) df \leq \int_{-\infty}^{\infty} \frac{1}{1 + (f/2W)^{2n}} \frac{N_0 f^2}{2S_R} df = \frac{N_0 W^3}{S_R} \int_0^{\infty} \frac{\lambda^2}{1 + \lambda^{2n}} d\lambda \\ \leq \frac{N_0 W^3}{S_R} \frac{\pi/2n}{\sin(3\pi/2n)} \approx \frac{N_0 W^3}{3S_R} \text{ if } n \gg 1 \end{aligned}$$

10.3-3

$$G_{n_q}(f) = 2 \frac{N_0}{2} |H_R(f + f_c)|^2 = \frac{N_0}{1 + (2f/B_T)^2}, G_{\xi}(f) = \frac{N_0}{2S_R} \frac{f^2}{1 + (2f/B_T)^2}$$

$$\begin{aligned} N_D = 2 \frac{N_0}{2S_R} \int_0^W \frac{f^2}{1 + (2f/B_T)^2} df = \frac{N_0 B_T^3}{8S_R} \left[ \frac{2W}{B_T} - \arctan \left( \frac{2W}{B_T} \right) \right] \\ = \frac{N_0 B_T^3}{8S_R} \left[ \frac{2W}{B_T} - \left[ \frac{2W}{B_T} - \frac{1}{3} \left( \frac{2W}{B_T} \right)^3 + \dots \right] \right] \approx \frac{N_0 B_T^3}{8S_R} \frac{1}{3} \left( \frac{2W}{B_T} \right)^3 = \frac{N_0 W^3}{3S_R} \text{ if } B_T \gg W \end{aligned}$$



## 10.3-4

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N / \mathcal{T}_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}, \quad D = f_\Delta / W = 2 \times 10^6 / 500 \times 10^3 = 4$$

$$\text{FM: } \left( \frac{S}{N} \right)_D = 3D^2 S_x \frac{S_R}{N_0 W} = 3 \times 4^2 \times 0.1 \frac{10^{-9}}{4 \times 10^{-20} \times 500 \times 10^3} = 240 \times 10^3 = 53.8 \text{ dB}$$

Deemphasized FM:

$$\left( \frac{S}{N} \right)_D \approx \left( \frac{f_\Delta}{B_{de}} \right)^2 S_x \frac{S_R}{N_0 W} = \left( \frac{2 \times 10^6}{5 \times 10^3} \right)^2 0.1 \frac{10^{-9}}{4 \times 10^{-20} \times 500 \times 10^3} = 800 \times 10^6 = 89.0 \text{ dB}$$

## 10.3-5

$$N_D = \int_{-W}^W \frac{1}{1 + (f / B_{de})^2} \frac{N_0}{2S_R} df = \frac{N_0 B_{de}}{S_R} \arctan \frac{W}{B_{de}}$$

$$\left( \frac{S}{N} \right)_D = \frac{\phi_\Delta^2 S_x S_R}{N_0 B_{de} \arctan(W / B_{de})} = \frac{W / B_{de}}{\arctan(W / B_{de})} \phi_\Delta^2 S_x \gamma \approx \left( \frac{2}{\pi} \frac{W}{B_{de}} \right) \phi_\Delta^2 S_x \gamma$$

## 10.3-6

$$S_D = f_\Delta^2 \overline{x^2} = f_\Delta^2 / 2 \text{ and } N_D = 2 \int_{100}^{300} \frac{N_0 f^2}{2S_R} df = \frac{N_0}{S_R} \times \frac{26}{3} \times 10^6 \text{ so}$$

$$\left( \frac{S}{N} \right)_D = \frac{1}{2} f_\Delta^2 \frac{3 \times 10^{-6}}{26} \frac{S_R}{N_0} = 290 = 24.6 \text{ dB}$$

## 10.3-7

$$N_D = 2 \int_0^W e^{-(f / B_{de})^2} \frac{N_0 f^2}{2S_R} df = \frac{N_0 B_{de}^3}{S_R} \int_0^{W / B_{de}} \lambda^2 e^{-\lambda^2} d\lambda < \frac{N_0 B_{de}^3}{S_R} \int_0^\infty \lambda^2 e^{-\lambda^2} d\lambda = \frac{N_0 B_{de}^3}{S_R} \frac{\sqrt{\pi}}{4}$$

$$\left( \frac{S}{N} \right)_D > \frac{4 f_\Delta^2 S_x S_R}{\sqrt{\pi} N_0 B_{de}^3} = \frac{4}{\sqrt{\pi}} \left( \frac{W}{B_{de}} \right)^3 \left( \frac{f_\Delta}{W} \right)^2 S_x \gamma \text{ so}$$

Improvement factor  $> (4 / \sqrt{\pi})(W / B_{de})^3 \approx 770$  when  $B_{de} = W / 7$ 

## 10.3-8

$$\text{PM: } (S / N)_D = \phi_\Delta^2 S_x \gamma = 10^3$$

$$\text{FM: } D = \phi_\Delta \text{ for same } B_T, \text{ so } (S / N)_D = (W / B_{de})^2 \phi_\Delta^2 S_x \gamma = 10^2 \times 10^3 = 50 \text{ dB}$$

10.3-9

$$(S/N)_{D_{th}} = (W/B_{de})^2 D^2 S_x \gamma_{th} \approx 20(W/B_{de})^2 D^2 (D+2) S_x, \quad D > 2$$

$$20 \times 5^2 D^2 (D+2)/2 = 10^5 \quad \Rightarrow \quad D^3 + 2D^2 = 400 \quad \Rightarrow \quad D \approx 6.7 \text{ (by trial and error)}$$

$$B_T \approx 2(6.7+2) \times 10 = 174 \text{ kHz}, \quad S_R \geq 20(6.7+2) \times 10^{-8} \times 10^4 = 17.4 \text{ mW}$$

10.3-10

$$\gamma_{th} = 20M(\phi_\Delta), \quad (S/N)_{D_{th}} = \phi_\Delta^2 S_x \gamma_{th} = 20M(\phi_\Delta) \phi_\Delta^2 S_x$$

$$\phi_\Delta \leq \pi, \quad M(\pi) \approx 2(\pi+2), \quad S_x \leq 1, \text{ so } (S/N)_{D_{th}} \leq 40(\pi+2)\pi^2 \approx 2030 \approx 33 \text{ dB}$$

10.3-11

$$\text{IF input} = A_c \cos[\omega_c t + \phi(t)] \times 2 \cos[(\omega_c - \omega_{IF})t + K\phi_D(t)] \text{ so}$$

$$v_{IF}(t) = A_c \cos[\omega_{IF} t + \phi_{IF}(t)] \quad \text{where} \quad \phi_{IF}(t) = \phi(t) - K\phi_D(t)$$

$$\text{Thus, } y_D(t) = \frac{1}{2\pi} \dot{\phi}_{IF}(t) = f_\Delta x(t) - Ky_D(t) \text{ so } y_D(t) = \frac{f_\Delta}{1+K} x(t) = \frac{1}{2\pi} \dot{\phi}_{IF}(t)$$

$$\text{and } D_{IF} = \frac{f_\Delta}{(1+K)W} = \frac{D}{1+K}$$

10.4-1

$$S_x = 1/2, \quad (S/N)_D = 10^4, \quad \gamma = S_T / LN_0 W = 10S_T$$

$$(a) \quad (S/N)_D = 10S_T \quad \Rightarrow \quad S_T = 1 \text{ kW}$$

$$(b) \quad \mu = 1, \quad \left(\frac{S}{N}\right)_D = \frac{1/2}{1+1/2} 10S_T \quad \Rightarrow \quad S_T = 3 \text{ kW}$$

$$\mu = \frac{1}{2}, \quad \left(\frac{S}{N}\right)_D = \frac{1/8}{1+1/8} 10S_T \quad \Rightarrow \quad S_T = 9 \text{ kW}$$

$$(c) \quad (S/N)_D = \pi^2 \times \frac{1}{2} \times 10S_T \quad \Rightarrow \quad S_T \approx 200 \text{ W}$$

$$(d) \quad (S/N)_D = 3D^2 \times \frac{1}{2} \times 10S_T \text{ provided that } \gamma \geq \gamma_{th} = 20M(D) \text{ so } S_T \geq 2M(D)$$

(cont.)

$D$	$10^4/15D^2$	$2M(D)$	$S_T$	
1	667	5	667 W	
5	26.7	14	26.7 W	
10	6.7	24	24 W	Threshold limited

#### 10.4-2

$$S_x = 1, (S/N)_D = 10^4, \gamma = S_T / LN_0 W = 5S_T$$

$$(a) (S/N)_D = 5S_T \Rightarrow S_T = 2 \text{ kW}$$

$$(b) \mu = 1, \left(\frac{S}{N}\right)_D = \frac{1}{1+1} 5S_T \Rightarrow S_T = 4 \text{ kW}$$

$$\mu = \frac{1}{2}, \left(\frac{S}{N}\right)_D = \frac{1/4}{1+1/4} 5S_T \Rightarrow S_T = 10 \text{ kW}$$

$$(c) (S/N)_D = \pi^2 \times 1 \times 5S_T \Rightarrow S_T \approx 200 \text{ W}$$

$$(d) (S/N)_D = 3D^2 \times 1 \times 5S_T \text{ provided that } \gamma \geq \gamma_{th} = 20M(D) \text{ so } S_T \geq 4M(D)$$

$D$	$10^4/15D^2$	$4M(D)$	$S_T$	
1	667	10	667 W	
5	26.7	28	28 W	Threshold limited
10	6.7	48	48 W	Threshold limited

#### 10.4-3

$$L = 10^{\alpha\ell/10} = 10^{\ell/10}, \gamma = S_T / LN_0 W = 10^{10} \times 10^{-\ell/10}, S_x = 1/2$$

$$(a) (S/N)_D = \gamma = 10^{10} \times 10^{-\ell/10} = 10^4 \Rightarrow \ell = 10(10 - 4) = 60 \text{ km}$$

$$(b) \left(\frac{S}{N}\right)_D = \frac{1/2}{1+1/2} \gamma = \frac{1}{3} \times 10^{10} \times 10^{-\ell/10} = 10^4 \Rightarrow \ell = 10(10 - 4 - \log_{10} 3) = 55.2 \text{ km}$$

$$(c) (S/N)_D = 3 \times 2^2 \times \frac{1}{2} \gamma = 6 \times 10^{10} \times 10^{-\ell/10} = 10^4 \Rightarrow \ell = 10(10 - 4 + \log_{10} 6) = 67.8 \text{ km}$$

$$(S/N)_D = 3 \times 8^2 \times \frac{1}{2} \gamma = 96 \times 10^{10} \times 10^{-\ell/10} = 10^4 \Rightarrow \ell = 10(10 - 4 + \log_{10} 96) = 79.8 \text{ km}$$

#### 10.4-4

$$L = \left( \frac{4\pi \times 3 \times 10^8}{3 \times 10^5} \ell \right)^2 = 1.58 \times 10^8 \ell^2 \quad \text{and} \quad g_T \times g_R = 52 \text{ dB} = 1.58 \times 10^5 \quad \text{so}$$

$$\gamma = \frac{g_T g_R S_T}{L N_0 W} \approx \frac{10^7}{\ell^2}$$

$$(a) (S/N)_D = \gamma = 10^7 / \ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{1000} = 31.6 \text{ km}$$

$$(b) \left( \frac{S}{N} \right)_D = \frac{1/2}{1 + 1/2} \gamma = 10^7 / 3\ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{1000/3} = 18.3 \text{ km}$$

$$(c) (S/N)_D = 3 \times 2^2 \times \frac{1}{2} \gamma = 6 \times 10^7 / \ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{6000} = 77.5 \text{ km}$$

$$(S/N)_D = 3 \times 8^2 \times \frac{1}{2} \gamma = 96 \times 10^7 / \ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{96,000} = 310 \text{ km}$$

#### 10.4-5

$$\text{AM: } \left( \frac{S}{N} \right)_D = \frac{1/2}{1 + 1/2} \gamma = 20 \quad \Rightarrow \quad \gamma = 60$$

$$\text{FM: } \gamma \geq \gamma_{th} = 20M(D) \quad \Rightarrow \quad M(D_{\max}) = 60/20 \quad \Rightarrow \quad D_{\max} \approx 1$$

$$(S/N)_D = 3D^2 \times \frac{1}{2} \gamma = 90 = 19.5 \text{ dB}$$

#### 10.4-6

At output of the  $k$ th BPF,

$$S_k = (f_{\Delta} \alpha_k)^2 \overline{x_k^2} = f_{\Delta}^2 \alpha_k^2, \quad N_k = 2 \int_{(k-1)W_0}^{kW_0} \frac{N_0 f^2}{2S_R} df = \frac{N_0}{3S_R} [k^3 W_0^3 - (k-1)^3 W_0^3]. \quad \text{Thus,}$$

$$\left( \frac{S}{N} \right)_k = \frac{f_{\Delta}^2 \alpha_k^2}{(3k^2 - 3k + 1)(N_0 W_0^3 / 3S_R)}$$

#### 10.4-7

$$\left( \frac{S}{N} \right)_k = \frac{\alpha_k^2}{3k^2 - 3k + 1} \frac{3S_R f_{\Delta}^2}{N_0 W_0^3} = \frac{S}{N} \quad \Rightarrow \quad \alpha_k^2 = C(3k^2 - 3k + 1) \quad \text{with} \quad C = \frac{N_0 W_0^3}{3S_R f_{\Delta}^2} \frac{S}{N}$$

$$\overline{x_b^2} = E \left[ \sum_{k=1}^K \sum_{i=1}^K \alpha_k x_k \alpha_i x_i \right] = \sum_{k=1}^K \sum_{i=1}^K \alpha_k \alpha_i \overline{x_k x_i} \quad \text{where} \quad \overline{x_k x_i} = \begin{cases} \overline{x_k^2} = 1 & i = k \\ \overline{x_k x_i} = 0 & i \neq k \end{cases} \quad (\text{cont.})$$

so

$$\begin{aligned}\overline{x_b^2} &= \sum_{k=1}^K \alpha_k^2 = 1 \text{ where } \sum_{k=1}^K \alpha_k^2 = C \sum_{k=1}^K [3k^2 - 3k + 1] \\ &= C \left[ 3 \frac{K(K+1)(2K+1)}{6} - 3 \frac{K(K+1)}{2} + K \right] = CK^3 = 1 \Rightarrow C = \frac{1}{K^3}\end{aligned}$$

$$\text{Thus, } \left( \frac{S}{N} \right)_k = \frac{S}{N} = \frac{3S_R f_\Delta^2}{N_0 W_0^3} \frac{1}{K^3} = 3 \left( \frac{f_\Delta}{W} \right)^2 \frac{S_R}{N_0 W}, \text{ where } W = K W_0$$

10.4-8

$$(a) \ G_{n_1}(f) = |H_{de}(f)|^2 G_{\xi_1}(f), \quad \overline{n_1^2} = \frac{N_0 B_{de}^3}{S_R} \left[ \frac{W}{B_{de}} - \arctan \frac{W}{B_{de}} \right] \approx 5.3 \times 10^{10} \frac{N_0}{S_R}$$

$$\begin{aligned}G_{n_2}(f) &= |H_{de}(f)|^2 \left[ G_{\xi_2}(f - f_0) + G_{\xi_2}(f + f_0) \right] \\ &= |H_{de}(f)|^2 \frac{N_0}{2S_R} [(f - f_0)^2 + (f + f_0)^2] = |H_{de}(f)|^2 \frac{N_0}{2S_R} (f^2 + f_0^2) \Pi\left(\frac{f}{2W}\right) \\ \overline{n_2^2} &= \int_{-W}^W \frac{N_0}{S_R} \frac{f^2 + f_0^2}{1 + (f/B_{de})^2} df = \frac{2N_0}{S_R} \left[ B_{de}^2 \int_0^{W/B_{de}} \frac{\lambda^2}{1 + \lambda^2} d\lambda + B_{de} f_0^2 \int_0^{W/B_{de}} \frac{d\lambda}{1 + \lambda^2} \right] \\ &= \frac{2N_0 B_{de}^3}{S_R} \left\{ \frac{W}{B_{de}} + \left[ \left( \frac{f_0}{B_{de}} \right)^2 - 1 \right] \arctan \frac{W}{B_{de}} \right\} \approx 8.8 \times 10^{12} \frac{N_0}{S_R} \gg \overline{n_1^2}\end{aligned}$$

$$(b) \ y_1 - y_2 = f_\Delta (x_L + x_R - x_L + x_R) + n_1 - n_2 \approx 2f_\Delta x_R - n_2, \text{ since } \overline{n_2^2} \gg \overline{n_1^2}$$

$$y_1 + y_2 = f_\Delta (x_L + x_R + x_L - x_R) + n_1 + n_2 \approx 2f_\Delta x_L + n_2. \text{ Thus,}$$

$$(S/N)_{D_L} = (S/N)_{D_R} \approx (2f_\Delta)^2 \overline{x_R^2} / \overline{n_2^2} = 1.5 \times 10^{-13} f_\Delta^2 S_R / N_0$$

For mono signal with  $\overline{x^2} = 1$ ,  $(S/N)_D = f_\Delta^2 \overline{x^2} / \overline{n_1^2} = 1.9 \times 10^{-11} f_\Delta^2 S_R / N_0$ . Thus,

$$(S/N)_D(\text{stereo}) / (S/N)_D(\text{mono}) \approx 8 \times 10^{-3} = -21 \text{ dB}$$

10.6-1

$$N_0 = kT_N = kT_0(T_N/T_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}, \quad \mu_p = t_0 = 0.1/f_s = 1/12 \times 10^6$$

(cont.)

$$\begin{aligned}\left(\frac{S}{N}\right)_D &= 4\mu_p^2 B_T \left(\frac{W}{f_s \tau_0}\right) \frac{S_x S_R}{N_0 W} = 4 \left(\frac{1}{12 \times 10^6}\right)^2 10^7 \left(\frac{500 \times 10^3}{1.2 \times 10^6 / 12 \times 10^6}\right) \frac{0.4 \times 10 \times 10^{-9}}{4 \times 10^{-20} \times 500 \times 10^3} \\ &= 2.78 \times 10^5 = 54.4 \text{ dB}\end{aligned}$$

10.6-2

$$\begin{aligned}N_0 &= kT_N = kT_0(T_N / T_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}, \mu_p = \mu \tau_0 = 0.2 / 50 \times 250 = 16 \times 10^{-6} \\ \left(\frac{S}{N}\right)_D &= 4\mu_p^2 B_T \left(\frac{W}{f_s \tau_0}\right) \frac{S_x S_R}{N_0 W} = 4(16 \times 10^{-6})^2 3 \times 10^3 \left(\frac{100}{250 / 50 \times 250}\right) \frac{0.1 S_R}{4 \times 10^{-20} \times 100} \\ &= 384 \times 10^{12} S_R \geq 10^4 \Rightarrow S_R \geq 26 \text{ pW}\end{aligned}$$

10.6-3

$$\begin{aligned}\left(\frac{S}{N}\right)_D &\leq \frac{1}{8} \times 20^2 S_x \gamma = 50 S_x \gamma. \text{ But with } \mu_p = t_0 = \frac{0.3}{f_s}, \tau_0 = \tau = \frac{0.2}{f_s}, \text{ and } B_T = 20W \\ \frac{4\mu_p^2 B_T W}{f_s \tau_0} &= 36 \left(\frac{W}{f_s}\right)^2 \Rightarrow \left(\frac{S}{N}\right)_D = 36 \left(\frac{1}{2.5}\right)^2 S_x \gamma \approx 5.8 S_x \gamma\end{aligned}$$

10.6-4

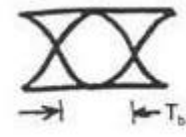
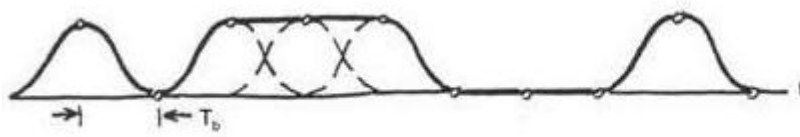
$$\begin{aligned}S_R &= 9f_s A^2 \tau + f_s A^2 3\tau \Rightarrow A^2 = \frac{S_R}{12f_s \tau}, \mu_p = t_0 = 3.6 \mu\text{s}, \tau_0 = \tau = 2.5 \mu\text{s}, \\ \text{and } B_T &= 400 \text{ kHz, so } \left(\frac{S}{N}\right)_D = \frac{4\mu_p^2 B_T A^2}{N_0} S_x = \frac{4t_0^2 B_T W}{12f_s \tau} S_x \frac{S_R}{N_0 W} = 2.49 \left(\frac{\gamma}{9}\right)\end{aligned}$$

10.6-5

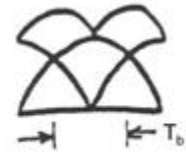
$$\begin{aligned}S_R &= Mf_s A^2 \tau \geq Mf_s A^2 / B_T \Rightarrow A^2 \leq B_T S_R / Mf_s \\ \mu_p = t_0 &= \frac{1}{2} \left( \frac{T_s}{M} - \tau - T_g \right) \leq \frac{1}{2} \left( \frac{1}{Mf_s} - \frac{2}{B_T} \right) = \frac{1}{20} \frac{B_T - 2Mf_s}{Mf_s B_T} \\ \left(\frac{S}{N}\right)_D &\leq \left( \frac{B_T - 2Mf_s}{Mf_s B_T} \right)^2 \frac{B_T^2 S_R}{Mf_s} S_x = \left( \frac{B_T - 2Mf_s}{Mf_s B_T} \right)^2 \left( \frac{W}{f_s} \right) S_x \frac{S_R}{MN_0 W} < \frac{1}{8} \left( \frac{B_T}{MW} \right)^2 S_x \frac{\gamma}{M}, f_s \geq 2W\end{aligned}$$

## Chapter 11

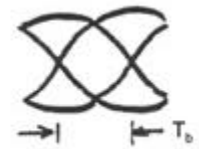
11.1-1



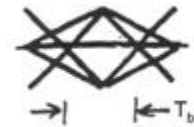
11.1-2



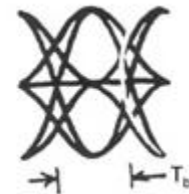
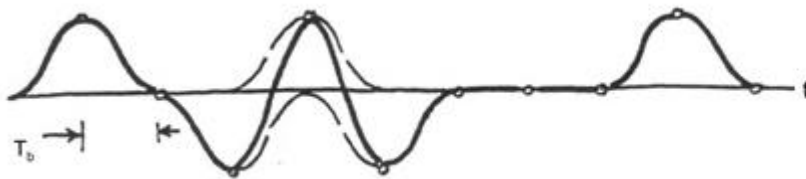
11.1-3



11.1-4



11.1-5



11.1-6

$a_k$	Nat. code	Gray codes
$7A/2$	111	100 010 001
$5A/2$	110	101 110 011
$3A/2$	101	111 100 111
$A/2$	100	110 101 101
$-A/2$	011	010 111 100
$-3A/2$	010	011 011 110
$-5A/2$	001	001 001 010
$-7A/2$	000	000 000 000

11.1-7

(a)  $r_b = 16 \times 20,000 = 320 \text{ kpbs}$ ,  $B \geq \frac{1}{2}r_b = 160 \text{ kHz}$

(b)  $r = 320 \text{ kpbs} / \log_2 M \leq 2B = 120 \text{ kbaud}$   
 $\log_2 M \geq 320/120 = 2.67 \Rightarrow M \geq 2^3 = 8$

11.1-8

(a)  $128 = 2^7 \Rightarrow 7 \text{ bits/character}$

$r_b = 7 \times 3000 = 21 \text{ kbps}$ ,  $B \geq \frac{1}{2}r_b = 10.5 \text{ kHz}$

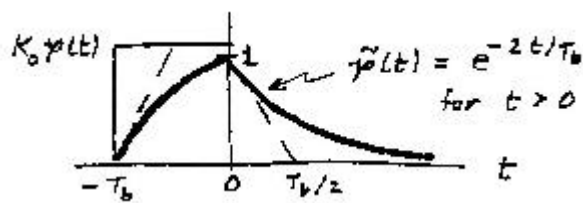
(b)  $r = 21 \text{ kbps} / \log_2 M \leq 2B = 6 \text{ kHz}$

$\log_2 M \geq 21/6 = 3.5 \Rightarrow M \geq 2^4 = 16$

11.1-9

(a)  $\tilde{p}(t) = g(t + T_b) - g(t)$

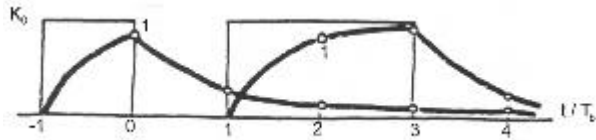
$\tilde{p}(0) = K_0(1 - e^{-2}) = 1 \Rightarrow K_0 = \frac{1}{1 - e^{-2}} = 1.157$





11.9 continued

(b)

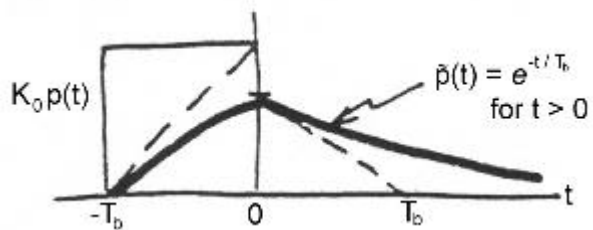


$t_k$	$y(t_k)$	ISI
0	1	0
1	$1e^{-2} = 0.135$	0.135
2	$1 + 1e^{-4} = 1.018$	0.018
3	$K_0(1 - e^{-4}) + 1e^{-6} = 1.138$	0.138
4	$K_0(1 - e^{-4})e^{-2} + 1e^{-8} = 0.154$	0.138

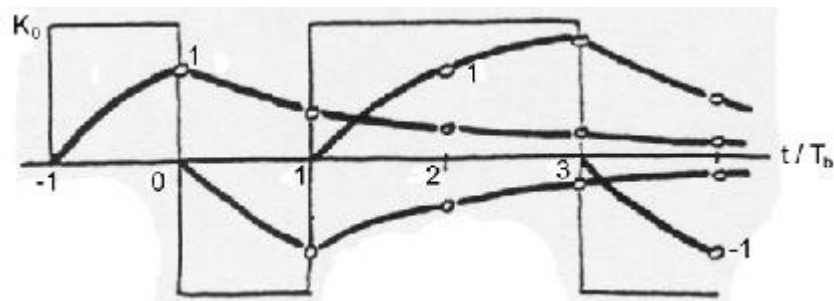
11.1-10

(a)  $\tilde{p}(t) = g(t + T_b) - g(t)$

$$\tilde{p}(0) = K_0(1 - e^{-1}) = 1 \Rightarrow K_0 = \frac{1}{1 - e^{-1}} = 1.582,$$



(b)



11.1-10 (b) continued

$t_k$	$y(t_k)$	ISI
0	1	0
1	$-1 + 1e^{-1} = -0.632$	+0.368
2	$1 - 1e^{-1} + 1e^{-2} = 0.767$	-0.233
3	$K_0(1 - e^{-2}) - 1e^{-2} + 1e^{-3} = 1.282$	+0.282
4	$-1 + K_0(1 - e^{-2})e^{-1} - 1e^{-3} + 1e^{-4} = -0.528$	+0.472

11.1-11

$$p(D) = e^{-\pi(B/b)^2} \leq 0.01 \Rightarrow \pi(bD)^2 \geq \ln 100 \text{ where } D = 1/r$$

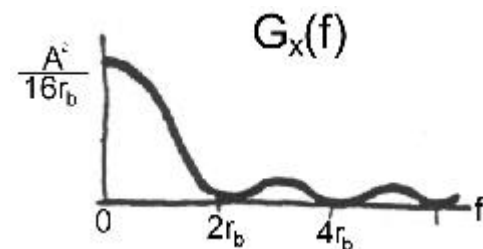
$$\frac{P(B)}{P(0)} = e^{-\pi(B/b)^2} \leq 0.01 \Rightarrow \pi(B/b)^2 \geq \ln 100$$

$$\text{thus } \pi(b/r)^2 \times \pi(B/b)^2 \geq (\ln 100)^2 \Rightarrow r \leq \frac{\pi}{\ln 100} B \approx 0.7B$$

11.1-12

$$m_a = \bar{a}_k = 0, \sigma_a^2 = \overline{a_k^2} = (A/2)^2, P(f) = \frac{1}{2r_b} \text{sinc} \frac{f}{2r_b}$$

$$\Rightarrow G_x(f) = \frac{A^2}{16r_b} \text{sinc}^2 \frac{f}{2r_b}$$



11.1-12 continued

$$\overline{x^2} = \int_{-\infty}^{\infty} G_x(f) df = 2 \frac{A^2}{16r_b} \int_0^{\infty} \text{sinc}^2 \frac{f}{2r_b} df = \frac{A^2}{8}$$

A waveform with polar RZ format, amplitude =  $\pm A/2$ , period =  $T_b$  and 50% duty cycle

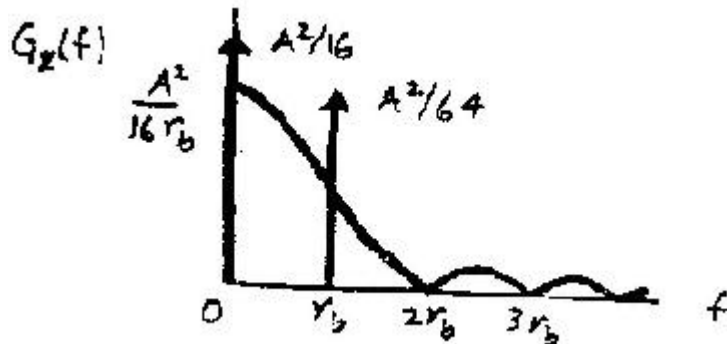
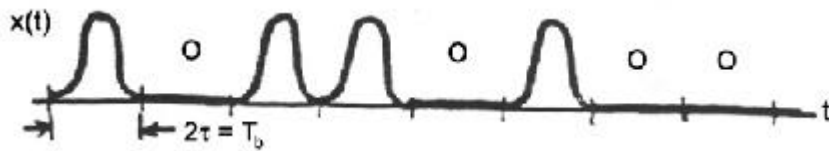
$$\Rightarrow \overline{x^2} = \frac{T_b/2}{T_b} \left( \frac{A}{2} \right)^2 = \frac{A^2}{8}$$

11.1-13

$$m_a = A/2, \quad \overline{a_k^2} = \frac{1}{2} A^2, \quad \sigma_a^2 = A^2/4, \quad P(f) = (\tau \text{sinc} 2f\tau) / [1 - (2f\tau)^2]$$

$$(a) \quad \tau = 1/2r_b, \quad P(0) = 1/2r_b, \quad P(\pm r_b) = 1/4r_b, \quad P(nr_b) = 0, \quad |n| \geq 2$$

$$G_x(f) = \frac{A^2}{4} r_b^2 |P(f)|^2 + \frac{A^2}{4} r_b^2 \left( \frac{1}{2r_b} \right)^2 \delta(f) + \frac{A^2}{4} r_b^2 \left( \frac{1}{4r_b} \right)^2 [\delta(f - r_b) + \delta(f + r_b)]$$

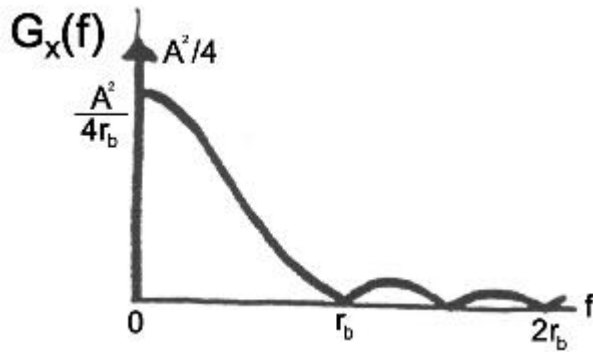


$$\tau = 1/r_b, \quad P(0) = 1/r_b, \quad P(nr_b) = 0, \quad n \neq 0$$

$$G_x(f) = \frac{A^2}{4} r_b^2 |P(f)|^2 + \frac{A^2}{4} r_b^2 \left( \frac{1}{r_b} \right)^2 \delta(f)$$



11.1-13 continued

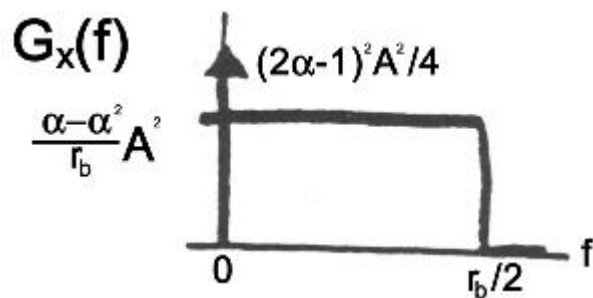


Note larger dc component and smoother waveform.

11.1-14

$$\left. \begin{aligned} m_a = \overline{a_k} &= \alpha \frac{A}{2} + (1-\alpha)(-A/2) = (2\alpha-1)A/2 \\ \overline{a_k^2} &= \alpha(A/2)^2 + (1-\alpha)(-A/2)^2 = A^2/4 \end{aligned} \right\} \sigma_a^2 = [1 - (2\alpha-1)^2] A^2/4 = (\alpha - \alpha^2) A^2$$

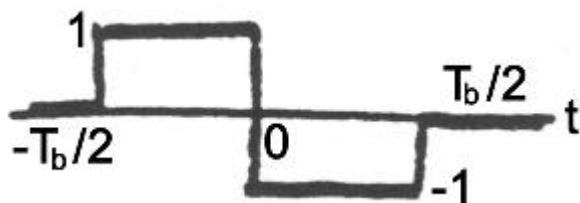
$$P(f) = \frac{1}{r_b} \Pi(f/r_b), \quad G_x(f) = \frac{(\alpha - \alpha^2)}{r_b} \Pi(f/r_b) + \left( \frac{2\alpha-1}{4} \right)^2 \delta(f)$$



$$\overline{x^2} = \int_{-\infty}^{\infty} G_x(f) df = \frac{(\alpha - \alpha^2)}{r_b} A^2 \times 2 r_b/2 + (2\alpha-1)^2 A^2/4 = A^2/4$$

11.1-15

$$p(t) = \Pi\left(\frac{t+T_b/4}{T_b/2}\right) - \Pi\left(\frac{t-T_b/4}{T_b/2}\right)$$

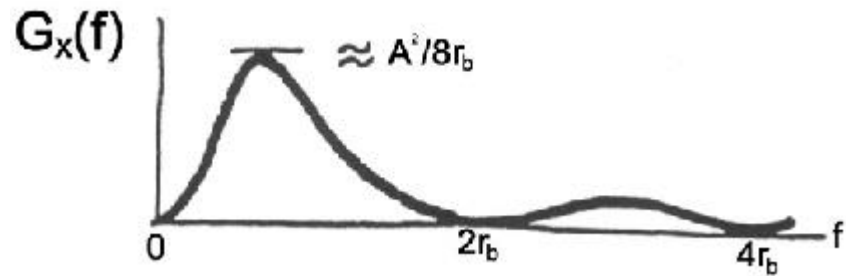


11.1-15 continued

$$P(f) = \frac{T_b}{2} \operatorname{sinc}\left(\frac{fT_b}{2}\right) (e^{j2\pi T_b/4} - e^{-j2\pi T_b/4}) = jT_b \operatorname{sinc}\frac{f}{2r_b} \sin\frac{\pi f}{2r_b}$$

$$a_k = \pm A/2 \Rightarrow m_a = 0, \quad \sigma_a^2 = \overline{a_k^2} = A^2/4$$

$$G_x(f) = \frac{A^2}{4r_b} \operatorname{sinc}^2 \frac{f}{2r_b} \sin^2 \frac{\pi f}{2r_b}$$



11.1-16

$$R_a(n) = E[a_{k-n}a_k]$$

$$n=0$$

$$a_k \quad P(a_k)$$

$$\begin{array}{cc} 0 & 1/2 \end{array}$$

$$\begin{array}{cc} \pm A & 1/4 \end{array}$$

$$\Rightarrow R_a(0) = \frac{1}{2} \times 0 + 2 \times \frac{1}{4} A^2 = A^2/2$$

11.1-16 continued

$n = 1$

$a_{k-1}$	$a_k$	$P(a_{k-1}a_k)$	
0	0	1/2 x 1/2	
0	A	1/2 x 1/4	
0	-A	1/2 x 1/4	
A	0	1/4 x 1/2	
-A	0	1/4 x 1/2	
A	-A	$1/2 \times (1/2)^2$	$\left. \vphantom{\begin{matrix} 1/2 \times (1/2)^2 \\ 1/2 \times (1/2)^2 \end{matrix}} \right\} = \frac{1}{2} P(1,1)$
-A	A	$1/2 \times (1/2)^2$	
A	A	0	$\left. \vphantom{\begin{matrix} 0 \\ 0 \end{matrix}} \right\} \text{not allowed}$
-A	-A	0	

$$R_a(\pm 1) = \frac{1}{4} \times 0 + 4 \times \frac{1}{8} \times 0 + 2 \times \frac{1}{8} (A)(-A) = -A^2 / 4$$

$n \geq 2$

$a_{k-n}$	$a_k$	$P(a_{k-n}a_k)$	
0	0	1/2 x 1/2	
0	A	1/2 x 1/4	
0	-A	1/2 x 1/4	
A	0	1/4 x 1/2	
-A	0	1/4 x 1/2	
A	-A	$(1/4)^2$	$\left. \vphantom{\begin{matrix} (1/4)^2 \\ (1/4)^2 \\ (1/4)^2 \\ (1/4)^2 \end{matrix}} \right\} = P(1)P(1)$
-A	A	$(1/4)^2$	
A	A	$(1/4)^2$	
-A	-A	$(1/4)^2$	

$$R_a(n) = \frac{1}{4} \times 0 + 4 \times \frac{1}{8} \times 0 + 2 \times \frac{1}{16} (A)(-A) + 2 \times \frac{1}{16} (\pm A)^2 = 0 \quad |n| \geq 2$$

11.1-17

$$\begin{aligned}
 \sum_{k=-K}^K \sum_{i=-K}^K g(k-i) &= \sum_{k=-K}^K [g(k-K) + g(k-K-1) + \dots + g(k+K)] \\
 &\left. \begin{aligned}
 &[g(-2K) + g(-2K+1) + \dots + g(0)] \\
 &+ [g(-2K+1) + \dots + g(0) + g(1)] \\
 &+ \dots + [g(0) + g(1) + \dots + g(2K)]
 \end{aligned} \right\} 2K+1 \text{ sums} \\
 &= g(-2K) + 2g(-2K+1) + \dots + (2K+1)g(0) + 2Kg(1) + \dots + 2g(2K-1) + g(2K) \\
 &= \sum_{n=-2K}^0 (2K+1+n)g(n) + \sum_{n=1}^{2K} (2K+1-n)g(n) \\
 &= \sum_{n=-2K}^{2K} (2K+1-|n|)g(n) = (2K+1) \sum_{n=-2K}^{2K} \left(1 - \frac{|n|}{2K+1}\right) g(n)
 \end{aligned}$$

$$\begin{aligned}
 E[a_k a_i] &= E[a_k a_{k-(k-i)}] = R_a(k-i) \\
 \text{thus, } p_k(f) &= \sum_{k=-K}^K \sum_{i=-K}^K g(k-i) \text{ where } g(k-i) = R_a(k-i)e^{-j\omega(k-i)D} \\
 &= (2K+1) \sum_{n=-2K}^{2K} \left(1 - \frac{|n|}{2K+1}\right) R_a(n) e^{-j\omega nD}
 \end{aligned}$$

11.1-18

$$\begin{aligned}
 \text{(a) } \overline{a_k} &= \frac{1}{2} \times 1 + \frac{1}{2} \times 0 = \frac{1}{2}, \quad \overline{a_k^2} = \frac{1}{2} \times 1^2 + \frac{1}{2} \times 0^2 = \frac{1}{2} \\
 \overline{b_k} &= 1 - \overline{a_k} = \frac{1}{2}, \quad \overline{b_k^2} = 1 - 2\overline{a_k} + \overline{a_k^2} = \frac{1}{2}, \quad \overline{a_k b_k} = \overline{a_k} - \overline{a_k^2} = 0 \\
 \text{for } i \neq k, \quad \overline{a_k a_i} &= \overline{a_k} \overline{a_i} = \frac{1}{4}, \quad \overline{b_k b_i} = \overline{b_k} \overline{b_i} = \frac{1}{4}, \quad \overline{a_k b_i} = \overline{a_k} \overline{b_i} = \frac{1}{4} \\
 \text{(b) } x_T(t) &= \sum_{k=-K}^K [a_k p_1(t - kT_b) + b_k p_0(t - kT_b)] \\
 X_T(f) &= \sum_{k=-K}^K [a_k P_1(f) + b_k P_0(f)] e^{-j\omega k T_b} \\
 |X_T(f)|^2 &= \sum_k \sum_i [a_k P_1 + b_k P_0] e^{-j\omega k T_b} [a_i P_1^* + b_i P_0^*] e^{+j\omega i T_b} \\
 &= \sum_k \sum_i [a_k a_i P_1 P_1^* + a_k b_i P_1 P_0^* + a_i b_k P_1^* P_0 + b_k b_i P_0 P_0^*] e^{+j\omega(k-i)T_b}
 \end{aligned}$$

11.1-18 (b) continued

let  $A = P_1 P_0^* + P_0 P_1^*$  and  $B = P_1 P_1^* + P_1 P_0^* + P_1^* P_0 + P_0 P_0^* = |P_1 + P_0|^2$ , so

$$E[|X_T(f)|^2] = \sum_{k=-K}^K \left\{ \frac{1}{2} A + \sum_{\substack{i=-K \\ i \neq k}}^K \frac{1}{4} B e^{-j\omega k - i)T_b} \right\} = \sum_{k=-K}^K \left\{ \frac{1}{2} A - \frac{1}{4} B + \sum_{i=-K}^K \frac{1}{4} B e^{-j\omega k - i)T_b} \right\}$$

$$\text{where } \frac{1}{2} A - \frac{1}{4} B = \frac{1}{4} [P_1 P_1^* + P_1 P_0^* + P_1^* P_0 + P_0 P_0^*] = \frac{1}{4} |P_1 - P_0|^2$$

$$\text{thus, } E[|X_T(f)|^2] = \frac{2K+1}{4} |P_1 - P_0|^2 + \frac{1}{4} |P_1 + P_0|^2 \sum_{k=-K}^K \sum_{i=-k}^K e^{-j\omega k - i)T_b}$$

$$\text{Hence, } G_x(f) = \frac{r_b}{4} |P_1(f) - P_0(f)|^2 + \frac{r_b^2}{4} \sum_{n=-\infty}^{\infty} |P_1(nr_b) + P_0(nr_b)|^2 \delta(f - nr_b)$$

$$(c) \sum_{k=-K}^K \sum_{i=-K}^K e^{-j\omega k - i)T_b} = (2K+1) \sum_{n=-2K}^{2K} \left( 1 - \frac{|n|}{2K+1} \right) e^{-j\omega n T_b}$$

$$\text{so } E[|X_T(f)|^2] = \frac{2K+1}{4} \left[ |P_1 - P_0|^2 + |P_1 + P_0|^2 \sum_{n=-2K}^{2K} \left( 1 - \frac{|n|}{2K+1} \right) e^{-j\omega n T_b} \right]$$

$$\begin{aligned} G_x(f) &= \lim_{K \rightarrow \infty} \frac{1}{(2K+1)T_b} E[|X_T(f)|^2] \\ &= \frac{1}{4T_b} |P_1 - P_0|^2 + \frac{1}{4T_b} |P_1 + P_0|^2 \left\{ \lim_{K \rightarrow \infty} \left[ \sum_{n=-2K}^{2K} e^{-j\omega n T_b} - \frac{1}{2K+1} \sum_{n=-2K}^{2K} |n| e^{-j\omega n T_b} \right] \right\} \\ &= \frac{1}{4T_b} |P_1 - P_0|^2 + \frac{1}{4T_b} |P_1 + P_0|^2 \sum_{n=-\infty}^{\infty} e^{-j\omega n T_b} \end{aligned}$$

$$\text{where } \frac{1}{T_b} = r_b \text{ and } \sum_{n=-\infty}^{\infty} e^{-j\omega n T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)$$

$$\text{Hence, } G_x(f) = \frac{r_b}{4} |P_1(f) - P_0(f)|^2 + \frac{r_b^2}{4} \sum_{n=-\infty}^{\infty} |P_1(nr_b) + P_0(nr_b)|^2 \delta(f - nr_b)$$

11.2-1

$$P_e = Q\left(\sqrt{\frac{1}{2}(S/N)_R}\right) = 10^{-3} \Rightarrow (S/N)_R \approx 2 \times 3.1^2 = 19.2$$

$$\text{Polar: } P_e = Q\left(\sqrt{(S/N)_R}\right) = Q(4.38) \approx 6 \times 10^{-6}$$



11.2-2

$$P_e = Q(A/2\sigma) \leq 10^{-6} \Rightarrow A/2\sigma \geq 4.76$$

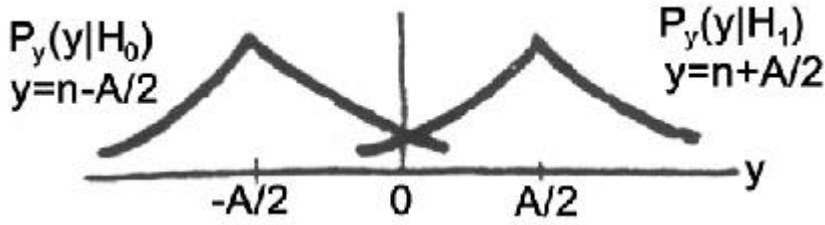
$$\text{Polar: } 4.76^2 \leq (A/2\sigma)^2 \leq S_R / (N_0 r_b / 2) \Rightarrow S_R \geq 4.76^2 \times \frac{1}{2} \times 10^{-8} = 0.113 \mu\text{W}$$

$$\text{Unipolar: } (A/2\sigma)^2 \leq \frac{1}{2} S_R / (N_0 r_b / 2) \Rightarrow S_R \geq 0.226 \mu\text{W}$$

11.2-3

(a) From symmetry

$$V_{opt} = 0 \Rightarrow P_e = P_{e0} = \int_0^\infty p_n(y + A/2) dy = \frac{1}{\sqrt{2\sigma^2}} \int_0^\infty e^{-\sqrt{2}(y+A/2)} dy = \frac{1}{2} e^{-A/\sqrt{2}\sigma}$$



$$(b) P_e \leq 10^{-3} \Rightarrow A/\sqrt{2}\sigma \geq \ln \frac{1}{2 \times 10^{-3}} \Rightarrow A \geq 8.8\sigma$$

$$\text{Gaussian noise: } P_e = Q(A/2\sigma) \leq 10^{-3} \Rightarrow A \geq 6.2\sigma$$

Note that impulse noise requires more signal power for same  $P_e$

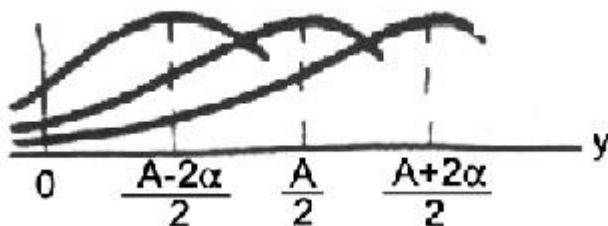
11.2-4

(a) From symmetry,  $P_e = P_{e1}$

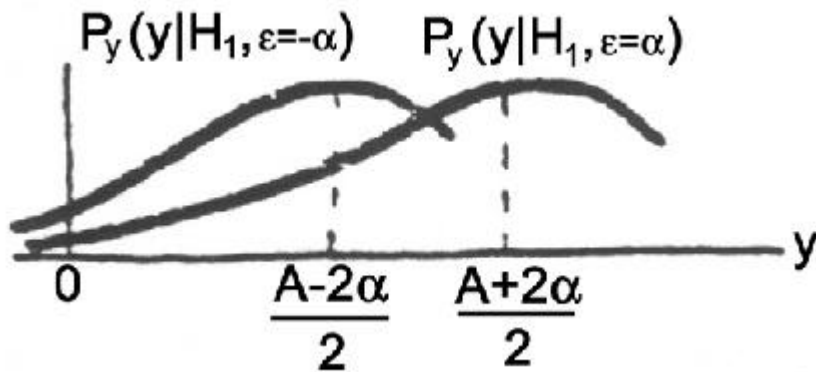
$$= P(y < 0 | \mathbf{e} = -\infty) + P(\mathbf{e} = -\infty) + P(y < 0 | \mathbf{e} = \infty) + P(\mathbf{e} = \infty)$$

$$= \frac{1}{2} Q\left(\frac{A-2a}{2\sigma}\right) + \frac{1}{2} Q\left(\frac{A+2a}{2\sigma}\right)$$

$$p_Y(y | H_1, \mathbf{e} = -a) \quad p_Y(y | H_1, \mathbf{e} = 0) \quad p_Y(y | H_1, \mathbf{e} = a)$$



11.2-4 continued



(b)  $(A \pm 2a) / 2s = (1 \pm 0.2)4,$

$$P_e = \frac{1}{2}Q(3.2) + \frac{1}{2}Q(4.8)$$

$$= \frac{1}{2}(7.4 \times 10^{-4} + 8.5 \times 10^{-7}) \approx 3.7 \times 10^{-4}$$

If  $e = 0$ , then  $P_e = Q(4.0) = 3.4 \times 10^{-5}$

11.2-5

(a) From symmetry,  $P_e = P_{e1}$

$$= P(y < 0 | e = -\infty)P(e = -\infty) + P(y < 0 | e = 0)P(e = 0) + P(y < 0 | e = a)P(e = a)$$

$$= \frac{1}{4}Q\left(\frac{A-2a}{2s}\right) + \frac{1}{2}Q\left(\frac{A}{2s}\right) + \frac{1}{4}Q\left(\frac{A+2a}{2s}\right)$$

$$p_Y(y | H_1, e = -a) \quad p_Y(y | H_1, e = 0) \quad p_Y(y | H_1, e = a)$$

(b)  $(A \pm 2a) / 2s = (1 \pm 0.2)4,$

$$P_e = \frac{1}{4}Q(3.2) + \frac{1}{2}Q(4) + \frac{1}{4}Q(4.8)$$

$$= \frac{1}{4}(7.4 \times 10^{-4} + \frac{1}{2} \times 3.4 \times 10^{-5} + \frac{1}{4} \times 8.5 \times 10^{-7}) \approx 2 \times 10^{-4}$$

If  $e = 0$ , then  $P_e = Q(4.0) = 3.4 \times 10^{-5}$

11.2-6

$$p_y(y | H_0) = p_n(y + A/2), p_y(y | H_1) = p_n(y - A/2), p_n(n) = \frac{1}{\sqrt{2ps^2}} e^{-n^2/2s^2}$$

$$\text{so } P_0 \frac{1}{\sqrt{2ps^2}} e^{-(V+A/2)^2/2s^2} = P_1 \frac{1}{\sqrt{2ps^2}} e^{-(V-A/2)^2/2s^2}$$

$$P_0 / P_1 = e^{[(V+A/2)^2 - (V-A/2)^2]/2s^2} = e^{VA/s^2}$$

$$\text{Hence, } VA/s^2 = \ln(P_0 / P_1) \Rightarrow V_{opt} = \frac{s^2}{A} \ln(P_0 / P_1)$$

11.2-7

$$\frac{dP_e}{dV} = P_0 \frac{dP_{e0}}{dV} + P_1 \frac{dP_{e1}}{dV} = 0 \quad \text{when } V = V_{opt}$$

$$\frac{dP_{e0}}{dV} = \frac{d}{dV} \int_{a(V)}^{b(V)} g(v, y) dy \quad \text{where } a(V) = V, \quad b(V) = \infty, \quad g(V, y) = p_y(y | H_0)$$

$$= 0 - p_y(V | H_0) + 0 \quad \text{since } \frac{db(V)}{dV} = 0, \quad \frac{\partial}{\partial V} [p_y(y | H_0)] = 0$$

$$\text{similarly, } \frac{dP_{e1}}{dV} = 0 + p_y(V | H_1) + 0$$

$$\text{hence, } -P_0 p_y(V | H_0) + P_1 p_y(V | H_1) = 0 \Rightarrow P_0 p_y(V | H_0) = P_1 p_y(V | H_1)$$

11.2-8

$$(S/N)_1 = 20 \text{ dB} = 100$$

$$\text{Regenerative: } P_e \approx 20Q\left[\sqrt{100}\right] \approx 20 \frac{1}{\sqrt{2p100}} e^{-100/2} \approx 1.5 \times 10^{-22}$$

$$\text{Nonregenerative: } P_e \approx Q\left[\sqrt{\frac{1}{20} \times 100}\right] = Q(2.24) \approx 1.2 \times 10^{-2}$$

11.2-9

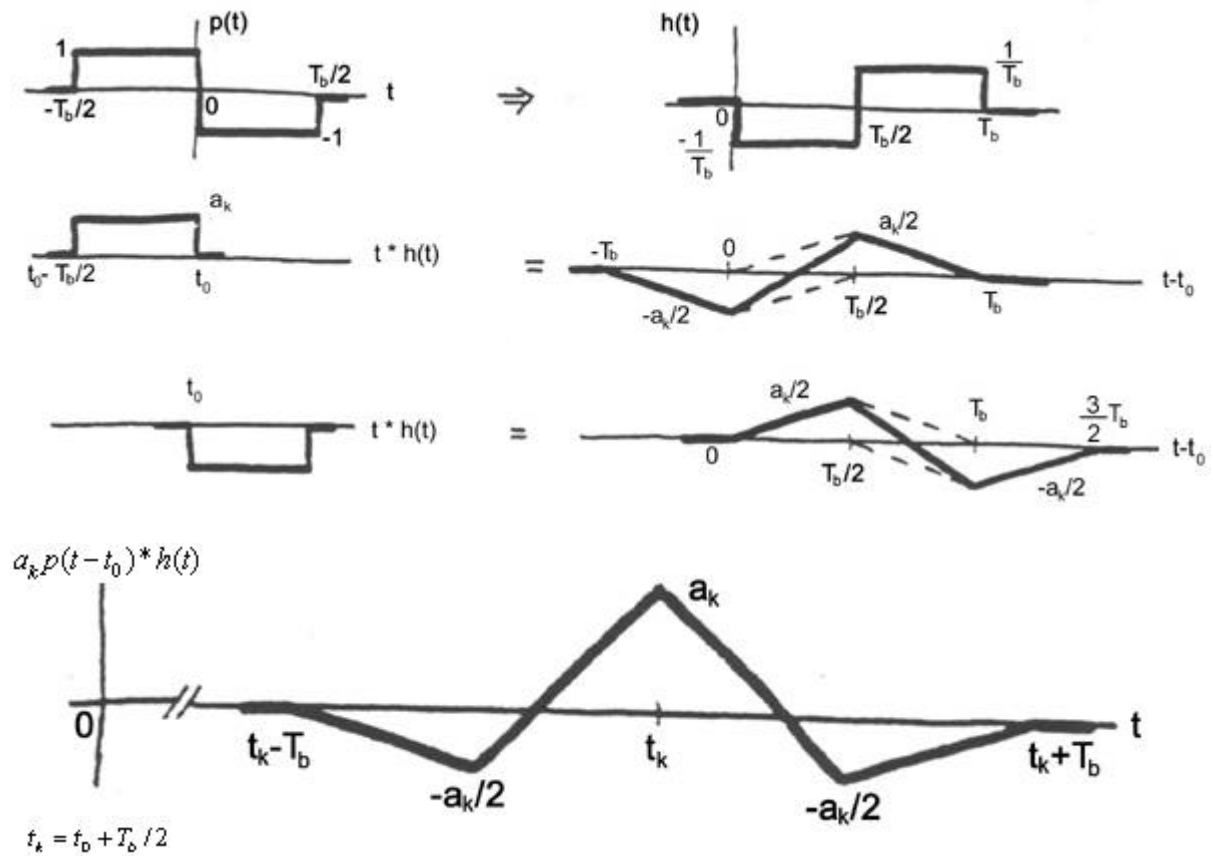
$$\text{Regenerative: } P_e = 50Q\left[\sqrt{(S/N)_1}\right] = 10^{-4} \Rightarrow \sqrt{(S/N)_1} \approx 4.62$$

$$\text{so } (S/N)_1 = 21.3 = 13.3 \text{ dB}$$

$$\text{Nonregenerative: } P_e = Q\left[\sqrt{\frac{1}{50}(S/N)_1}\right] = 10^{-4} \Rightarrow \sqrt{\frac{1}{50}(S/N)_1} \approx 3.73$$

$$\text{so } (S/N)_1 = 50 \times 3.73^2 = 696 = 28.4 \text{ dB}$$

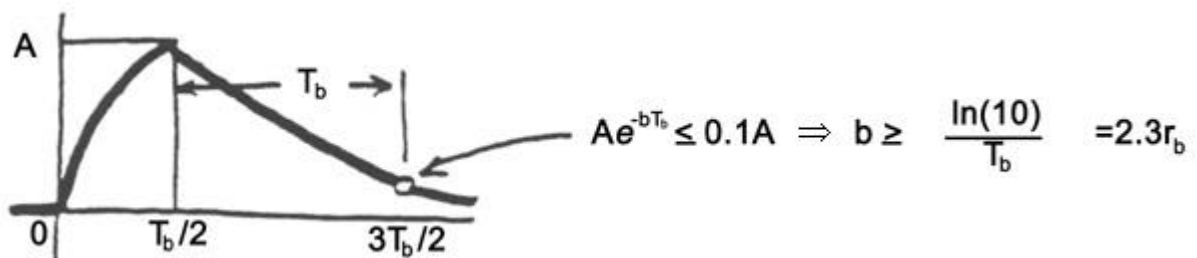
11.2-10



11.2-11

$$(a) \quad 0 \leq t \leq T_b/2: Ap(t) * h(t) = AK_0 \int_0^t e^{-b\lambda} d\lambda = \frac{AK_0}{b} (1 - e^{-bt}) = \frac{A}{(1 - e^{-bT_b/2})} (1 - e^{-bt})$$

$$t \geq T_b/2: Ap(t) * h(t) = AK_0 \int_{t-T_b/2}^t e^{-b\lambda} d\lambda = \frac{AK_0}{b} [e^{-b(t-T_b/2)} - e^{-bt}] = Ae^{-b(t-T_b/2)}$$



11.2-11 continued

$$(b) \quad \sigma^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{K_0^2 N_0}{2} \int_0^{\infty} e^{-2bt} dt = \frac{K_0^2 N_0}{4b}$$

$$E_b = \overline{a_k^2} \int_{-\infty}^{\infty} p(t)^2 dt = \frac{A^2}{2} \frac{T_b}{2} = \frac{A^2}{4r_b}$$

$$\text{thus, } \left( \frac{A}{2\sigma} \right)^2 = \frac{4E_b r_b}{4(K_0^2 N_0 / 4b)} = \frac{4b r_b}{K_0^2} \gamma_b = \frac{4r_b}{b} (1 - e^{-bT_b/2})^2 \gamma_b \leq 0.812 \gamma_b \text{ if } b \geq 2.3r_b$$

11.2-12

$$(a) \quad Q(\sqrt{2\gamma_b}) = 10^{-4} \Rightarrow \gamma_b = \frac{1}{2} \times 3.7^2, \quad S_R \geq N_0 r_b \gamma_b = 34.2 \text{ pW}$$

$$(b) \quad 2 \frac{7}{8 \times 3} Q\left(\sqrt{\frac{6 \times 3}{63} \gamma_b}\right) = 10^{-4} \Rightarrow Q(\sqrt{0.286 \gamma_b}) = 1.7 \times 10^{-4}$$

$$\gamma_b = \frac{1}{0.286} \times 3.6^2, \quad S_R \geq N_0 r_b \gamma_b = 227 \text{ pW}$$

11.2-13

$$r = \frac{500 \times 10^3}{\log_2 M} \leq 2B = 160 \times 10^3 \Rightarrow \log_2 M \geq \frac{50}{16} = 3.125 \text{ so } M_{\min} = 2^4 = 16$$

$$2 \frac{15}{16 \times 4} Q\left(\sqrt{\frac{6 \times 4}{255} \gamma_b}\right) = 10^{-4} \Rightarrow Q(\sqrt{0.0941 \gamma_b}) = 2.13 \times 10^{-4}$$

$$\text{so } \gamma_b = \frac{1}{0.0941} \times 3.55^2, \text{ and } S_R \geq N_0 r_b \gamma_b = 670 \text{ pW}$$

11.2-14

With matched filtering;  $P_{be} \approx 2 \frac{M-1}{M \log_2 M} Q \left( \sqrt{\frac{600 \log_2 M}{M^2 - 1}} \right)$

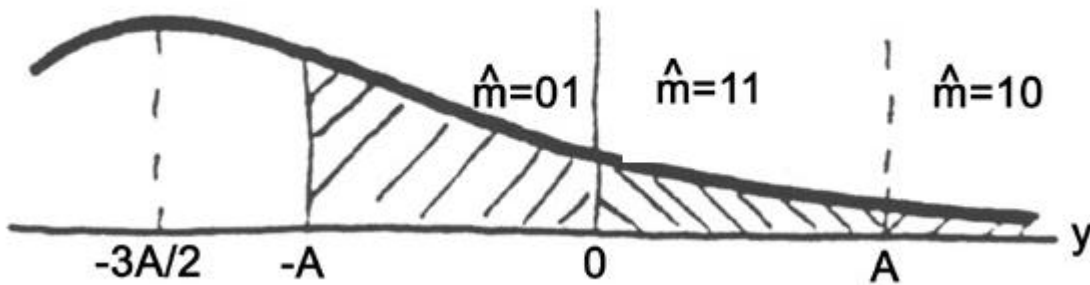
$M$	$2 \frac{M-1}{M \log_2 M}$	$\sqrt{\frac{600 \log_2 M}{M^2 - 1}}$	$Q$	$P_{be}$
4	0.75	8.94	$< 10^{-12}$	$< 10^{-12}$
8	0.583	5.35	$4.5 \times 10^{-8}$	$2.6 \times 10^{-8} \Rightarrow \text{take } M = 8$
16	0.469	3.07	$> 10^{-3}$	$> 10^{-4}$

11.2-15

$$\begin{aligned}
 \overline{a_k^2} &= \frac{1}{M} \left\{ \left[ -\frac{(M-1)A}{2} \right]^2 + \dots + \left( \frac{-3A}{2} \right)^2 + \left( -\frac{A}{2} \right)^2 + \left( \frac{3A}{2} \right)^2 + \dots + \left[ \frac{(M-1)A}{2} \right]^2 \right\} \\
 &= 2 \times \frac{1}{M} \left[ 1^2 + 3^2 + \dots + (M-1)^2 \right] \left( \frac{A}{2} \right)^2 \\
 &= \frac{A^2}{2M} \sum_{i=1}^{M/2} (2i-1)^2 = \frac{A^2}{2M} \sum_{i=1}^{M/2} (4i^2 - 4i + 1) \\
 &= \frac{A^2}{2M} \left[ 4 \frac{(M/2)(1+M/2)(M+1)}{6} - 4 \frac{(M/2)(1+M/2)}{2} + \frac{M}{2} \right] = (M^2 - 1) \frac{A^2}{12}
 \end{aligned}$$

11.2-16

Let  $\tilde{m}$  = regenerated  $m$  and consider  $m = 00$



$$P(\tilde{m} = 01) = Q(A/2\sigma) - Q(3A/2\sigma) \quad 1 \text{ bit error}$$

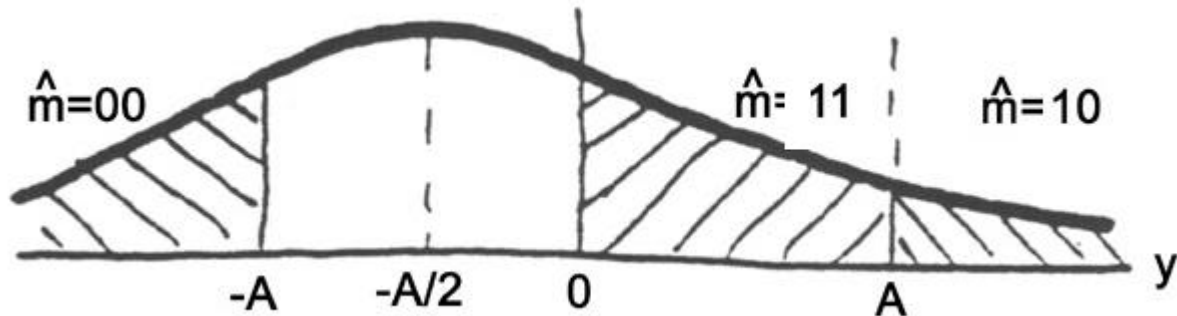
$$P(\tilde{m} = 11) = Q(3A/2\sigma) - Q(5A/2\sigma) \quad 2 \text{ bit errors}$$

$$P(\tilde{m} = 10) = Q(5A/2\sigma) \quad 1 \text{ bit error}$$

Similarly when  $m = 10$ .

11.2-16 continued

Now consider  $m = 01$ , and similarly  $m = 11$ .



$$P(\tilde{m} = 00) = Q(A/2\sigma) \quad 1 \text{ bit error}$$

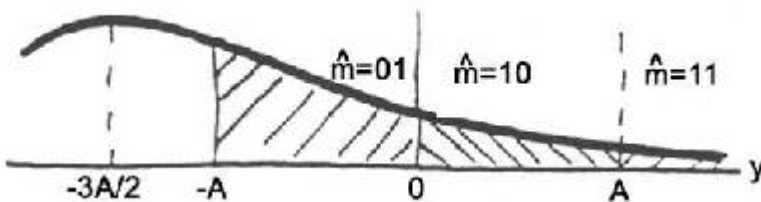
$$P(\tilde{m} = 11) = Q(A/2\sigma) - Q(3A/2\sigma) \quad 1 \text{ bit error}$$

$$P(\tilde{m} = 10) = Q(3A/2\sigma) \quad 2 \text{ bit errors}$$

$$\begin{aligned} \text{Thus, } P_{be} &= 2 \times \frac{1}{4} \{ [Q(k) - Q(3k)] + 2[Q(3k) - Q(5k)] + Q(5k) \} \\ &\quad + 2 \times \frac{1}{4} \{ Q(k) + [Q(k) - Q(3k)] + 2Q(3k) \} \\ &= \frac{3}{2}Q(k) + Q(3k) - \frac{1}{2}Q(5k) \approx \frac{3}{2}Q(k) \quad \text{when } k > 1 \text{ since } Q(5k) \ll Q(3k) \ll Q(k) \end{aligned}$$

11.2-17

Let  $\tilde{m}$  = regenerated  $m$  and consider  $m = 00$  (similarly for  $m = 11$ )



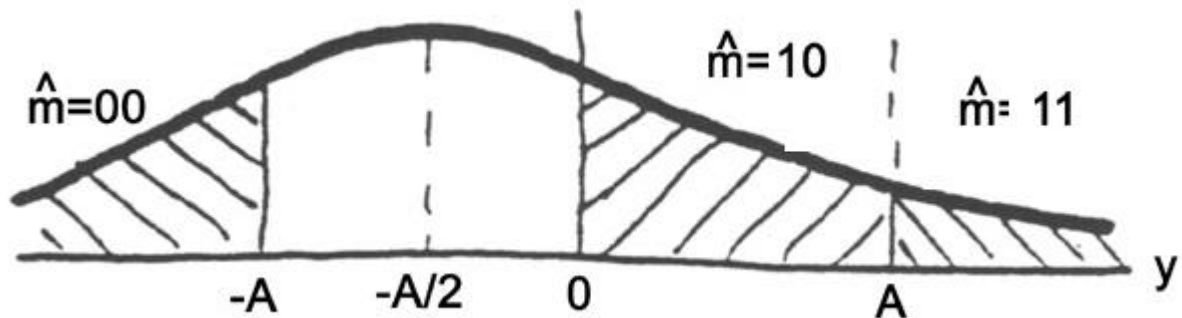
$$P(\tilde{m} = 01) = Q(A/2\sigma) - Q(3A/2\sigma) \quad 1 \text{ bit error}$$

$$P(\tilde{m} = 10) = Q(3A/2\sigma) - Q(5A/2\sigma) \quad 1 \text{ bit error}$$

$$P(\tilde{m} = 11) = Q(5A/2\sigma) \quad 2 \text{ bit errors}$$

11.2-17 continued

Now consider  $m = 01$  (similarly  $m = 10$ )



$$P(\tilde{m} = 00) = Q(A/2\sigma) \quad 1 \text{ bit error}$$

$$P(\tilde{m} = 10) = Q(A/2\sigma) - Q(3A/2\sigma) \quad 2 \text{ bit errors}$$

$$P(\tilde{m} = 11) = Q(3A/2\sigma) \quad 1 \text{ bit error}$$

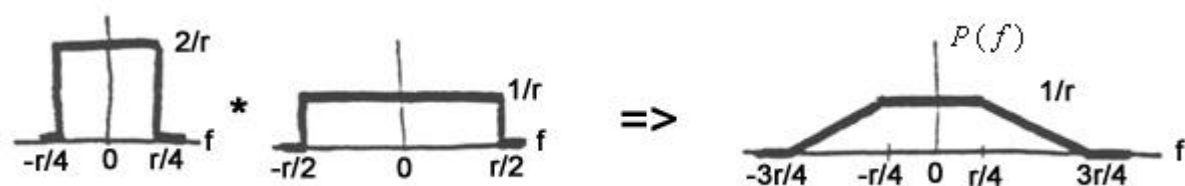
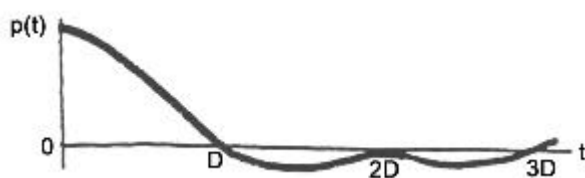
$$\begin{aligned} \text{Thus, } P_{be} &= 2 \times \frac{1}{4} \{ [Q(k) - Q(3k)] + 2[Q(3k) - Q(5k)] + 2Q(5k) \} \\ &\quad + 2 \times \frac{1}{4} \{ Q(k) + 2[Q(k) - Q(3k)] + 2Q(3k) \} \\ &= 2Q(k) - \frac{1}{2}Q(3k) + \frac{1}{2}Q(5k) \approx 2Q(k) \quad \text{when } k > 1 \text{ since } Q(5k) \ll Q(3k) \ll Q(k) \end{aligned}$$

11.3-1

$$p_{\beta}(t) = \text{sinc} \pi r t / 2$$

$$p(t) = \text{sinc} \frac{rt}{2} \text{sinc} \pi r t, \frac{1}{r} = D$$

No additional zero crossings.



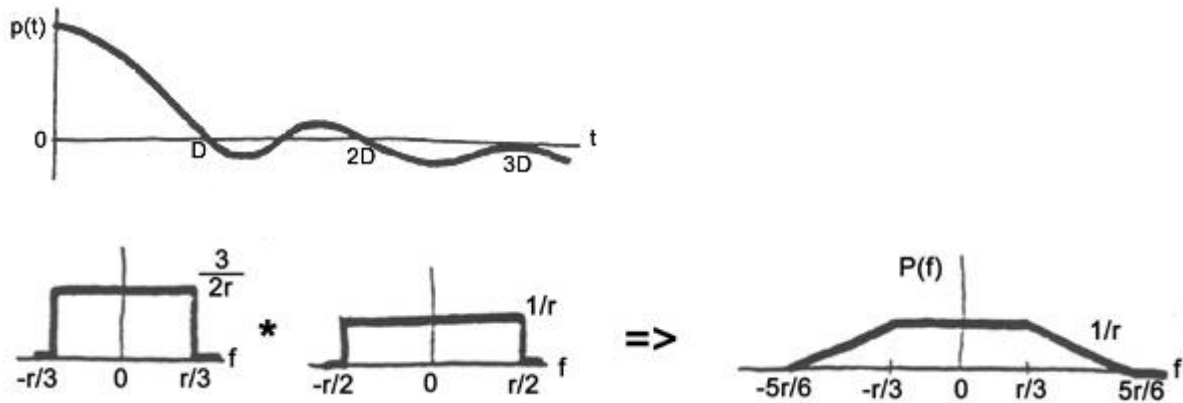


### 11.3-2

$$p_{\beta}(t) = \text{sinc}(2rt/3)$$

$$p(t) = \text{sinc} \frac{2rt}{3} \text{sinc} rt, \frac{1}{r} = D$$

Additional zero crossings at  $t = \pm 3D/2, \pm 9D/2, \pm 15D/2, \dots$



### 11.3-3

Given  $B = 3$  kHz

$$(a) B = \frac{r}{2} + \beta, 100\% \Rightarrow \beta = r/2 \Rightarrow B = \frac{r}{2} + \frac{r}{2} = r \Rightarrow r = 3 \text{ kbps.}$$

$$(b) 50\% \Rightarrow \beta = \frac{r}{4} \Rightarrow B = \frac{r}{2} + \frac{r}{4} = \frac{3}{4}r \Rightarrow r = 4 \text{ kbps.}$$

$$(c) 25\% \Rightarrow \beta = \frac{r}{8} \Rightarrow B = \frac{r}{2} + \frac{r}{8} = \frac{5}{8}r \Rightarrow r = 4.8 \text{ kbps}$$

### 11.3-4

Figure P11.3-4 are baseband waveforms for 10110100 using Nyquist pulses with  $\beta=r/2$  (dotted plot),  $\beta=r/4$  (solid plot). Note that the plot with  $\beta=r/2$  is the same as the plot of Figure 11.3-2.

In comparing the two waveforms, the signal with  $\beta=r/4$  exhibits higher intersymbol interference (ISI) than the signal with  $\beta=r/4$ .

### 11.3-4 continued

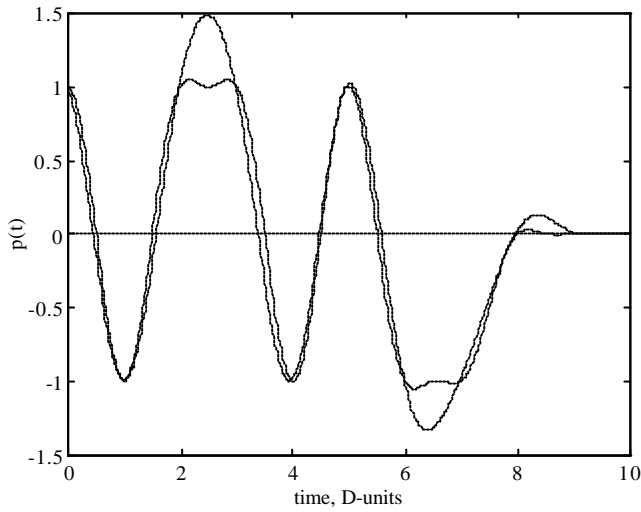


Figure P11.3-5

### 11.3-5

(a) Given  $B = r/2 + \beta$  where  $\beta$  represents excess bandwidth.

With a data rate of  $r = 56$  kbps  $\Rightarrow$  theoretical bandwidth of  $B = r/2 = 28$  kHz.

But with 100 rolloff  $\Rightarrow$  100% excess bandwidth  $\Rightarrow \beta = r/2 \Rightarrow B = r/2 + r/2 = r = 56$  kHz.

(b) 50% excessive bandwidth  $\Rightarrow \beta = r/4 \Rightarrow B = r/2 + r/4 = 3/4 r \Rightarrow B = 42$  kHz.

(c) 25% excessive bandwidth  $\Rightarrow \beta = r/8 \Rightarrow B = r/2 + r/8 = 5/8 r \Rightarrow B = 35$  kHz.

### 11.3-6

$$p(t) = \frac{\cos \pi r t}{1 - (2rt)^2} \cdot \frac{\sin \pi r t}{\pi r t} = \frac{\frac{1}{2} \sin 2\pi r t}{[1 - (2rt)^2] \pi r t} = \frac{\text{sinc } 2rt}{1 - (2rt)^2}$$

At  $t = \pm D/2 = \pm 1/2r$ ,  $\text{sinc } 2rt = 0$ , and  $1 - (2rt)^2 = 0$

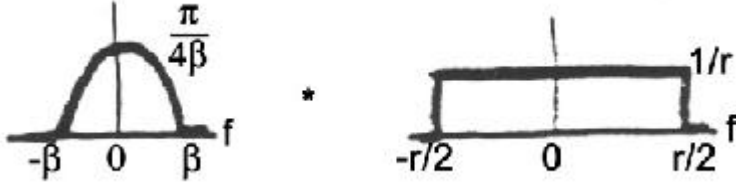
So use L'Hopital's rule with  $p(t) = \frac{\sin 2\pi r t}{2\pi r t [1 - (2rt)^2]}$

$$\frac{d}{dt} \sin 2\pi r t = 2\pi r \cos 2\pi r t, \quad \frac{d}{dt} \{2\pi r t [1 - (2rt)^2]\} = 2\pi r - 24\pi r^3 t^2$$

$$\text{thus } p\left(\pm \frac{1}{2r}\right) = \frac{2\pi r \cos(\pm \pi)}{2\pi r - 6\pi r} = \frac{1}{2}$$

11.3-7

$$P_{\beta}(f) = \frac{\pi}{4\beta} \cos \frac{\pi f}{2\beta} \Pi\left(\frac{f}{2\beta}\right), \quad P(f) =$$



$$|f| < \frac{r}{2} - \beta, \quad P(f) = \int_{-\beta}^{\beta} \frac{\pi}{4\beta} \cos \frac{\pi \lambda}{2\beta} \frac{1}{r} d\lambda = \frac{1}{2r} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = 1/r$$

$$\begin{aligned} \frac{r}{2} - \beta < f < \frac{r}{2} + \beta, \quad P(f) &= \int_{f-r/2}^{\beta} \frac{\pi}{4\beta} \cos \frac{\pi \lambda}{2\beta} \frac{1}{r} d\lambda = \frac{1}{2r} \int_{\frac{\pi}{2\beta}(f-r/2)}^{\pi/2} \cos \theta d\theta \\ &= \frac{1}{2r} \left[ 1 - \sin \frac{\pi}{2\beta} (f - r/2) \right] = \frac{1}{2r} \left[ 1 + \cos \frac{\pi}{2\beta} (f - r/2 + \beta) \right] \\ &= \frac{1}{r} \cos^2 \frac{\pi}{4\beta} (f - r/2 + \beta) \end{aligned}$$

$$f > \frac{r}{2} + \beta, \quad P(f) = 0$$

Since  $P(f)$  has even symmetry,

$$P(f) = \begin{cases} 1/r & |f| < r/2 + \beta \\ \frac{1}{r} \cos^2 \frac{\pi}{4\beta} (|f| - r/2 + \beta) & r/2 - \beta < |f| < r/2 + \beta \\ 0 & |f| > r/2 + \beta \end{cases}$$

$$\begin{aligned} p_{\beta}(t) &= \mathfrak{F}^{-1} [P_{\beta}(f)] = \frac{\pi}{4\beta} \int_{-\beta}^{\beta} \cos \frac{\pi f}{2\beta} e^{j\omega f} df = \frac{\pi}{2\beta} \int_0^{\beta} \cos \frac{\pi f}{2\beta} \cos 2\pi f t df \\ &= \frac{\pi}{2\beta} \left[ \frac{\sin(2\pi r t - \pi/2\beta)\beta}{2(2\pi r t - \pi/2\beta)\beta} + \frac{\sin(2\pi r t + \pi/2\beta)\beta}{2(2\pi r t + \pi/2\beta)\beta} \right] = \frac{1}{2} \left[ \frac{-\cos 2\pi \beta t}{4\beta t - 1} + \frac{\cos 2\pi \beta t}{4\beta t + 1} \right] \end{aligned}$$

$$\text{Thus, } p(t) = p_{\beta}(t) \text{sinc } rt = \frac{\cos 2\pi \beta t}{1 - (4\beta t)^2} \text{sinc } rt$$

11.3-8

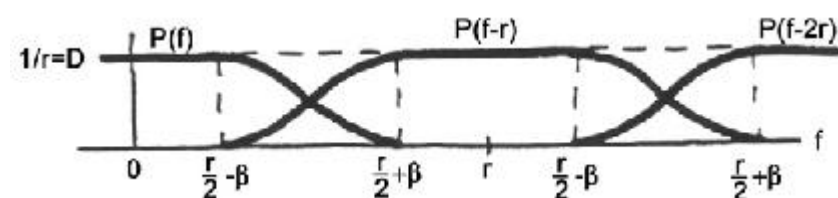
$$(a) \quad \mathfrak{I} \left[ p(t) \sum_k \delta(t - kD) \right] = P(f) * \left[ \sum_k e^{-j2\pi f k D} \right] = P(f) * \left[ \frac{1}{D} \sum_n \delta(f - n/D) \right]$$

$$= r \sum_n P(f - nr) = 1$$

$$\mathfrak{I} \left[ \sum_k p(kD) \delta(t - kD) \right] = \sum_k p(kD) e^{-j2\pi f k D}$$

Thus,  $\sum_k p(kD) e^{-j2\pi f k D} = 1$  for all  $f$  so  $p(kD) = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}$

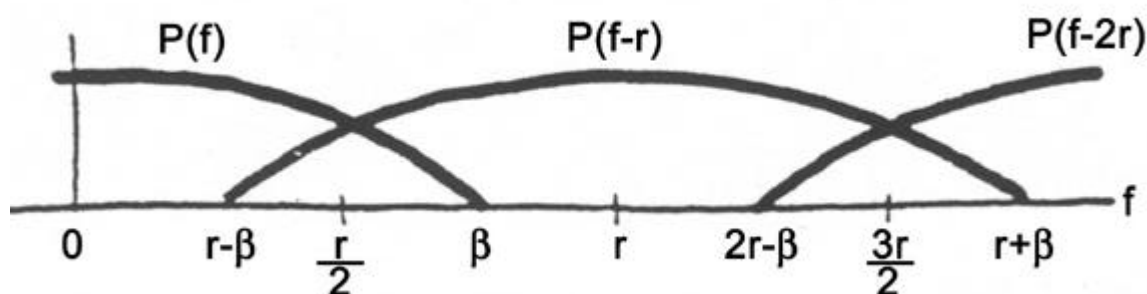
(b)  $\frac{1}{r} = D$



Clearly,  $\sum_{n=-\infty}^{\infty} P(f - nr) = D$

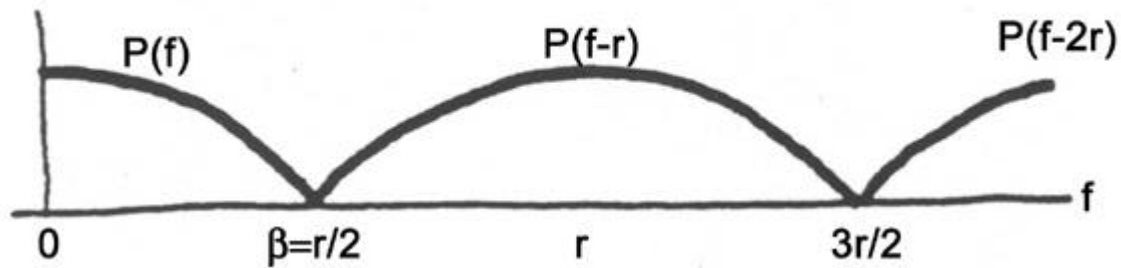
11.3-9

(a) Conditions:  $P(f) = 1/r$  for  $|f| < r - B$  and  $P(f) + P(f - r) = 1/r$  for  $r - B < |f| < B$

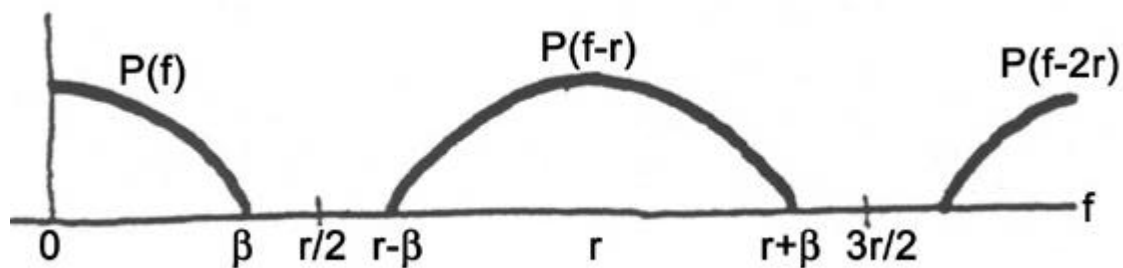


11.3-9 continued

(b) Condition:  $P(f) = 1/r$   $|f| \leq B = r/2$  so  $P(f) = \frac{1}{r} \Pi\left(\frac{f}{r}\right)$



(c) Can't satisfy the theorem when  $B < r/2$ .



11.3-10

We want smallest possible  $M$  to minimize  $S_T$

$$r = \frac{r_b}{\log_2 M} \leq 2B \Rightarrow \log_2 M \geq \frac{r_b}{2B} = 1.5 \Rightarrow \text{take } M = 2^2 = 4$$

$$r = r_b / 2 = 300 \text{ kbaud}, \beta = B - r/2 = 50 \text{ kHz}$$

$$P_{be} = 2 \frac{3}{4 \times 2} Q\left(\frac{A}{2\sigma}\right) \leq 10^{-5} \Rightarrow Q\left(\frac{A}{2\sigma}\right) \leq \frac{4}{3} \times 10^{-5} \Rightarrow \frac{A}{2\sigma} \geq 4.23$$

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{6 \times 2}{15} \frac{S_R}{N_0 r_b} \text{ so } S_T = L S_R \geq \frac{15}{12} 4.23^2 L N_0 r_b = 1.34 \text{ W}$$

## 11.3-11

We want smallest possible  $M$  to minimize  $S_T$

$$r = \frac{r_b}{\log_2 M} \leq 2B \Rightarrow \log_2 M \geq \frac{r_b}{2B} = 2.5 \Rightarrow \text{take } M = 2^3 = 8$$

$$r = r_b / 3 = 200 \text{ kbaud}, \beta = B - r / 2 = 20 \text{ kHz}$$

$$P_{be} = 2 \frac{7}{8 \times 3} Q\left(\frac{A}{2\sigma}\right) \leq 10^{-5} \Rightarrow Q\left(\frac{A}{2\sigma}\right) \leq \frac{24}{14} \times 10^{-5} \Rightarrow \frac{A}{2\sigma} \geq 4.15$$

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{6 \times 3}{63} \frac{S_R}{N_0 r_b} \text{ so } S_T = LS_R \geq \frac{63}{18} 4.15^2 LN_0 r_b = 3.61 \text{ W}$$

## 11.3-12

We want smallest possible  $M$  to minimize  $S_T$

$$r = \frac{r_b}{\log_2 M} \leq 2B \Rightarrow \log_2 M \geq \frac{r_b}{2B} = 3.75 \Rightarrow \text{take } M = 2^4 = 16$$

$$r = r_b / 4 = 150 \text{ kbaud}, \beta = B - r / 2 = 5 \text{ kHz}$$

$$P_{be} = 2 \frac{15}{16 \times 4} Q\left(\frac{A}{2\sigma}\right) \leq 10^{-5} \Rightarrow Q\left(\frac{A}{2\sigma}\right) \leq \frac{64}{30} \times 10^{-5} \Rightarrow \frac{A}{2\sigma} \geq 4.10$$

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{6 \times 4}{255} \frac{S_R}{N_0 r_b} \text{ so } S_T = LS_R \geq \frac{255}{24} 4.10^2 LN_0 r_b = 10.7 \text{ W}$$

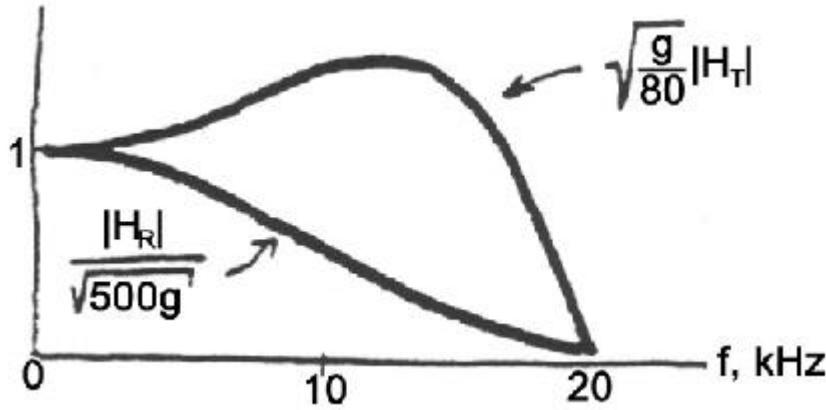
## 11.3-13

$$(a) P(f) = 1/r \cos^2(\pi f / 2r) \Pi(f / 2r), \quad P_x(f) = \frac{1}{2r} \text{sinc}(f / 2r), \quad r = 20,000$$

$$|H_R(f)|^2 = 500g \frac{\cos^2(\pi f / 2r)}{1 + 3 \times 10^{-4} |f|} \Pi(f / 2r)$$

$$|H_T(f)|^2 = \frac{80}{g} \frac{(1 + 3 \times 10^{-4} |f|) \cos^2(\pi f / 2r)}{\text{sinc}^2(f / 2r)} \Pi(f / 2r)$$

11.3-13 continued



$$(b) P_e = Q \left( \frac{A}{2\sigma} \right) = 10^{-6} \Rightarrow \frac{A}{2\sigma} = 4.75, \left( \frac{A}{2\sigma} \right)_{\max}^2 = \frac{3S_T}{(4-1)r} I^{-2}$$

$$\begin{aligned} \text{where } I &= \int_{-\infty}^{\infty} \frac{|P| \sqrt{G_n}}{|H_C|} df = \int_{-r}^r \frac{10^{-5}}{0.01r} \cos^2 \left( \frac{\pi f}{2r} \right) (1 + 3 \times 10^{-4} |f|) df \\ &= 2 \frac{10^{-3}}{r} \int_0^r (1 + 3 \times 10^{-4} f) \cos^2 \left( \frac{\pi f}{2r} \right) df = 10^{-3} \left( 4 - \frac{12}{\pi^2} \right) \approx 2.78 \times 10^{-3} \end{aligned}$$

$$\text{Thus, } S_T \geq r I^2 \left( \frac{A}{2\sigma} \right)^2 = 2 \times 10^4 \times (2.78 \times 10^{-3})^2 \times 4.75^2 \approx 3.49 \text{ W}$$

11.3-14

$$(a) P(f) = \frac{1}{r} \Pi(f/r), \quad P_x(f) = \frac{1}{10r} \text{sinc} \left( \frac{f}{10r} \right), \quad r = 100$$

$$|H_R(f)|^2 = 10^6 g \sqrt{1 + 32 \times 10^{-4} f^2} \Pi \left( \frac{f}{r} \right)$$

$$|H_T(f)|^2 = \frac{100}{g} \frac{\sqrt{1 + 32 \times 10^{-4} f^2}}{\text{sinc}^2(f/10r)} \Pi \left( \frac{f}{r} \right)$$

$$\approx \frac{1}{10^4 g^2} |H_R(f)|^2 \quad \text{since } \text{sinc}^2(f/10r) \approx 1 \quad \text{for } |f| \leq r/2$$

11.3-14 continued



$$(b) P_e = 2 \left(1 - \frac{1}{4}\right) Q \left( \frac{A}{2\sigma} \right) = 10^{-6} \Rightarrow \frac{A}{2\sigma} = 4.85, \left( \frac{A}{2\sigma} \right)^2 = \frac{3S_T}{(16-1)r} I^{-2}$$

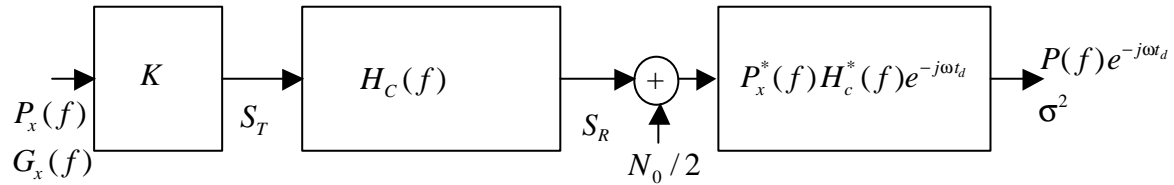
$$\text{where } I = \int_{-\infty}^{\infty} \frac{|P| \sqrt{G_n}}{|H_c|} df = \int_{-r/2}^{r/2} \frac{10^{-5}}{r} \frac{1}{10^{-3}} \sqrt{1 + 32 \times 10^4 f^2} df$$

$$= \frac{2 \times 10^{-2}}{r} \int_0^{r/2} \sqrt{1 + 32 \times 10^4 f^2} df \approx 2 \times 10^{-4} \left[ 75 + \frac{100}{2\sqrt{32}} \ln \left( \frac{\sqrt{32}}{2} + 3 \right) \right] \approx 1.81 \times 10^{-2}$$

$$\text{Thus, } S_T \geq 5rI^2 \left( \frac{A}{2\sigma} \right)^2 = 500(1.81 \times 10^{-2})^2 (4.85)^2 = 3.85 \text{ W}$$

11.3-15

(a)



$$P(f) e^{-j\omega t_d} = K H_c P_x^* H_c^* e^{-j\omega t_d} P_x \Rightarrow P(f) = K |P_x H_c|^2$$

since  $P(f)$  is real, even, and non-negative,  $p(t)$  is even an maximum at  $t = 0$ .

$$p(0) = \int_{-\infty}^{\infty} P(f) df = K \int_{-\infty}^{\infty} |P_x H_c|^2 df = 1 \Rightarrow K = \left[ \int_{-\infty}^{\infty} |P_x H_c|^2 df \right]^{-1}$$

$$\sigma^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |P_x H_c|^2 df = \frac{N_0}{2K}$$

$$G_x(f) = \sigma_a^2 r |P_x H_c|^2, \quad \sigma_a^2 = (M^2 - 1)A^2 / 12$$



11.3-15 continued

$$S_T = K^2 \int_{-\infty}^{\infty} G_x(f) df = K^2 \frac{M^2 - 1}{12} A^2 r \int_{-\infty}^{\infty} |P_x(f)|^2 df$$

$$\text{Thus, } \left( \frac{A}{2\sigma} \right)^2 = \frac{1}{4} \frac{12S_T}{(M^2 - 1)K^2 r \int_{-\infty}^{\infty} |P_x(f)|^2 df} \frac{2K}{N_0} = \frac{3S_T}{(M^2 - 1)r} I_{HR}^{-1}$$

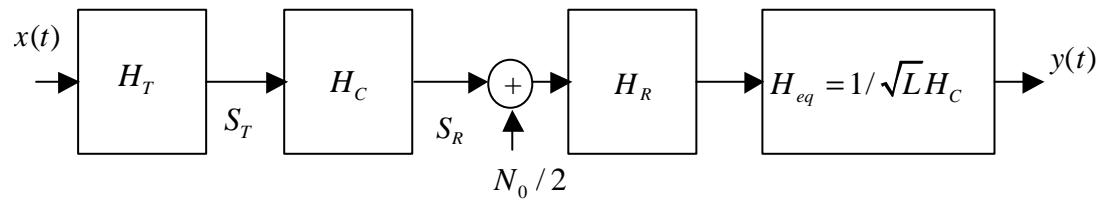
$$\text{where } I_{HR} = \frac{N_0}{2} K \int_{-\infty}^{\infty} |P_x(f)|^2 df = \frac{N_0 \int_{-\infty}^{\infty} |P_x(f)|^2 df}{2 \int_{-\infty}^{\infty} |P_x(f)H_c(f)|^2 df}$$

$$(b) S_R = \int_{-\infty}^{\infty} |H_c(f)|^2 K^2 G_x(f) df \Rightarrow \frac{S_T}{S_R} = \frac{\int_{-\infty}^{\infty} G_x df}{\int_{-\infty}^{\infty} |H_c(f)|^2 G_x df} = \frac{\int_{-\infty}^{\infty} |P_x|^2 df}{\int_{-\infty}^{\infty} |H_c|^2 |P_x|^2 df}$$

$$\text{Thus, } \left( \frac{A}{2\sigma} \right)^2 = \frac{3}{(M^2 - 1)r} \frac{\int_{-\infty}^{\infty} |P_x|^2 df}{\int_{-\infty}^{\infty} |H_c|^2 |P_x|^2 df} S_R \frac{2 \int_{-\infty}^{\infty} |P_x H_c|^2 df}{N_0 \int_{-\infty}^{\infty} |P_x|^2 df} = \frac{6S_R}{(M^2 - 1)N_0 r}$$

11.3-16

(a)



$$|H_T|^2 = \sqrt{\frac{N_0 L}{2}} \frac{|P|}{g |P_x|^2} \quad |H_R|^2 = \sqrt{\frac{2L}{N_0}} g |P|$$

$$S_T = \frac{M^2 - 1}{12} A^2 r \int_{-\infty}^{\infty} |H_T P_x|^2 df = \frac{M^2 - 1}{12} A^2 r \sqrt{\frac{N_0 L}{2}} \frac{1}{g} \int_{-\infty}^{\infty} |P| df \quad \text{where } \int_{-\infty}^{\infty} |P| df = 1$$

11.3-16 continued

$$\sigma^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_R H_{eq}|^2 df = \frac{N_0}{2} \sqrt{\frac{2L}{N_0}} \frac{g}{L} \int_{-\infty}^{\infty} \frac{|P|}{|H_C|^2} df$$

$$\text{Thus, } \left( \frac{A}{2\sigma} \right)^2 = \frac{1}{4} \frac{12gS_T}{(M^2-1)r} \sqrt{\frac{2}{N_0 L}} \frac{2}{N_0} \sqrt{\frac{N_0}{2L}} \frac{L}{g} \left[ \int_{-\infty}^{\infty} \frac{|P|}{|H_C|^2} df \right]^{-1} = \frac{6S_T / L}{K(M^2-1)N_0 r}$$

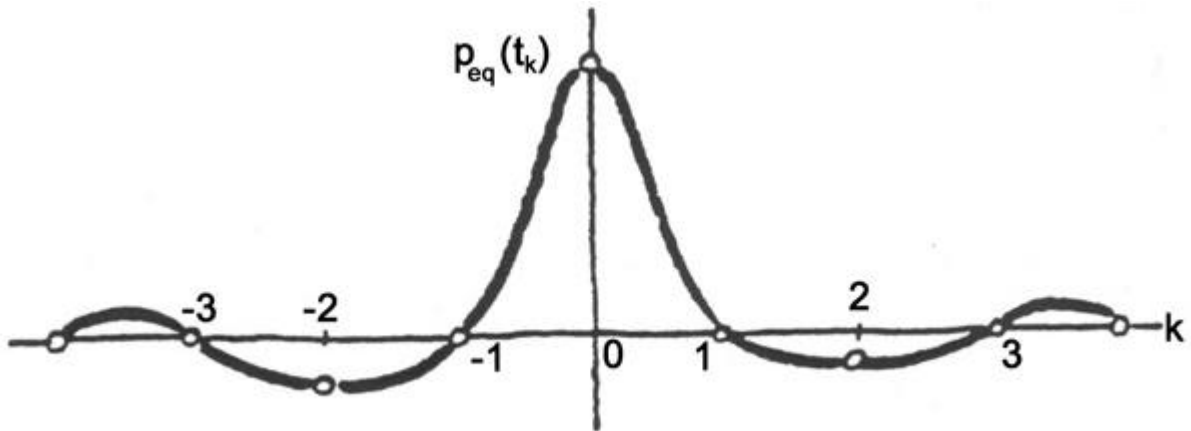
$$\text{where } K = \frac{1}{L} \int_{-\infty}^{\infty} \frac{|P(f)|}{|H_C(f)|^2} df$$

$$\begin{aligned} \text{(b) } K &= \frac{1}{L} \int_{-r}^r \frac{1}{r} \cos^2 \left( \frac{\pi f}{2r} \right) L \left[ 1 + \left( \frac{2f}{r} \right)^2 \right] df = \frac{2}{r} \int_0^r \cos^2 \left( \frac{\pi f}{2r} \right) \left[ 1 + \left( \frac{2f}{r} \right)^2 \right] df \\ &= \frac{4}{\pi} \int_0^{\pi/2} \cos^2 \lambda d\lambda + \frac{64}{\pi^3} \int_0^{\pi/2} \lambda^2 \cos^2 \lambda d\lambda \\ &= 1 + \frac{64}{\pi^3} \left( \frac{\pi^3}{48} - \frac{\pi}{8} \right) = 1.52 = 1.83 \text{ dB} \end{aligned}$$

11.3-17

$$\begin{bmatrix} 1.0 & 0.4 & 0.0 \\ 0.2 & 1.0 & 0.4 \\ 0.0 & 0.2 & 1.0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow c_{-1} = -10/21, \quad c_0 = 25/21, \quad c_1 = -5/21$$

$$p_{eq}(t_k) = -\frac{10}{21} \tilde{p}_{k+1} + \frac{25}{21} \tilde{p}_k - \frac{5}{21} \tilde{p}_{k-1} = \begin{cases} 1.0 & k=0 \\ -0.19 & k=-2 \\ -0.15 & k=2 \\ 0 & \text{otherwise} \end{cases}$$



11.3-18

$$\begin{bmatrix} 1 & \epsilon & 0 \\ \delta & 1 & \epsilon \\ 0 & \delta & 1 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{Let } \Delta = \begin{vmatrix} 1 & \epsilon & 0 \\ \delta & 1 & \epsilon \\ 0 & \delta & 1 \end{vmatrix} = 1 - 2\epsilon\delta$$

$$c_{-1} = \frac{1}{\Delta} \begin{vmatrix} 0 & \epsilon & 0 \\ 1 & 1 & \epsilon \\ 0 & \delta & 1 \end{vmatrix} = \frac{-\epsilon}{1 - 2\epsilon\delta}, \quad c_0 = \frac{1}{\Delta} \begin{vmatrix} 1 & 0 & 0 \\ \delta & 1 & \epsilon \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{1 - 2\epsilon\delta}, \quad c_1 = \frac{1}{\Delta} \begin{vmatrix} 1 & \epsilon & 0 \\ \delta & 1 & 1 \\ 0 & \delta & 0 \end{vmatrix} = \frac{-\delta}{1 - 2\epsilon\delta}$$

11.3-19

$$\begin{bmatrix} 1 & 0.1 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.1 & 0.0 & 0.0 & 0.0 \\ -0.2 & 1.0 & 0.1 & 0.0 & 0.0 \\ 0.1 & -0.2 & 1.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.1 & -0.2 & 1.0 \end{bmatrix} \begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Solving yields  $c_{-2} = 0.0094$ ,  $c_{-1} = -0.0941$ ,  $c_0 = 0.9596$ ,  $c_1 = 0.2068$ ,  $c_2 = -0.0549$

$$p_{eq}(t_k) = c_{-2}\tilde{p}_{k+2} + c_{-1}\tilde{p}_{k+1} + c_0\tilde{p}_k + c_1\tilde{p}_{k-1} + c_2\tilde{p}_{k-2}$$

$$= \begin{cases} 0 & k \leq -4 \\ 0.00094 & k = -3 \\ 0 & k = -2, -1 \\ 1 & k = 0 \\ 0 & k = 1, 2 \\ -0.0468 & k = 3 \\ -0.0055 & k = 4 \\ 0 & k \geq 5 \end{cases}$$

$$t_k = k + 2D$$

worst-case ISI now occurs at  $k = 3$  ( $t = 5D$ ) but has not been reduced significantly in magnitude.

11.3-20

(a)  $h(t) = \text{sinc } r_b t + 2 \text{sinc } r_b (t - T_b) + \text{sinc } r_b (t - 2T_b)$

$$\begin{aligned} H(f) &= \frac{1}{r_b} \Pi \left( \frac{f}{r_b} \right) (1 + 2e^{-j\omega T_b} + e^{-j\omega 2T_b}) = \frac{1}{r_b} \Pi \left( \frac{f}{r_b} \right) (2 + \cos \omega T_b) e^{-j\omega T_b} \\ &= \frac{1}{r_b} \Pi \left( \frac{f}{r_b} \right) 2 \left( 1 + \cos 2 \frac{\pi f}{r_b} \right) e^{-j\omega T_b} = \frac{4}{r_b} \cos^2 \frac{\pi f}{r_b} \Pi \left( \frac{f}{r_b} \right) e^{-j\omega T_b} \end{aligned}$$



(b)  $\dot{a}_k = \dot{a}_k + 2\dot{a}_{k-1} + \dot{a}_{k-2}$

$$= \left( \dot{m}_k - \frac{1}{2} + 2\dot{m}_{k-1} - 1 + \dot{m}_{k-2} - \frac{1}{2} \right) A = (\dot{m}_k + 2\dot{m}_{k-1} + \dot{m}_{k-2} - 2) A$$

$$\begin{array}{cccc} \dot{m}_k & \dot{m}_{k-1} & \dot{m}_{k-2} & \dot{a}_k / A \end{array}$$

0	0	0	-2
0	0	1	-1
0	1	0	0
0	1	1	1
1	0	0	-1
1	0	1	0
1	1	0	1
1	1	1	2

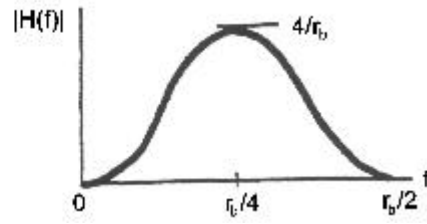
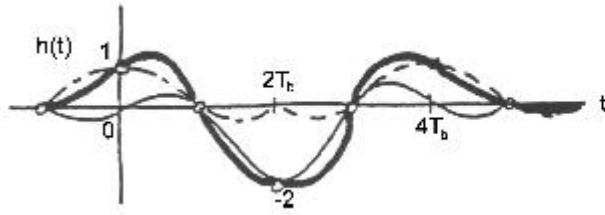
Thus,

$$y(t_k) = \begin{cases} 2A & \dot{m}_k = \dot{m}_{k-1} = \dot{m}_{k-2} = 1 \\ A & \dot{m}_{k-1} = 1, \dot{m}_k \neq \dot{m}_{k-2} \\ 0 & \dot{m}_{k-1} \neq \dot{m}_k, \dot{m}_k = \dot{m}_{k-2} \\ -A & \dot{m}_{k-1} = 0, \dot{m}_k \neq \dot{m}_{k-2} \\ -2A & \dot{m}_k = \dot{m}_{k-1} = \dot{m}_{k-2} = 0 \end{cases}$$

11.3-21

(a)  $h(t) = \text{sinc } r_b t + 2 \text{sinc } r_b (t - T_b) + \text{sinc } r_b (t - 2T_b)$

$$\begin{aligned} H(f) &= \frac{1}{r_b} \Pi \left( \frac{f}{r_b} \right) (1 - 2e^{-j\omega T_b} + e^{-j\omega 2T_b}) = \frac{1}{r_b} \Pi \left( \frac{f}{r_b} \right) (2\cos 2\omega T_b - 2) e^{-j\omega 2T_b} \\ &= \frac{1}{r_b} \Pi \left( \frac{f}{r_b} \right) (-2) \left( 1 - \cos 2\frac{\pi f}{r_b} \right) e^{-j\omega 2T_b} = -\frac{4}{r_b} \sin^2 \frac{2\pi f}{r_b} \Pi \left( \frac{f}{r_b} \right) e^{-j\omega 2T_b} \end{aligned}$$



(b)  $a_k' = a_k - 2a_{k-2} + a_{k-4}$

$$= \left( m_k' - \frac{1}{2} - 2m_{k-2}' + 1 + m_{k-4}' - \frac{1}{2} \right) A = (m_k' - 2m_{k-2}' + m_{k-4}') A$$

$$m_k' \quad m_{k-1}' \quad m_{k-4}' \quad a_k' / A$$

0	0	0	0
0	0	1	1
0	1	0	-2
0	1	1	-1
1	0	0	1
1	0	1	2
1	1	0	-1
1	1	1	0

Thus,

$$y(t_k') = \begin{cases} 2A & m_{k-2}' = 0, \quad m_k' = m_{k-4}' = 1 \\ A & m_{k-2}' = 0, \quad m_k' \neq m_{k-4}' \\ 0 & m_k' = m_{k-2}' = m_{k-4}' \\ -A & m_{k-2}' = 1, \quad m_k' \neq m_{k-4}' \\ -2A & m_{k-2}' = 1, \quad m_k' = m_{k-4}' = 0 \end{cases}$$

11.3-22

$$\frac{1}{\sqrt{L}} |H_T(f) H_R(f)| = |H(f)| = \frac{2}{r_b} \cos \frac{\pi f}{r_b} \Pi \left( \frac{f}{r_b} \right)$$

$$|H_T|^2 = L \frac{|H|^2}{|H_R|^2}, \quad P_x(f) = 1, \quad G_n(f) = N_0 / 2$$

$$S_T = \frac{A^2 r_b}{4} \int_{-\infty}^{\infty} |H_T P_x|^2 df \quad \sigma^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_R|^2 df$$

$$\text{Thus, } \left( \frac{A}{2\sigma} \right)^2 = \frac{2S_T}{N_0 r_b L} \left[ \int_{-\infty}^{\infty} \frac{|H|^2}{|H_R|^2} df \int_{-\infty}^{\infty} |H_R|^2 df \right]^{-1}$$

$$\text{where } \frac{2S_T}{N_0 r_b L} = \frac{2S_R}{N_0 r_b} = 2\gamma_b$$

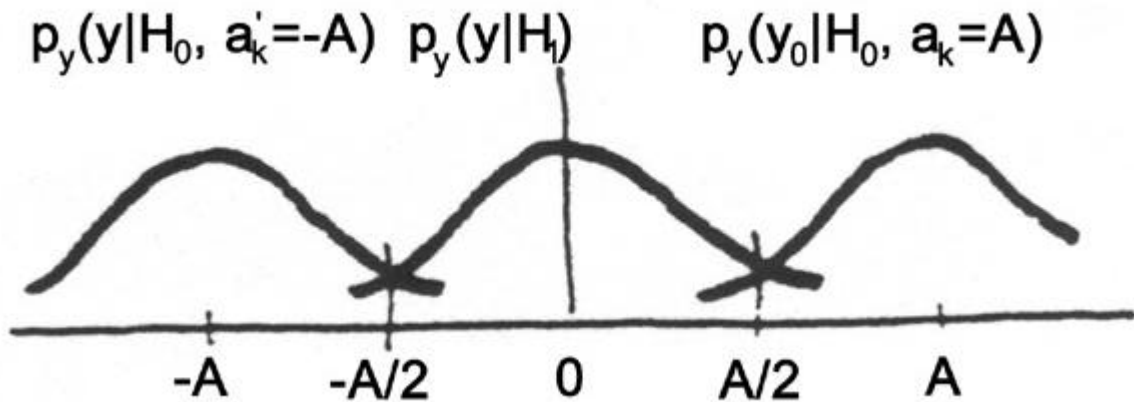
$$\int_{-\infty}^{\infty} \frac{|H|^2}{|H_R|^2} df \int_{-\infty}^{\infty} |H_R|^2 df \geq \left| \int_{-\infty}^{\infty} \frac{|H|}{|H_R|} |H_R| df \right|^2$$

$$\text{with equality when } \frac{|H|}{|H_R|} = g |H_R|$$

$$\text{So take } |H_R(f)|^2 = \frac{1}{g} |H(f)| \quad \text{and} \quad |H_T(f)|^2 = gL |H(f)|$$

$$\text{Then, since } \int_{-\infty}^{\infty} |H(f)| df = \frac{4}{r_b} \int_0^{r_b/2} \cos \frac{\pi f}{r_b} df = \frac{4}{\pi}$$

$$\left( \frac{A}{2\sigma} \right)_{\max}^2 = \frac{2\gamma_b}{(4/\pi)^2}$$



11.2-22 continued

$$P_{e1} = 2Q\left(\frac{A}{2\sigma}\right) \text{ and}$$

$$P_{e0} = \frac{1}{2}Q\left(\frac{A}{2\sigma}\right) + \frac{1}{2}Q\left(\frac{A}{2\sigma}\right) \text{ since } P(a_k' = \pm A | H_0) = \frac{1}{2}$$

$$\text{Hence, } P_e = \frac{1}{2}(P_{e1} + P_{e0}) = \frac{3}{2}Q\left(\frac{A}{2\sigma}\right) = \frac{3}{2}Q\left(\frac{\pi}{4}\sqrt{2\gamma_b}\right)$$

11.3-23

(a) Assume  $m_{-1} = \hat{m}_{-1} = 0$ .

Use  $y(t_k) = (m_k + m_{k-1} - 1)A$  to calculate  $y(t_k)$  given  $m_k$ , and  $m_{k-1}$  and to calculate  $\hat{m}_k$  given  $y(t_k)$  and  $\hat{m}_{-1}$ .

$k$	0	1	2	3	4	5	6	7	8
$m_k$	1	0	1	0	1	1	1	0	1
$m_{k-1}$	0	1	0	1	0	1	1	1	0
$y(t_k)$	0	0	0	0	0	2	2	0	0
$\hat{m}_{k-1}$	0	1	0	1	0	1	1	1	0
$\hat{m}_k$	1	0	1	0	1	1	1	0	1

(b) dc value:  $\overline{y(t_k)} = (2 + 2)/9 = 0.44$

(c) As the table below indicates, if bit  $\hat{m}_2$  is received in error such that

$\hat{m}_2 = 0 \Rightarrow y(t_2) = -2$  instead of 0.

Because  $m_k = f(m_{k-1}) \Rightarrow$  errors in  $\hat{m}_{k=3}$  will affect all subsequent values of  $\hat{m}_k$  as indicated in the table below.

$k$	0	1	2	3	4	5	6	7	8
$y(t_k)$	0	0	-2	0	0	2	2	0	0
$\hat{m}_{k-1}$	0	1	0	0	1	0	2	0	1
$\hat{m}_k$	1	0	0	1	0	2	0	1	0
			x	x	x	x	x	x	x = errors

## 11.3-24

(a) Use the precoder of Fig. 11.3-9 to convert  $m_k \rightarrow m'_k$ , Eq. (23) for  $y(t_k)$ , Eq. (24) to determine  $\hat{m}_k$  from the received value of  $y(t_k)$ . Note that with precoding  $\hat{m}_k$  is not a function of  $\hat{m}_{k-1}$ . Also, assume  $m'_{-1} = 0$ .

$k$	0	1	2	3	4	5	6	7	8
$m_k$	1	0	1	0	1	1	1	0	1
$m'_{k-1}$	0	1	1	0	0	1	0	1	1
$m'_k$	1	1	0	0	1	0	1	1	0
$y(t_k)$	0	2	0	-2	0	0	0	2	0
$\hat{m}_k$	1	0	1	0	1	1	1	0	1

(b) dc value:  $\overline{y(t_k)} = (2 - 2 + 2)/9 = 0.22$

(c) If bit  $\hat{m}_2$  is received in error, only that bit is affected since with precoding  $\hat{m}_k$  is not a function of  $\hat{m}_{k-1}$ .

## 11.3-25

(a) Use the precoder of Fig. 11.3-9 to convert  $m_k \rightarrow m'_k$  except use two stages of delay  $\Rightarrow m'_k = m_k \oplus m'_{k-2}$ .

Then use Eq. (27) to determine  $y(t_k)$  from  $m'_k$  and  $m'_{k-2}$  and  $\hat{m}_k$  from  $y(t_k)$ .

Assume  $m'_{-1} = m'_{-2} = 0$ .

$k$	0	1	2	3	4	5	6	7	8
$m_k$	1	0	1	0	1	1	1	0	1
$m'_{k-2}$	0	0	1	0	0	0	1	1	0
$m'_k$	1	0	0	0	1	1	0	1	1
$y(t_k)$	2	0	-2	0	2	2	-2	0	2
$\hat{m}_k$	1	0	1	0	1	1	1	0	1

(b) dc value:  $\overline{y(t_k)} = (2 - 2 + 2 + 2 - 2 + 2)/9 = 0.44$

(c) If bit  $\hat{m}_2$  is received in error, only that bit is affected since we can obtain  $\hat{m}_k$  directly from  $y(t_k)$ .



#### 11.4-1

(a) Using structure of Fig. 11.4-6a to scramble the input sequence we get:

$m_k$	0	1	1	1	1	1	1	0	1	1	1	0	1	1	1
$m'_{k-1}$	0	0	1	0	0	1	1	1	0	1	1	0	1	1	0
$m'_{k-2}$	0	0	0	1	0	0	1	1	1	0	1	1	0	1	1
$m'_{k-3}$	0	0	0	0	1	0	0	1	1	1	0	1	1	0	1
$m'_{k-4}$	0	0	0	0	0	1	0	0	1	1	1	0	1	1	0
$m'_{k-5}$	0	0	0	0	0	0	1	0	0	1	1	1	0	1	1
$m''_k$	0	0	1	1	0	0	0	0	0	0	1	1	0	1	1
$m'_k$	0	1	0	0	1	1	1	0	1	1	0	1	1	0	0

dc values of unscrambled and scrambled sequences:

$$\bar{m}_k = (1+1+1+1+1+1+1+1+1+1+1+1+1)/15 = 12/15 = 0.80$$

$$\bar{m}'_k = (1+1+1+1+1+1+1+1+1)/15 = 8/15 = 0.53$$

(b) Using the structure of Fig 11.4-6b to unscramble  $m'_k$  we get:

$m'_k$	0	1	0	0	1	1	1	0	1	1	0	1	1	0	0
$m'_{k-1}$	0	0	1	0	0	1	1	1	0	1	1	0	1	1	0
$m'_{k-2}$	0	0	0	1	0	0	1	1	1	0	1	1	0	1	1
$m'_{k-3}$	0	0	0	0	1	0	0	1	1	1	0	1	1	0	1
$m'_{k-4}$	0	0	0	0	0	1	0	0	1	1	1	0	1	1	0
$m'_{k-5}$	0	0	0	0	0	0	1	0	0	1	1	1	0	1	1
$m''_k$	0	0	1	1	0	0	0	0	0	0	1	1	0	1	1
$m_k$	0	1	1	1	1	1	1	0	1	1	1	0	1	1	1

#### 11.4-2

Using the results from Exercise 11.4-1, we get the output sequence and its shifted versions to generate the following table used to calculate the autocorrelation function.

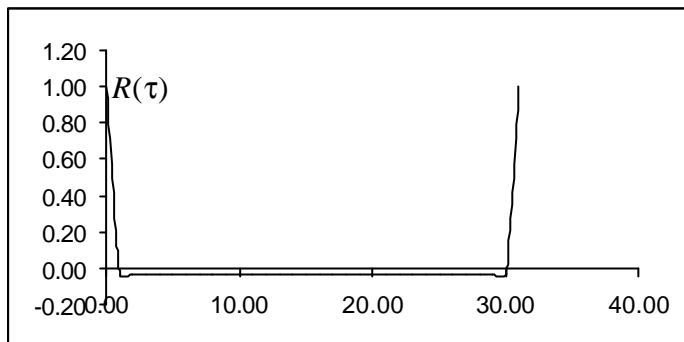
# 11.4-2 continued

$\tau$	original/shifted	$v(\tau)$	$R(\tau) = v(\tau) / N$
0	1111100100110000101101010001110 1111100100110000101101010001110	31	1
1	1111100100110000101101010001110 0111110010011000010110101000111	-1	-0.032
2	1111100100110000101101010001110 1011111001001100001011010100011	-1	-0.032
3	1111100100110000101101010001110 1101111100100110000101101010001	-1	-0.032
4	1111100100110000101101010001110 1110111110010011000010110101000	-1	-0.32
5	1111100100110000101101010001110 0111011111001001100001011010100	-1	-0.032
6	1111100100110000101101010001110 0011101111100100110000101101010	-1	-0.032
7	1111100100110000101101010001110 0001110111110010011000010110101	-1	-0.032
8	1111100100110000101101010001110 1000111011111001001100001011010	-1	-0.032
9	1111100100110000101101010001110 0100011101111100100110000101101	-1	-0.032
10	1111100100110000101101010001110 1010001110111110010011000010110	-1	-0.032
11	1111100100110000101101010001110 0101000111011111001001100001011	-1	-0.032
12	1111100100110000101101010001110 1010100011101111100100110000101	-1	-0.032
13	1111100100110000101101010001110 1101010001110111110010011000010	-1	-0.032
14	1111100100110000101101010001110 0110101000111011111001001100001	-1	-0.032

# 11.4-2 continued

$\tau$	original/shifted	$v(\tau)$	$R(\tau) = v(\tau) / N$
15	1111100100110000101101010001110 1011010100011101111100100110000	-1	-0.032
16	1111100100110000101101010001110 0101101010001110111110010011000	-1	-0.032
17	1111100100110000101101010001110 0010110101000111011111001001100	-1	-0.032
28	1111100100110000101101010001110 0001011010100011101111100100110	-1	-0.032
19	1111100100110000101101010001110 0000101101010001110111110010011	-1	-0.032
20	1111100100110000101101010001110 1000010110101000111011111001001	-1	-0.032
21	1111100100110000101101010001110 1100001011010100011101111100100	-1	-0.032
22	1111100100110000101101010001110 0110000101101010001110111110010	-1	-0.032
23	1111100100110000101101010001110 0011000010110101000111011111001	-1	-0.032
24	1111100100110000101101010001110 1001100001011010100011101111100	-1	-0.032
25	1111100100110000101101010001110 0100110000101101010001110111110	-1	-0.032
26	1111100100110000101101010001110 0010011000010110101000111011111	-1	-0.032
27	1111100100110000101101010001110 1001001100001011010100011101111	-1	-0.032
28	1111100100110000101101010001110 1100100110000101101010001110111	-1	-0.032
29	1111100100110000101101010001110 1110010011000010110101000111011	-1	-0.032
30	1111100100110000101101010001110 1111001001100001011010100011101	-1	-0.032
31	1111100100110000101101010001110 1111100100110000101101010001110	31	1

## 11.4-2 continued



$R(\tau)$  is periodic with period = 31.

Is the output a ml sequence? Apply ML rules: (1) #1s = 16, #0s = 15  $\Rightarrow$  obeys balance property; (2) obeys the run property; (3) has a single autocorrelation peak; (4) obeys Mod-2 property, and (5) all 32 states exist during sequence generation.

## 11.4-3

$m_1 = m_2 + m_4$  and the output sequence =  $m_4$

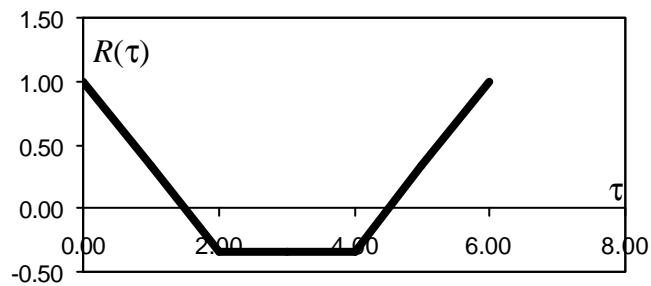
shift	$m_1$	$m_2$	$m_3$	$m_4$
0	1	1	1	1
1	0	1	1	1
2	0	0	1	1
3	1	0	0	1
4	1	1	0	0
5	1	1	1	0
6	1	1	1	1

### 11.4-3 continued

The autocorrelation function is calculated as follows:

$\tau$	original/shifted	$v(\tau)$	$R(\tau) = v(\tau) / N$
0	1 1 1 1 0 0 1 1 1 1 0 0	6	6/6=1
1	1 1 1 1 0 0 0 1 1 1 1 0	2	0.333
2	1 1 1 1 0 0 0 0 1 1 1 1	-2	-0.333
3	1 1 1 1 0 0 1 0 0 1 1 1	-2	-0.333
4	1 1 1 1 0 0 1 1 0 0 1 1	-2	-0.333
5	1 1 1 1 0 0 1 1 1 0 0 1	2	0.333
6	1 1 1 1 0 0 1 1 1 1 0 0	2	1.

$R(\tau)$  is periodic with period = 6.



The [4,2] register does not produce a ml sequence since there are only 5/16 possible states exist in the output sequence and the period is not  $2^4 - 1$ .

## Chapter 12

12.1-1

$$\left. \begin{array}{l} \frac{1}{2} v f_s \leq B_T \\ f_s \geq 2W \end{array} \right\} v \leq \frac{2B_T}{2W} = 3.33 \Rightarrow v = 3, f_s \leq \frac{2B_T}{v} = 33.3 \text{ kHz}$$

$$q = M^3 \geq 200 \Rightarrow M = 2^n \geq 200^{1/3} = 5.85 \Rightarrow n = 3$$

12.1-2

$$\left. \begin{array}{l} \frac{1}{2} v f_s \leq B_T \\ f_s \geq 2W \end{array} \right\} v \leq \frac{2B_T}{2W} = 5.33 \Rightarrow v = 5, f_s \leq \frac{2B_T}{v} = 32 \text{ kHz}$$

$$q = M^5 \geq 200 \Rightarrow M = 2^n \geq 200^{1/5} = 2.88 \Rightarrow n = 2$$

12.1-3

Transmit one quantized pulse, having  $q^N$  possible values, for every  $N$  successive quantized samples. The output pulse rate is  $f_s / N$  so  $B_T \geq \frac{1}{2}(f_s / N) \geq W / N$ , allowing  $B_T < W$ .

12.1-4

$$\left. \begin{array}{l} |\epsilon_k| \leq 1/q \\ 2|x(t)| = 2 \end{array} \right\} \frac{1/q}{2} \leq \frac{P}{100} \Rightarrow q = M^v \geq \frac{50}{P}$$

Thus,  $v \geq \log_M (50/P)$

12.1-5

$$(S/N)_D = 3q^2 \times 0.25 \geq 10^4 \Rightarrow q \geq 116, v \leq B_T / W = 5.3, q = M^v$$

$M$        $v$        $q$       comment

---

2	5	32	$q$ too small, try $M > 2$
3	5	243	ok
3	4	81	$q$ too small

$\Rightarrow$  Take  $M = 3, v = 5, f_s \leq 2B_T / v = 6.4 \text{ kHz}$

## 12.1-6

$$(S/N)_D = 3q^2 \times 0.25 \geq 4000 \Rightarrow q \geq 73, \nu \leq B_T / W = 6.7, q = M^\nu$$

$M$	$\nu$	$q$	comment
-----	-------	-----	---------

2	6	64	$q$ too small, try $M > 2$
---	---	----	----------------------------

3	6	729	$q$ excessively large
---	---	-----	-----------------------

3	5	81	ok
---	---	----	----

$\Rightarrow$  Take  $M = 3, \nu = 5, f_s \leq 2B_T / \nu = 10$  kHz

## 12.1-7

$$W = 5 \text{ kHz}, (S/N)_D = 40 - 50 \text{ dB} = 10^4 - 10^5$$

$$(a) (S/N)_D = 0.9q^2 \geq 10^4 - 10^5 \Rightarrow q = 2^\nu = 105 - 333 \text{ so } \nu = 7 \text{ or } 8$$

$$B_T \geq \nu W = 35 - 40 \text{ kHz}$$

$$(b) \nu \leq 4, q = M^4 = 105 - 333 \Rightarrow M \geq 4$$

## 12.1-8

$$W = 20 \text{ kHz}, (S/N)_D = 55 - 65 \text{ dB} = 3.2 \times 10^5 - 3.2 \times 10^6$$

$$(a) (S/N)_D = 0.9q^2 \geq 3.2 \times 10^5 - 3.2 \times 10^6 \Rightarrow q = 2^\nu = 596 - 1886 \text{ so } \nu = 10 \Rightarrow B_T \geq \nu W = 200 \text{ kHz}$$

$$(b) \nu \leq 4, q = M^4 = 596 - 1886 \Rightarrow M \geq 5$$

## 12.1-9

$$x_{\min} = 5 \times 10^{-6} \text{ V and } x_{\max} = 200 \times 10^{-3} \text{ V} \Rightarrow \text{normalize} \Rightarrow x_{\min} = 25 \times 10^{-6} \text{ V and } x_{\max} = 1 \text{ V}$$

$$\text{Assume a sinusoidal input, the power of the smallest signal is } \Rightarrow S_x = (25 \times 10^{-6})^2 / 2 = 3.12 \times 10^{-10}$$

$$\text{Using Eq. (7)} \Rightarrow 40 \text{ dB} = 4.8 + 10 \log(2^{2\nu} \times 3.12 \times 10^{-10}) \Rightarrow \nu = 21.7 \Rightarrow \nu = 22.$$

## 12.1-10

$$\text{Scale } \pm 10 \text{ V input by a factor of 10 to make input } \pm 1 \text{ V, then because its sinusoidal} \Rightarrow S_x = 0.5$$

$$q = 2^{12} \Rightarrow \text{quantum size} = 2/q = 2/4096 = 0.488 \text{ mV.}$$

$$(S/N)_D = 3q^2 S_x = 3(2^{12})^2 \times 0.5 = 2.52 \times 10^7 \Rightarrow 74 \text{ dB.}$$

12.1-11

Scale +/- 10V input by a factor of 10 to make input +/- 1V, then because its sinusoidal  $\Rightarrow S_x = 0.5$

$q = 2^{16} \Rightarrow \text{quantum size} = 2/q = 2/65,536 = 30.5 \text{ uV}.$

$(S/N)_D = 3q^2 S_x = 3(2^{16})^2 \times 0.5 = 6.44 \times 10^9 \Rightarrow 98 \text{ dB}.$

12.1-12

Let  $v_{\max}, v_{\min}$  = maximum and minimum input voltages

$\Rightarrow \text{Dynamic range} = 20\log(v_{\max}/v_{\min})$

Assuming the largest signal = 1 volt  $\Rightarrow$  the smallest signal =  $1/q$  volts

$\Rightarrow \text{Dynamic range} = 20\log(q) = 20\log(2^v)$

Dynamic range =  $20\log(2^v) = 120 \text{ dB} \Rightarrow v = 20 \text{ bits}.$

12.1-13

If  $(S/N)_D = 35 \text{ dB}$  and assuming  $S_x = 1$

$\Rightarrow 35 = 4.8 + 6v \Rightarrow v = 5.03$

Memory must hold:

10 min x 60 secs/min x 8000 samples/sec = 4.8 Msamples @ 5 bits/sample = 24 Mbits

12.1-14

(a)  $v = 12 \text{ bits} \Rightarrow q = 2^{12} = 4096.$  With  $|x|_{\max} = 10 \text{ V} \Rightarrow$  each step size =  $10 \times 2/q = 4.88 \times 10^{-3} \text{ V}$

Maximum input =  $x(kT_s)_{\max} = (q-1)/q = 4095/4096 \times 10 = 9.9976 \text{ V}$

For positive inputs from 0 to 10 V,  $x_q(kT_s) = 0.00244, 0.00732, 0.01221, 0.01709, 0.02197, \dots$

$\Rightarrow x(t) = 0.02 \text{ V} \Rightarrow x_q(kT_s) = 0.02197 \Rightarrow |\epsilon_k| = |0.02 - 0.02197| = 0.00197$

Quantization error% =  $0.00197/0.02 \times 100\% = 9.85\%$

(b) For  $x(t) = 0.2 \text{ V} \Rightarrow x_q(kT_s) = 0.19775 \Rightarrow |\epsilon_k| = |0.2 - 0.19775| = 0.00225$

Quantization error% =  $0.00225/0.2 \times 100\% = 1.125\%$

12.1-15

Let  $v = 2 + n, n \geq 1$  then  $h_i = 2/q = \frac{2}{2^n} v = \frac{1}{2^n} \times \frac{1}{2} \leq \frac{1}{4}$

so  $p_x(x)$  is constant over each band.

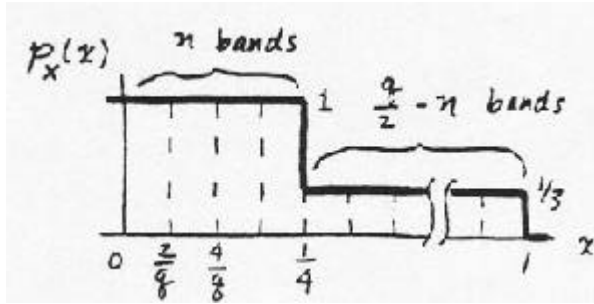
$a_i = x_i - 1/q, b_i = x_i + 1/q$



12.1-15 continued

$$n \frac{2}{q} = \frac{1}{4} \Rightarrow n = q/8$$

$$q/2 - n = 3q/8$$



$$\overline{\epsilon_i^2} = p_x(x_i) \int_{x_i - 1/q}^{x_i + 1/q} (x_i - x)^2 dx = \frac{2p_x(x_i)}{3q^3}$$

$$\sigma_q^2 = 2 \frac{2}{3q^3} \left[ n \times 1 + \left( \frac{q}{2} - n \right) \times \frac{1}{3} \right] = \frac{1}{3q^2}$$

$$S_x = 2 \left[ \int_0^{1/4} x^2 dx + \int_{1/4}^1 x^2 \times \frac{1}{3} dx \right] = \frac{22}{96} = 0.229$$

$$\text{Thus, } (S/N)_D = 3q^2 \times 0.229 \approx 0.7 q^2$$

12.1-16

$$h_i = 2/q, \quad a_i = x_i - 1/q, \quad b_i = x_i + 1/q, \quad p_x(x) \approx p_x(x_i) \text{ for } a_i < x < b_i$$

$$\text{Thus, } \overline{\epsilon_i^2} \approx p_x(x_i) \int_{x_i - 1/q}^{x_i + 1/q} (x_i - x)^2 dx = \frac{2p_x(x_i)}{3q^3}$$

$$\text{so } \sigma_q^2 \approx \frac{2}{3q^3} \sum_{i=-q/2}^{q/2} p_x(x_i)$$

$$\text{But } \int_{-1}^1 p_x(x) dx = \sum_{i=-q/2}^{q/2} \frac{2}{q} p_x(x_i) \text{ and } \int_{-1}^1 p_x(x) dx = P[|x| < 1] \approx 1$$

$$\text{Hence, } \sum_{i=-q/2}^{q/2} p_x(x_i) \approx \frac{q}{2} \text{ so } \sigma_q^2 \approx \frac{2}{3q^3} \frac{q}{2} = \frac{1}{3q^2}$$

12.1-17

$$\left. \begin{aligned} \bar{x} = 0 &\Rightarrow \sigma_x = \sqrt{S_x} \\ P[|x| > 1] = 2Q(1/\sigma_x) \leq 0.01 &\Rightarrow \frac{1}{\sigma_x} \geq 2.6 \end{aligned} \right\} S_x \leq \frac{1}{2.6^2} \approx 0.148$$

$$\text{Thus, } (S/N)_D \leq 10 \log_{10}(3 \times 2^{2v} \times 0.148) = -3.5 + 6v \text{ dB}$$

12.1-18

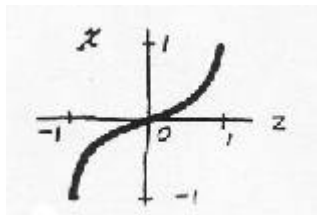
$$P[|x| > 1] = 2 \int_1^\infty \frac{\alpha}{2} e^{-\alpha x} dx = e^{-\alpha} \leq 0.01 \Rightarrow \alpha \geq -\ln 0.01$$

$$\text{so } S_x = 2/\alpha^2 \leq 2/(\ln 100)^2 = 0.0943$$

$$\text{and } (S/N)_D \leq 10 \log_{10}(3 \times 2^{2v} \times 0.0943) = -5.5 + 6v \text{ dB}$$

12.1-19

$$(a) \ x = \begin{cases} z^2 & z > 0 \\ -z^2 & z < 0 \end{cases} \Rightarrow x(z) = (\text{sgn } z) z^2$$



$$(b) \ z'(x) = \frac{1}{2} |x|^{-3/2},$$

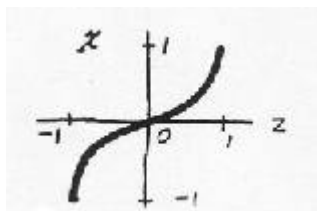
$$K_z = 2 \int_0^1 4x^3 p_x(x) dx = 8 \left[ \int_0^{1/4} x^3 dx + \frac{1}{3} \int_{1/4}^1 x^3 dx \right] = 0.672$$

12.1-20

$$(a) \ z > 0: \ln(1+\mu x) = z \ln(1+\mu) = \ln(1+\mu)^2 \Rightarrow x = [(1+\mu)^2 - 1]/\mu$$

$$z < 0: \ln(1-\mu x) = -z \ln(1+\mu) = \ln(1+\mu)^{-2} \Rightarrow x = -[(1+\mu)^{-2} - 1]/\mu$$

$$\text{Thus, } x(z) = (\text{sgn } z) \frac{(1+\mu)^{|z|} - 1}{\mu}$$



12.1-20 continued

$$(b) K_z = 2 \left[ \frac{\ln(1+\mu)}{\mu} \right]^2 \int_0^1 (1+\mu x)^2 p_x(x) dx$$

$$= 2 \frac{\ln^2(1+\mu)}{\mu^2} \left[ \int_0^1 p_x(x) dx + 2\mu \int_0^1 x p_x(x) dx + \mu^2 \int_0^1 x^2 p_x(x) dx \right]$$

$$\text{where } \int_0^1 p_x(x) dx = 1/2, \quad 2 \int_0^1 x p_x(x) dx = \int_{-1}^1 |x| p_x(x) dx = \overline{|x|}$$

$$\int_0^1 x^2 p_x(x) dx = \frac{1}{2} \int_{-1}^1 x^2 p_x(x) dx = \frac{1}{2} \overline{x^2} = \frac{1}{2} S_x$$

$$\text{Thus, } K_z = \frac{\ln^2(1+\mu)}{\mu^2} (1 + 2\mu \overline{|x|} + \mu^2 S_x)$$

12.1-21

(a) With  $x(t) = 0.02$  V  $\Rightarrow$  with companding using Eq. (12) we have

$$z(x) = 9.9976 \left[ \frac{\ln(1 + 255 \times 0.02/9.9976)}{\ln(256)} \right] = 0.7432$$

$z(x)$  is fed to the quantizer giving  $z(kT_s) = 0.74463$

Using Eq. (13) with  $x_q(kT_s) = z(kT_s)$  gives

$$\hat{x} = \frac{9.9976}{255} \left[ (1 + 255)^{0.74463/9.9976} - 1 \right] = 0.02005 \Rightarrow |\varepsilon_k| = |0.02 - 0.02005| \approx 0 \Rightarrow 0\% \text{ quantization error.}$$

(b) With  $x(t) = 0.2$  V  $\Rightarrow$  with companding using Eq. (12) we have

$$z(x) = 9.9976 \left[ \frac{\ln(1 + 255 \times 0.2/9.9976)}{\ln(256)} \right] = 3.26059$$

$z(x)$  is fed to the quantizer giving  $z(kT_s) = 3.25928$

Using Eq. (13) with  $x_q = z(kT_s)$  gives

$$\hat{x} = \frac{9.9976}{255} \left[ (1 + 255)^{3.25928/9.9976} - 1 \right] = 0.19983 \Rightarrow |\varepsilon_k| = |0.2 - 0.19983| \approx 0 \Rightarrow 0\% \text{ quantization error.}$$

12.1-22

(a)  $z'(x) = 3e^{-3x}, \quad x > 0$

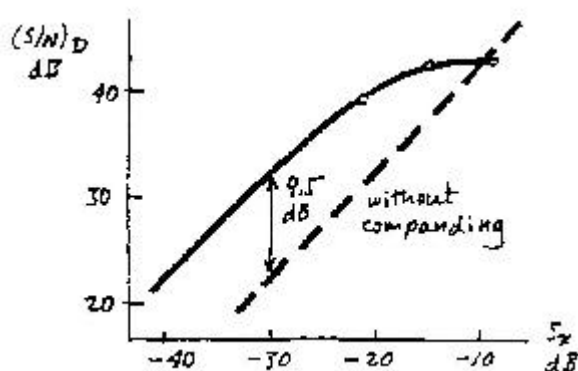
$$K_z = 2 \int_0^1 \frac{1}{9} e^{6x} \frac{\alpha}{2} e^{-\alpha x} dx = \frac{\alpha}{9(6-\alpha)} (e^{6-\alpha} - 1) \approx \frac{\alpha}{9(-\alpha)} (0 - 1) = \frac{1}{9} \text{ for } \alpha \gg 1$$

$$(S/N)_D \approx 10 \log_{10} (9 \times 3 \times 2^{2\nu} S_x) = 14.3 + 6.0\nu + S_x \text{ dB}$$

12.1-22 continued

$$(b) \ (S/N)_D = 52.9 + S_x - K_z \text{ dB}$$

$\alpha$	$S_x$ (dB)	$K_z$ (dB)	$(S/N)_D$
4	-9	1.5	42.4
8	-15	-4.2	42.1
16	-21	-7.5	39.4
$\infty$	1	-9.5	$62.4 + S_x$



12.1-23

$$(a) \ z'(x) = \begin{cases} \frac{A}{1 + \ln A} & 0 \leq x \leq 1/A \\ \frac{A}{1 + \ln A} \frac{1}{x} & 1/A \leq x \leq 1 \end{cases}$$

$$K_z = (1 + \ln A)^2 \left[ 2 \int_0^{1/A} \frac{1}{A^2} p_x(x) dx + 2 \int_{1/A}^1 x^2 p_x(x) dx \right]$$

$$= (1 + \ln A)^2 \left[ 2 \int_0^1 x^2 p_x(x) dx + 2 \int_0^{1/A} \left( \frac{1}{A^2} - x^2 \right) p_x(x) dx \right]$$

$$\text{where } 2 \int_0^1 x^2 p_x(x) dx = \int_{-1}^1 x^2 p_x(x) dx = S_x$$

12.1-23 continued

$$(b) \quad 2 \int_0^{1/A} \left( \frac{1}{A^2} - x^2 \right) p_x(x) dx = \frac{\alpha}{A^2} \int_0^{1/A} e^{-\alpha x} dx - \alpha \int_0^{1/A} x^2 e^{-\alpha x} dx$$

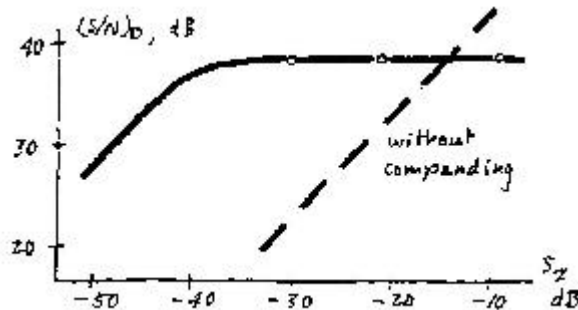
$$= \frac{2}{\alpha^2} \left( \frac{\alpha}{A} + 1 \right) e^{-\alpha/A} + \frac{1}{A^2} - \frac{2}{\alpha^2}, \quad S_x = \frac{2}{\alpha^2}$$

$$\text{Thus, } K_z = (1 + \ln A)^2 \left[ \frac{2}{\alpha^2} \left( \frac{\alpha}{A} + 1 \right) e^{-\alpha/A} + \frac{1}{A^2} \right]$$

$$= \left( \frac{1 + \ln A}{A} \right)^2 \left[ 1 + 2 \frac{A}{\alpha} \left( \frac{A}{\alpha} + 1 \right) e^{-\alpha/A} \right] \approx \left( \frac{1 + \ln A}{A} \right)^2 \quad \text{if } \alpha \gg A$$

$$(c) \quad (S/N)_D = 52.9 + S_x - K_z \quad \text{dB}$$

$\alpha$	$S_x$ (dB)	$K_z$ (dB)	$(S/N)_D$
4	-9	5.9	38
16	-21	-6.1	38
64	-33	-17.9	37.8
$\infty$ 100		-25	$77.9 + S_x$



12.2-1

$$(S/N)_D = 3q^2 \times 1/2 \geq 4 \times 10^3 \Rightarrow q > 51, \quad v \leq B_T/W = 2.5$$

$$\text{so } q \leq M^2 > 51 \Rightarrow M > 7. \text{ Thus, take } M = 8, \quad v = 2, \quad q = 64$$

$$\gamma = \frac{S_R}{N_0 W} \geq \gamma_{th} = 6(B_T/W)(M^2 - 1) = 945 \Rightarrow S_R \geq 945 N_0 W = 56.7 \text{ mW}$$

12.2-2

$$(S/N)_D = 3q^2 \times 1/2 \geq 4 \times 10^3 \Rightarrow q > 51, v \leq B_T/W = 3.33$$

so  $q \leq M^3 > 51 \Rightarrow M > 3.7$  Thus, take  $M = 4, v = 3, q = 64$

$$\gamma = \frac{S_R}{N_0 W} \geq \gamma_{th} = 6(B_T/W)(M^2 - 1) = 300 \Rightarrow S_R \geq 300N_0 W = 18 \text{ mW}$$

12.2-3

$$(S/N)_D = 3q^2 \times 1/2 \geq 4 \times 10^3 \Rightarrow q > 51, v \leq B_T/W = 8.33$$

so  $q \leq M^8 > 51 \Rightarrow M > 1.2$  Thus, take  $M = 2, v = 6, q = 64$

$$\gamma = \frac{S_R}{N_0 W} \geq \gamma_{th} = 6(B_T/W)(M^2 - 1) = 150 \Rightarrow S_R \geq 150N_0 W = 9 \text{ mW}$$

12.2-4

$$\text{PCM: } P_e = 20Q\left[\sqrt{(S/N)_1}\right] \leq 10^{-5} \Rightarrow (S/N)_1 \geq 4.9^2$$

$$B_T/W \geq v = 8, \gamma = (B_T/W)(S/N)_1 \geq 192 \approx 22.8 \text{ dB}$$

$$\text{Analog: } (S/N)_R = \frac{1}{20}(S/N)_1 = 37 \text{ dB} \approx 5000, \gamma = (S/N)_1 = 10^5 = 50 \text{ dB}$$

PCM advantage:  $50 - 22.8 = 27.2 \text{ dB}$

12.2-5

$$\text{PCM: } P_e = 100Q\left[\sqrt{(S/N)_1}\right] \leq 10^{-5} \Rightarrow (S/N)_1 \geq 5.2^2$$

$$B_T/W \geq v = 8, \gamma = (B_T/W)(S/N)_1 \geq 216 = 23.4 \text{ dB}$$

$$\text{Analog: } (S/N)_R = \frac{1}{100}(S/N)_1 = 37 \text{ dB} \approx 5000, \gamma = (S/N)_1 = 5 \times 10^5 = 57 \text{ dB}$$

PCM advantage:  $57 - 23.4 = 33.6 \text{ dB}$

## 12.2-6

$$10 \log_{10}(1 + 4q^2 P_e) = 1 \text{ dB} \Rightarrow 1 + 4q^2 P_e = 10^{0.1} = 1.259$$

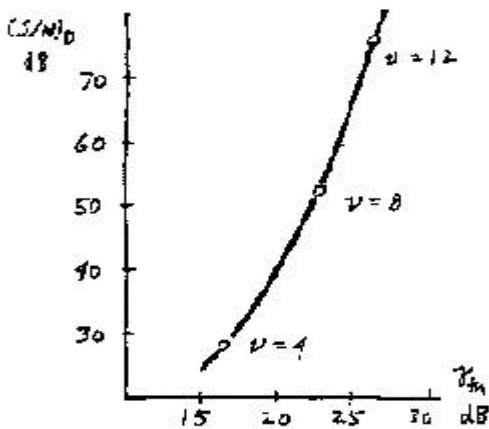
$$\text{so } P_e = 0.259/4q^2 \approx 1/15q^2$$

$$P_e = Q\left[\sqrt{(S/N)_R}\right] \approx \frac{1}{15 \times 2^{2v}}$$

$$(S/N)_R = \frac{W}{B_T} \gamma \leq \frac{1}{v} \gamma \Rightarrow \gamma_{th} \approx v(S/N)_R$$

$$(S/N)_D = 4.8 + 6.0v \text{ dB}$$

$v$	$P_e$	$\sqrt{(S/N)_R}$	$\gamma_{th} \text{ (dB)}$	$(S/N)_D, \text{ dB}$
4	$2.6 \times 10^{-4}$	3.5	16.9	28.8
8	$1.0 \times 10^{-6}$	4.8	22.7	52.8
12	$4.0 \times 10^{-9}$	5.8	26.1	76.8



## 12.2-7

Errors in magnitude bits have same effect as before, and there are  $q/2$  equiprobable values for  $i$ . Thus

$$\begin{aligned} \overline{\epsilon_m^2} &= \frac{1}{v} \left[ \sum_{m=0}^{v-2} \left( \frac{2}{q} 2^m \right)^2 + \frac{1}{q/2} \sum_{i=0}^{q/2} \left( \frac{2}{q} \right)^2 (2i-1)^2 \right] = \frac{4}{vq^2} \left[ \sum_{m=0}^{v-2} 4^m + \frac{2}{q} \sum_{i=0}^{q/2} (4i^2 - 4i + 1) \right] \\ &= \frac{4}{vq^2} \left\{ \frac{4^{v-1} - 1}{3} + \frac{2}{q} \left[ 4 \frac{q/2(q/2+1)(q+1)}{6} - 4 \frac{q/2(q/2+1)}{2} + \frac{q}{2} \right] \right\}, \quad 4^v = q^2 \\ &= \frac{4}{vq^2} \left( \frac{5q^2}{12} - \frac{2}{3} \right) = \frac{5q^2 - 8}{3vq^2} \approx \frac{5}{3v} \quad \text{if } 5q^2 \gg 8 \end{aligned}$$

12.2-8

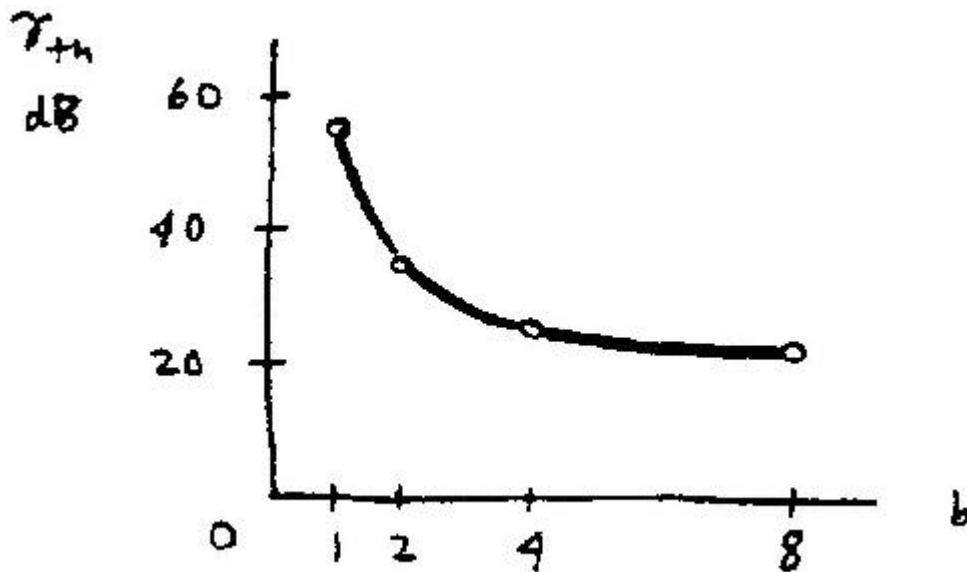
$$\gamma \geq \gamma_{th} \approx 6b(M^2 - 1) \Rightarrow M^2 \leq \frac{\gamma}{6b} + 1, \quad v \leq b$$

$$\text{Thus, } q_{\max} = M_{\max}^{v_{\max}} = \left( \frac{\gamma}{6b} + 1 \right)^{v/2}$$

12.2-9

$$M^b = 256, \quad \gamma_{th} = 6b(M^2 - 1)$$

M	b	$\gamma_{th}$	dB
2	8	144	21.6
4	4	360	25.6
16	2	3060	34.9
256	1	$3.93 \times 10^5$	55.9



12.3-1

$$\text{Using Eq. (5) and a sine wave} \Rightarrow f_s \Delta \geq |\dot{x}(t)|_{\max} = 2\pi f_m A_m \Rightarrow \Delta \geq \frac{2\pi f_m A_m}{f_s}$$

$$\text{If } W = 3 \text{ kHz and normalized input with } A_m = 1 \Rightarrow f_m = 3 \text{ kHz} \Rightarrow \Delta \geq \frac{2\pi \cdot 3}{30} = 0.628$$



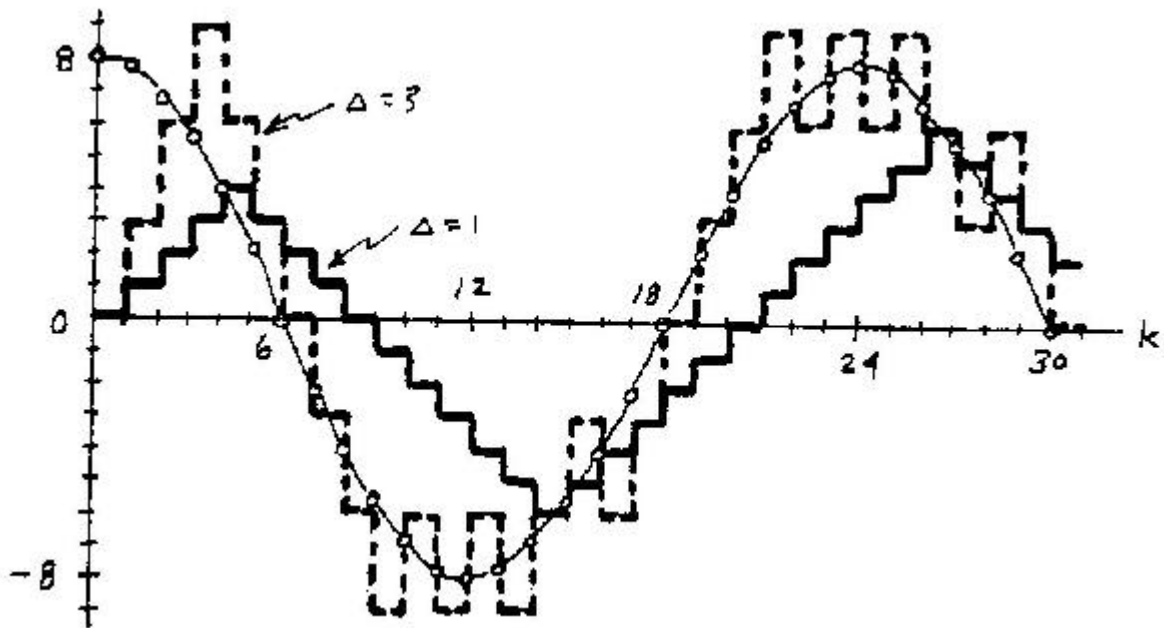
12.3-2

Using Eq. (5) and a sine wave  $\Rightarrow f_s \Delta \geq |\dot{x}(t)|_{\max} = 2\pi f_m A_m \Rightarrow A_m \leq \frac{f_s \Delta}{2\pi f_m}$

If  $W = 1$  kHz and Nyquist sampling  $\Rightarrow f_s = 20$  kHz

$$\Rightarrow A_m \leq \frac{20 \times 10^3 \times 0.117}{2\pi \times 1000} = 0.372$$

12.3-3



Note slope overload when  $\Delta=1$ .

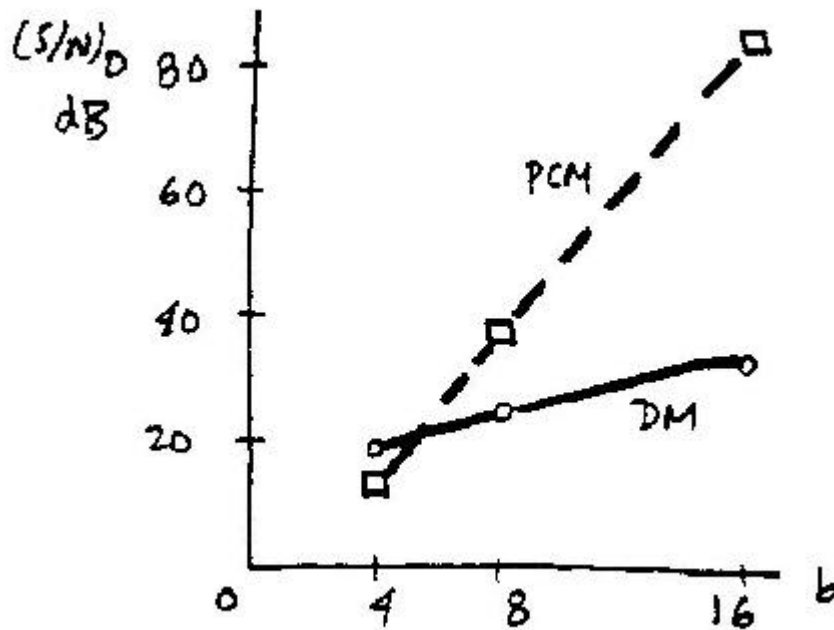
12.3-4

$$\text{DM: } (S/N)_D = 5.8b^3 / (\ln 2b)^2 = 7.6 + 10 \log_{10} \frac{b^3}{(\ln 2b)^2} \text{ dB}$$

$$\text{PCM: } (S/N)_D = 3 \times 2^{2b} \times \frac{1}{30} = 6.0b - 10 \text{ dB}$$

$b$	DM	PCM
4	19.3	14
8	25.8	38
16	32.9	86

12.3-4 continued



12.3-5

$$f_s = 2Wb \approx 8b \text{ kHz}, \quad \sigma = \sqrt{S_x} = 1/3$$

$$s_{opt} = \frac{f_s \Delta_{opt}}{2\pi\sigma W_{rms}} \approx \ln 2b$$

$$\Delta_{opt} \approx \frac{1.3\pi \ln 2b}{12 b}$$

$b$	$f_s$ (kHz)	$\Delta_{opt}$
4	32	0.177
8	64	0.118
16	128	0.0737

12.3-6

$$f_s \Delta \geq 2\pi f_0 \Rightarrow \Delta \geq 2\pi f_0 / f_s, \quad f_s = 2Wb$$

$$(S/N)_D = \frac{3f_s}{\Delta^2 W} S_x \leq \frac{3f_s^2}{4\pi^2 f_0^2 W} S_x = \frac{3}{4\pi^2} \frac{8W^3 b^3}{f_0^2 W} S_x = \frac{6}{\pi^2} \left( \frac{W}{f_0} \right)^2 b^3 S_x$$

12.3-7

$$S_x = \int_{-\infty}^{\infty} G_x(f) df = 2K \int_0^W \frac{df}{f_0^2 + f^2} = \frac{2K}{f_0} \arctan \frac{W}{f_0} \Rightarrow K = \frac{f_0 S_x}{2 \arctan(W / f_0)}$$

$$\begin{aligned} W_{rms}^2 &= \frac{1}{S_x} \int_{-\infty}^{\infty} f^2 G_x(f) df = \frac{2K}{S_x} \int_0^W \frac{f^2}{f_0^2 + f^2} df \\ &= \frac{2K}{S_x} \left[ \int_0^W \frac{f_0^2 + f^2}{f_0^2 + f^2} df - \int_0^W \frac{f_0^2}{f_0^2 + f^2} df \right] = \frac{2K}{S_x} \left[ W - f_0 \arctan \frac{W}{f_0} \right] \end{aligned}$$

$$\text{Thus, } W_{rms} = \left\{ \frac{f_0}{\arctan(W / f_0)} \left[ W - f_0 \arctan \frac{W}{f_0} \right] \right\}^{1/2} = 1.3 \text{ kHz}$$

when  $W = 4 \text{ kHz}$ ,  $f_0 = 0.8 \text{ kHz}$

12.3-8

$$\Delta = 2\pi\sigma W_{rms} s / f_s, \quad b = f_s / 2W, \quad \sigma = \sqrt{S_x}$$

$$N_g = \frac{W}{f_s} \frac{\Delta^2}{3} = \frac{\pi^2}{6b^3} \left( \frac{W_{rms}}{W} \right)^2 s^2 S_x$$

$$N_g + N_{so} = K \left[ s^2 + a(3s+1)e^{-3s} \right] \quad \text{where } K = \frac{\pi^2}{6b^3} \left( \frac{W_{rms}}{W} \right)^2 S_x, \quad a = \frac{16}{9} b^3$$

$$\frac{d}{ds} (N_g + N_{so}) = K \left[ 2s + 3ae^{-3s} - 3a(3s+1)e^{-3s} \right] = 0 \Rightarrow 2 - 9ae^{-3s} = 0$$

$$\text{Thus, } s_{opt} = \frac{1}{3} \ln \frac{9a}{2} = \frac{1}{3} \ln 8b^3 = \ln 2b$$

12.3-9

$$\rho_0 = R_x(0) / S_x = 1$$

$$n=1: \quad \rho_0 c_1 = \rho_1 \Rightarrow c_1 = \rho_1, \quad G_p = [1 - \rho_1^2]^{-1} = 2.77 = 4.4 \text{ dB}$$

$$n=2: \quad \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \Rightarrow c_1 = 8/9, \quad c_2 = -1/9$$

$$G_p = \left[ 1 - \frac{8}{9} \times 0.8 + \frac{1}{9} \times 0.6 \right]^{-1} = 2.81 = 4.5 \text{ dB}$$

12.3-10

$$\rho_0 = R_x(0)/S_x = 1$$

$$n=1: \rho_0 c_1 = \rho_1 \Rightarrow c_1 = \rho_1, G_p = [1 - \rho_1^2]^{-1} = 10.26 = 10.1 \text{ dB}$$

$$n=2: \begin{bmatrix} 1 & 0.95 \\ 0.95 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0.90 \end{bmatrix} \Rightarrow c_1 = 0.9744, c_2 = -0.0256$$

$$G_p = [1 - 0.9744 \times 0.95 + 0.0256 \times 0.9]^{-1} = 10.27 = 10.1 \text{ dB}$$

12.3-11

$$x[(k-1)T_s] \approx x_q(k-1)$$

$$\frac{dx(t)}{dt} \approx \frac{1}{T_s} [x(t) - x(t - T_s)] \Rightarrow T_s \left. \frac{dx(t)}{dt} \right|_{(k-1)T_s} \approx x_q(k-1) - x_q(k-2)$$

$$\text{Thus, take } \bar{x}_q(k) = x_q(k-1) + [x_q(k-1) - x_q(k-2)] = 2x_q(k-1) - x_q(k-2)$$

$$\text{so } c_1 = 2, c_2 = -1$$

12.3-12

$$\tilde{x}_q(k) = cx_q(k-1) \approx cx(k-1) \text{ so } \varepsilon_q(k) \approx x(k) - cx(k-1)$$

$$\text{Then } \overline{\varepsilon^2} = E[x^2(k) - 2cx(k)x(k-1) + c^2x^2(k-1)]$$

$$\text{where } E[x^2(k)] = E[x^2(k-1)] = E[x^2(t)] = S_x$$

$$E[x(k)x(k-1)] = E[x(t)x(t-T_s)] = R_x(T_s)$$

$$\text{so } \overline{\varepsilon^2} = S_x - 2cR_x(T_s) + c^2S_x = (1 + c^2)S_x - 2cR_x(T_s)$$

$$\frac{d\overline{\varepsilon^2}}{dc} = 2cS_x - 2R_x(T_s) = 0 \Rightarrow c = R_x(T_s)/S_x = \rho_1$$

12.3-13

$$\tilde{x}_q(k) = c_1x_q(k-1) + c_2x_q(k-2) \approx c_1x(k-1) + c_2x(k-2)$$

$$\text{so } \varepsilon_q(k) \approx x(k) - c_1(k-1) - c_2(k-2)$$

$$\overline{\varepsilon^2} = E \left[ \begin{aligned} &x^2(k) + c_1^2x^2(k-1) + c_2^2x^2(k-2) - 2c_1x(k)x(k-1) \\ &- 2c_2x(k)x(k-2) + 2c_1c_2x(k-1)x(k-2) \end{aligned} \right]$$

$$\text{where } E[x^2(k-n)] = E[x^2(t)] = S_x$$

$$E[x(k-n)x(k-m)] = E[x(t-nT_s)x(t-mT_s)] = R_x[(m-n)T_s]$$

$$\text{Hence, } \overline{\varepsilon^2} = (1 + c_1^2 + c_2^2)S_x + 2c_1(c_2 - 1)R_x(T_s) - 2c_2R_x(2T_s)$$

### 12.3-13 continued

We want

$$\partial \overline{\epsilon^2} / \partial c_1 = 2c_1 S_x + 2(c_2 - 1) R_x(T_s) = 0$$

$$\partial \overline{\epsilon^2} / \partial c_2 = 2c_2 S_x + 2c_1 R_x(T_s) - 2R_x(2T_s) = 0$$

so

$$\left. \begin{aligned} c_1 + \frac{R_x(T_s)}{S_x} c_2 &= \frac{R_x(T_s)}{S_x} \\ \frac{R_x(T_s)}{S_x} c_1 + c_2 &= \frac{R_x(2T_s)}{S_x} \end{aligned} \right\} \Rightarrow \begin{bmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

Same result as Eq. (16b) with  $n = 2$  since  $\rho_0 = R_x(0)/S_x = 1$

### 12.4-1

Assume just music samples and no parity or control information

$$70 \text{ min/CD} \times 1.4112 \text{ Mbits/sec} \times 60 \text{ sec/min} = 5.927 \text{ Gbits.}$$

### 12.4-2

$$981 \text{ pages} \times 2 \text{ columns/page} \times 57 \text{ lines/column} \times 45 \text{ characters/line} \times 7 \text{ bits/character} = 35 \text{ Mbits.}$$

$$\text{Based on problem 12.4-1, a CD can store } 5.9 \text{ Gbits} = 5900 \text{ Mbytes} \Rightarrow 35/5900 \times 100\% = 0.59\%$$

### 12.4-3

Assume with the 2 Gbyte hard drive there is no need to store extra control or parity bits.

$$\text{Music} \Rightarrow 1.4112 \text{ Mbits/sec} \times 1 \text{ byte/8bits}$$

$$\frac{1 \text{ sec}}{1.4112 \times 10^6 \text{ bits}} \times 8 \text{ bits/byte} \times 1 \text{ min/60secs} \times 2 \times 10^9 \text{ bytes/hard drive} = 189 \text{ minutes}$$

If we do incorporate the same error control used on the CD, the recording time is:

$$\frac{1 \text{ sec}}{7350 \text{ frames}} \times 1 \text{ min/60 secs} \times 1 \text{ frame/561 bits} \times 8 \text{ bits/byte} \times 2 \times 10^9 \text{ bytes/hard drive} = 65 \text{ minutes.}$$

### 12.5-1

$$r_b = v f_s \geq 12 \times 2 \times 15 \text{ kHz} = 360 \text{ kbps}$$

$$N \leq \frac{1.544 \text{ Mbps}}{r_b} = 4.2 \Rightarrow N = 4$$

$$\left. \begin{array}{l} \text{Digital } B_T \geq \frac{1}{2} \times 1.544 \text{ Mbps} = 772 \text{ kHz} \\ \text{Analog } B_T \geq NW = 60 \text{ kHz} \end{array} \right\} \text{Eff} = \frac{60}{772} = 7.8\%$$

### 12.5-2

$$r_b = v f_s \geq 12 \times 2 \times 15 \text{ kHz} = 360 \text{ kbps}$$

$$N \leq \frac{2.048 \text{ Mbps}}{r_b} = 5.6 \Rightarrow N = 5$$

$$\left. \begin{array}{l} \text{Digital } B_T \geq \frac{1}{2} \times 2.048 \text{ Mbps} = 1.024 \text{ MHz} \\ \text{Analog } B_T \geq NW = 75 \text{ kHz} \end{array} \right\} \text{Eff} = \frac{75}{1024} = 7.3\%$$

### 12.5-3

From Fig. 12.5-8, if we subtract Transport and Path overhead,

a SONET frame has 9 rows x 86 bytes/row = 774 bytes of user data.

Thus a STS-1 has a capacity of 774 bytes/frame x 8 bits/byte x 8000 bits/frame = 49.536 Mbps.

A DS0 line is 64 kbps  $\Rightarrow$  and STS-1 can handle  $49536/64 = 774$  DS0 lines.

However, in practice, a VT is used to interface DS0 and DS1 lines to a STS-1.

The additional overhead of the VT reduces the number of DS0 inputs to 672 and the number of DS1 inputs to 28. See Bellamy (1991) for more information.

### 12.5-4

$$(600 \text{ dots/inch})^2 \times (8 \text{ inches} \times 11 \text{ inches})/\text{page} = 31,680 \text{ kbits/page.}$$

$$2 \text{ BRI channel} \Rightarrow 128 \text{ kbps} \Rightarrow 31,680 \text{ kbits}/128 \text{ kbps} = 247 \text{ seconds/page.}$$

Obviously, some image compression is needed for this to be practical!

### 12.5-5

$$(600 \text{ dots/inch})^2 \times (8 \text{ inches} \times 11 \text{ inches})/\text{page} = 31,680 \text{ kbits/page.}$$

$$1\text{-}56 \text{ kbps channel} \Rightarrow 31,680 \text{ kbits}/56 \text{ kbps} = 566 \text{ seconds/page.}$$

## Chapter 13

### 13.1-1

$$P(\text{no errors}) = P(0,4) = (1 - 0.1)^4 = 0.6561$$

$$P(\text{detected errors}) = P(1,4) = 4 \times 0.1 \times 0.9^3 = 0.2916$$

$$P(\text{undetected errors}) = 1 - P(0,4) - P(1,4) = 0.0523$$

### 13.1-2

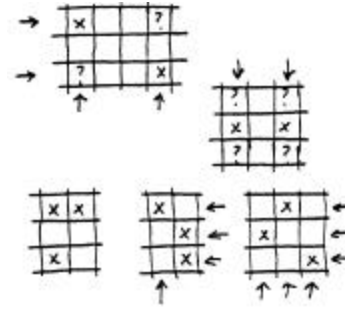
$$P(\text{no errors}) = P(0,9) = (1 - 0.05)^9 = 0.5971$$

$$P(\text{detected errors}) = P(1,9) = 9 \times 0.05 \times 0.95^8 = 0.2985$$

$$P(\text{undetected errors}) = 1 - P(0,9) - P(1,9) = 0.0712$$

### 13.1-3

- (a) Two errors not in the same row or column yields 4 intersections as possible error locations. Two errors in the same row (or column) yields two columns (or rows) as possible error locations.
- (b) L shaped error pattern yields no parity failures and is undetectable. Other patterns yield 4 or 6 parity failures and are detectable.



### 13.1-4

$$(31,26) \ t = 1: \ Q\left(\sqrt{\frac{52}{31}}g_b\right) \leq \left(\frac{1}{30} \times 10^{-4}\right)^{1/2} \Rightarrow g_b \geq \frac{31}{52} \times 2.9^2 = 7.0\text{dB}$$

$$(31,21) \ t = 2: \ Q\left(\sqrt{\frac{42}{31}}g_b\right) \leq \left(\frac{2}{30 \times 29} \times 10^{-4}\right)^{1/3} \Rightarrow g_b \geq \frac{31}{42} \times 2.53^2 = 6.7\text{dB}$$

$$(31,16) \ t = 3: \ Q\left(\sqrt{\frac{32}{31}}g_b\right) \leq \left(\frac{3 \times 2}{30 \times 29 \times 28} \times 10^{-4}\right)^{1/4} \Rightarrow g_b \geq \frac{31}{32} \times 2.25^2 = 6.9\text{dB}$$

$$\text{Uncoded} : \ Q(\sqrt{2}g_b) \leq 10^{-4} \Rightarrow g_b \geq \frac{1}{2} \times 3.73^2 = 8.4\text{dB}$$

Thus, use (31, 21) code to save 1.7 dB.

### 13.1-5

$$(31,26) \quad t=1: \quad Q\left(\sqrt{\frac{52}{31}}g_b\right) \leq \left(\frac{1}{30} \times 10^{-6}\right)^{1/2} \Rightarrow g_b \geq \frac{31}{52} \times 3.6^2 = 8.9\text{dB}$$

$$(31,21) \quad t=2: \quad Q\left(\sqrt{\frac{42}{31}}g_b\right) \leq \left(\frac{2}{30 \times 29} \times 10^{-6}\right)^{1/3} \Rightarrow g_b \geq \frac{31}{42} \times 3.0^2 = 8.2\text{dB}$$

$$(31,16) \quad t=3: \quad Q\left(\sqrt{\frac{32}{31}}g_b\right) \leq \left(\frac{3 \times 2}{30 \times 29 \times 28} \times 10^{-6}\right)^{1/4} \Rightarrow g_b \geq \frac{31}{32} \times 2.65^2 = 8.3\text{dB}$$

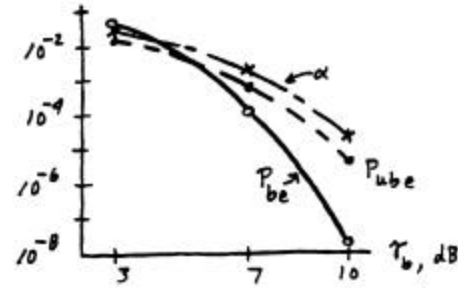
$$\text{Uncoded} \quad : \quad Q(\sqrt{2}g_b) \leq 10^{-6} \Rightarrow g_b \geq \frac{1}{2} \times 4.76^2 = 10.5\text{dB}$$

Thus, use (31, 21) code to save 2.3 dB.

### 13.1-6

$$(31,26) \quad t=1:$$

$$P_{\text{ube}} = Q(\sqrt{2}g_b), a = Q\left(\sqrt{\frac{52}{31}}g_b\right), P_{\text{be}} = 30a^2$$



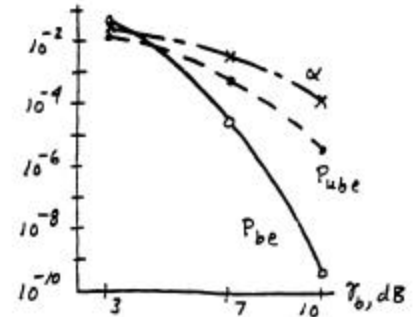
$\gamma_b$	dB	$P_{\text{ube}}$	$\alpha$	$P_{\text{be}}$
2	3	$2.3 \times 10^{-2}$	$3.5 \times 10^{-2}$	$3.7 \times 10^{-2}$
5	7	$8.5 \times 10^{-4}$	$2 \times 10^{-3}$	$1.2 \times 10^{-4}$
10	10	$4 \times 10^{-6}$	$2.2 \times 10^{-5}$	$1.5 \times 10^{-8}$

### 13.1-7

$$(31,21) \quad t=2:$$

$$P_{\text{ube}} = Q(\sqrt{2}g_b), a = Q\left(\sqrt{\frac{42}{31}}g_b\right), P_{\text{be}} = \frac{30 \times 29}{2} a^3$$

$\gamma_b$	DB	$P_{\text{ube}}$	$\alpha$	$P_{\text{be}}$
2	3	$2.3 \times 10^{-2}$	$5 \times 10^{-2}$	$5.4 \times 10^{-2}$
5	7	$8.5 \times 10^{-4}$	$4.7 \times 10^{-3}$	$4.5 \times 10^{-5}$
10	10	$4 \times 10^{-6}$	$1.2 \times 10^{-4}$	$7.5 \times 10^{-10}$





13.1-8

Coded transmission has  $r_b / r = R_c'$  and  $Q(\sqrt{2R_c'g_b}) = a$

(12,11)  $l = 1$ :

$$a = \left( \frac{1}{11} \times 10^{-5} \right)^{1/2} = 9.5 \times 10^{-4}, R_c' = \frac{11}{12}(1 - 12a) = 0.906, g_b = 3.12^2 / 2R_c' = 7.3\text{dB}$$

(15,11)  $l = 2$ :

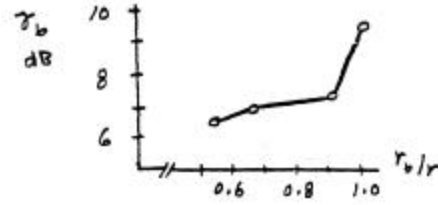
$$a = \left( \frac{2}{14 \times 13} \times 10^{-5} \right)^{1/3} = 4.8 \times 10^{-3}, R_c' = \frac{11}{15}(1 - 15a) = 0.681, g_b = 2.6^2 / 2R_c' = 7.0\text{dB}$$

(16,11)  $l = 3$ :

$$\alpha = \left( \frac{3 \times 2}{15 \times 14 \times 13} \times 10^{-5} \right)^{1/4} = 1.22 \times 10^{-2}, R_c' = \frac{11}{16}(1 - 16\alpha) = 0.554, \gamma_b = 2.25^2 / 2R_c' = 6.6\text{dB}$$

Uncoded transmission  $r_b / r = 1$

$$Q(\sqrt{2g_b}) = 10^{-5} \Rightarrow g_b \approx \frac{1}{2} \times 4.27^2 = 9.6\text{dB}$$



13.1-9

Coded transmission has  $r_b / r = R_c'$  and  $Q(\sqrt{2R_c'g_b}) = a$

(12,11)  $l = 1$ :

$$a = \left( \frac{1}{11} \times 10^{-6} \right)^{1/2} = 3.0 \times 10^{-4}, R_c' = \frac{11}{12}(1 - 12a) = 0.913, g_b = 3.45^2 / 2R_c' = 8.1\text{dB}$$

(15,11)  $l = 2$ :

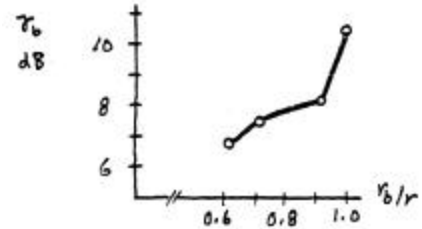
$$a = \left( \frac{2}{14 \times 13} \times 10^{-6} \right)^{1/3} = 2.2 \times 10^{-3}, R_c' = \frac{11}{15}(1 - 15a) = 0.709, g_b = 2.85^2 / 2R_c' = 7.6\text{dB}$$

(16,12)  $l = 3$ :

$$a = \left( \frac{3 \times 2}{15 \times 14 \times 13} \times 10^{-6} \right)^{1/4} = 6.9 \times 10^{-3}, R_c' = \frac{11}{16}(1 - 16a) = 0.612, g_b = 2.45^2 / 2R_c' = 6.9\text{dB}$$

Uncoded transmission  $r_b / r = 1$

$$Q(\sqrt{2g_b}) = 10^{-6} \Rightarrow g_b \approx \frac{1}{2} \times 4.77^2 = 10.6\text{dB}$$



13.1-10

$$\mathbf{a} = Q(\sqrt{2 \times 4}) = 2.3 \times 10^{-3}, l = 2, P_{be} = \frac{14 \times 13}{2} \mathbf{a}^3 \approx 10^{-6}$$

$$\left. \begin{array}{l} T_w = n/r = 30 \mu s, \\ t_d \geq 45 \text{ km} / 3 \times 10^5 \text{ km/s} = 150 \mu s \end{array} \right\} \Rightarrow N \geq \frac{2t_d}{T_w} = 10$$

$$p = n\alpha \approx 0.035, R_c' = \frac{11}{15} \frac{1 - 0.035}{1 + 9 \times 0.035} = 0.538 \Rightarrow r_b = 269 \text{ kbps}$$

13.1-11

$$\mathbf{a} = Q(\sqrt{2 \times 4}) = 2.3 \times 10^{-3}, l = 3, P_{be} = \frac{15 \times 14 \times 13}{3 \times 2} \mathbf{a}^4 \approx 10^{-8}$$

$$\left. \begin{array}{l} T_w = n/r = 32 \mu s, \\ t_d \geq 45 \text{ km} / 3 \times 10^5 \text{ km/s} = 150 \mu s \end{array} \right\} \Rightarrow N \geq \frac{2t_d}{T_w} = 9.38 \Rightarrow N = 10$$

$$p = n\alpha \approx 0.037, R_c' = \frac{11}{16} \frac{1 - 0.037}{1 + 9 \times 0.037} = 0.497 \Rightarrow r_b = 248 \text{ kbps}$$

13.1-12

$$P_{be} = k\alpha^2 \leq 10^{-6} \Rightarrow \alpha < 10^{-3} \text{ and } p = (k+1)\alpha < 1 \text{ if } k < 100$$

$$\left. \begin{array}{l} D \geq 2 \times 18 \text{ km} / 3 \times 10^5 \text{ km/s} = 120 \mu s \\ T_w = (k+1)/r = 100(k+1) \mu s \end{array} \right\} \Rightarrow D/T_w \geq \frac{1.2}{k+1}$$

$$R_c' \leq \frac{k}{k+1} \times \frac{1-p}{1+D/T_w} \approx \frac{k}{k+1+1.2} \geq \frac{7200}{10,000} \Rightarrow k \geq 5.66 \Rightarrow k = 6$$

Then,

$$R_c' \approx 0.732, \mathbf{a} = Q(\sqrt{2R_c' \mathbf{g}_b}) \leq (\frac{1}{6} \times 10^{-6})^{1/2} = 4.1 \times 10^{-4}$$

$$\text{So, } \mathbf{g}_b \geq 3.35^2 / 2R_c' = 8.8 \text{ dB}$$

13.1-13

$$P_{be} = k\alpha^2 \leq 10^{-6} \Rightarrow \alpha < 10^{-3} \text{ and } p = (k+1)\alpha < 1 \text{ if } k < 100$$

$$\left. \begin{array}{l} D \geq 2 \times 60 \text{ km} / 3 \times 10^5 \text{ km/s} = 400 \mu s \\ T_w = (k+1)/r = 100(k+1) \mu s \end{array} \right\} \Rightarrow D/T_w \geq \frac{4}{k+1}$$

$$R_c' \leq \frac{k}{k+1} \times \frac{1-p}{1+D/T_w} \approx \frac{k}{k+1+4} \geq \frac{7200}{10,000} \Rightarrow k \geq 12.9 \Rightarrow k = 13$$

$$\text{Then, } R_c' \approx 0.737, \alpha = Q(\sqrt{2R_c' \gamma_b}) \leq (\frac{1}{13} \times 10^{-6})^{1/2} = 2.8 \times 10^{-4}$$

$$\text{So, } \mathbf{g}_b \geq 3.45^2 / 2R_c' = 9.1 \text{ dB}$$

13.1-14

$$\begin{aligned}\bar{m} &= 1(1-p) + (1+N)p(1-p) + (1+2N)p^2(1-p) + (1+3N)p^3(1-p) + \dots \\ &= (1-p)[1 + (1+N)p + (1+2N)p^2 + (1+3N)p^3 + \dots] \\ &= (1-p)[(1+p+p^2+p^3+\dots) + Np(1+2p+3p^2+\dots)]\end{aligned}$$

But  $(1+p+p^2+p^3+\dots) = (1-p)^{-1}$ ,

and  $(1+2p+3p^2+\dots) = (1-p)^{-2}$

$$\text{Thus, } \bar{m} = (1-p) \left[ \frac{1}{1-p} + \frac{Np}{(1-p)^2} \right] = 1 + \frac{Np}{1-p}$$

13.1-15

A word has a detected error and must be retransmitted if the number of bit error is  $t < i \leq l$ , so

$$p = \sum_{i=t+1}^l P(i, n) \approx P(t+1, n) = \binom{n}{t+1} a^{t+1} (1-a)^{n-t-1} \approx \binom{n}{t+1} a^{t+1}$$

$$\text{With } d_{\min} = 4, l = 2 \text{ and } t = 1 \text{ so } p \approx \frac{n(n-1)}{2} \alpha^2$$

13.1-16

$$\begin{aligned}\text{(a) } p &= P(t+1, n) = \binom{n}{t+1} a^{t+1} (1-a)^{n-t-1} \approx \binom{n}{t+1} a^{t+1} \\ P_{we} &= \sum_{i=t+2}^n P(i, n) \approx P(t+2, n) = \binom{n}{t+2} \alpha^{t+2} (1-\alpha)^{n-t-2} \approx \binom{n}{t+2} \alpha^{t+2} \\ \dot{P}_{be} &= \frac{t+2}{n} P_{we} \approx \frac{t+2}{n} \frac{n(n-1) \cdots (n-t-1)}{(t+2)!} \alpha^{t+2} = \binom{n-1}{t+1} \alpha^{t+2}\end{aligned}$$

$$\text{(b) } d_{\min} = 2t + 2 = 8 \Rightarrow t = 3, R_c' \approx \frac{12}{24} = \frac{1}{2}, \text{ assuming } p \ll 1$$

$$P_{be} \approx \frac{23 \times 22 \times 21 \times 20}{4 \times 3 \times 2} a^5 = 10^{-5} \Rightarrow a = 1.62 \times 10^{-2}$$

$$p \approx \frac{24 \times 23 \times 22 \times 21}{4 \times 3 \times 2} a^4 = 7.3 \times 10^{-4} \ll 1 \text{ as assumed}$$

$$a = Q\left(\sqrt{2 \times \frac{1}{2} g_b}\right) = 1.62 \times 10^{-2} \Rightarrow g_b = 2.15^2 = 4.62$$

$$\text{Uncoded: } P_{be} = Q(\sqrt{2\gamma_b}) \approx 1.3 \times 10^{-3}$$

### 13.2-1

(a) Let  $n_u$  = number of 1s in U,  $n_v$  = number of 1s in V, and

$n_{uv}$  = number of 1s position in U and V. Then,

$$W(U) + W(V) = n_u + n_v$$

$$d(U, V) = W(U + V) = n_u + n_v - n_{uv} \leq W(U) + W(V)$$

(b)  $U + V = X + Y + Y + Z$

$$= (x_1 \oplus y_1 \oplus y_1 \oplus z_1, \dots), \text{ where } y_1 \oplus y_1 = 0, \text{ etc.}$$

$$= (x_1 \oplus z_1, \dots)$$

$$= X + Z$$

Thus,

$$d(U, V) = W(U + V) = W(X + Z) = d(X, Z)$$

$$\text{and } W(U) + W(V) = W(X + Y) + W(Y + Z) = d(X, Y) + d(Y, Z)$$

$$\text{so } d(X, Z) \leq W(U) + W(V) = d(X, Y) + d(Y, Z)$$

### 13.2-2

$$d(X, Y) = i \leq l,$$

$$d(X, Y) + d(Y, Z) \geq d(X, Z) \geq d_{\min} \geq l + 1$$

so,

$$l + 1 \leq d_{\min} \leq i + d(Y, Z) \leq l + d(Y, Z) \Rightarrow d(Y, Z) \geq 1$$

Thus, Y cannot be a vector in the code, and the errors are detectable.

### 13.2-3

$$d(X, Y) = i \leq t,$$

$$d(X, Y) + d(Y, Z) \geq d(X, Z) \geq d_{\min} \geq 2t + 1$$

so,

$$2t + 1 \leq d_{\min} \leq i + d(Y, Z) \leq t + d(Y, Z) \text{ and } d(Y, Z) \geq t + 1 > d(X, Y)$$

Thus, Y is closer to X than to any other valid code vector, and the errors can be corrected.

### 13.2-4

$$n = 3, k = 1, q = 2$$

$$H^T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S = EH^T \Rightarrow$$

E	S
1 0 0	1 1
0 1 0	1 0
0 0 1	0 1

13.2-4 continued

$Y$	$S$	$\hat{E}$	$Y + \hat{E}$
0 0 0	0 0	0 0 0	0 0 0
0 0 1	0 1	0 0 1	0 0 0
0 1 0	1 0	0 1 0	0 0 0
0 1 1	1 1	1 0 0	1 1 1
1 0 0	1 1	1 0 0	0 0 0
1 0 1	1 0	0 1 0	1 1 1
1 1 0	0 1	0 0 1	1 1 1
1 1 1	0 0	0 0 0	1 1 1

13.2-5

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = EH^T \Rightarrow$$

$\hat{E}$	$S$
1 0 0 0 0 0	1 1 0
0 1 0 0 0 0	0 1 1
0 0 1 0 0 0	1 0 1
0 0 0 1 0 0	1 0 0
0 0 0 0 1 0	0 1 0
0 0 0 0 0 1	0 0 1

13.2-6

$$H^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S = EH^T \Rightarrow$$

$E$	$S$
0 0 0 0 0	0 0
1 0 0 0 0	1 1
0 1 0 0 0	0 1
0 0 1 0 0	1 0
0 0 0 1 0	1 0
0 0 0 0 1	0 1

$\Rightarrow$  Correctly indicates no errors.  
 Different  $E$ s yield same  $S$ , so error correction is impossible.

13.2-7

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

$$\begin{aligned}
c_1 &= m_5 \oplus m_6 \oplus m_7 \oplus m_8 \oplus m_9 \oplus m_{10} \oplus m_{11} \\
c_2 &= m_2 \oplus m_3 \oplus m_4 \oplus m_8 \oplus m_9 \oplus m_{10} \oplus m_{11} \\
c_3 &= m_1 \oplus m_3 \oplus m_4 \oplus m_6 \oplus m_7 \oplus m_{10} \oplus m_{11} \\
c_4 &= m_1 \oplus m_2 \oplus m_4 \oplus m_5 \oplus m_7 \oplus m_9 \oplus m_{11}
\end{aligned}$$

$S$	Error bit
0 0 0 0	None
0 0 1 1	1
0 1 0 1	2
0 1 1 0	3
0 1 1 1	4
1 0 0 1	5
1 0 1 0	6
1 0 1 1	7
1 1 0 0	8
1 1 0 1	9
1 1 1 0	10
1 1 1 1	11
1 0 0 0	12
0 1 0 0	13
0 0 1 0	14
0 0 0 1	15

13.2-8

$$(a) \left. \begin{array}{l} z^q - 1 \geq n \\ q = n - k \end{array} \right\} \frac{z^n}{n+1} \geq z^k = 64 \Rightarrow n \geq 10 \text{ (by trial and error)}$$

Take  $n = 10$ , so  $q = 4$  and  $(q + n) \times z^q = 224$  bits

(b) Any  $6 \times 4$  matrix whose rows are all different and contain at least 2 ones.

Example:

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

13.2-9

$$(a) \left. \begin{array}{l} z^q - 1 \geq n \\ q = n - k \end{array} \right\} \frac{z^n}{n+1} \geq z^k = 256 \Rightarrow n \geq 12 \text{ (by trial and error)}$$

Take  $n = 12$ , so  $q = 4$  and  $(q + n) \times z^q = 256$  bits

(b) Any  $8 \times 4$  matrix whose rows are all different and contain at least 2 ones.

Example:

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

13.2-10

$$a_{ij} = \sum_{m=1}^n g_{im} h_{mj}^T \pmod{2}, \text{ where}$$

$$g_{im} = m^{th} \text{ element of } i^{th} \text{ row of } G \quad h_{mj}^T = m^{th} \text{ element of } j^{th} \text{ row of } H^T$$

$$= \begin{cases} 0 & 1 \leq m \leq i-1 \\ 1 & m = i \\ 0 & i+1 \leq m \leq k \\ p_{i(m-k)} & k+1 \leq m \leq n \end{cases} \quad = \begin{cases} p_{mj} & 1 \leq m \leq n-k \\ 0 & n-k+1 \leq m \leq j+k-1 \\ 1 & m = j+k \\ 0 & j+k+1 \leq m \leq n \end{cases}$$

13.2-10 continued

Thus,  $a_{ij} = 1 \cdot p_{ij} \oplus p_{i(j+k-k)} \cdot 1 = p_{ij} \oplus p_{ij} = 0$

13.2-11

(a) The binary number  $s_1 s_2 s_3$  equals the error location,

i.e.,

000  $\Rightarrow$  no error

001  $\Rightarrow$  1<sup>st</sup> bit

010  $\Rightarrow$  2<sup>nd</sup> bit

etc.

$s_1$	$s_2$	$s_3$	$E$							
0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0
1	0	1	0	0	0	0	1	0	0	0
1	1	0	0	0	0	0	0	1	0	0
1	1	1	0	0	0	0	0	0	1	0

(b)

$$s_1 = x_4 \oplus x_5 \oplus x_6 \oplus x_7 = 0$$

$$s_2 = x_2 \oplus x_3 \oplus x_6 \oplus x_7 = 0$$

$$s_3 = x_1 \oplus x_3 \oplus x_5 \oplus x_7 = 0$$

Since  $x_1, x_2$ , and  $x_4$  appear only once, they must be the check bits.

Thus,

$$X = (c_1 \ c_2 \ m_1 \ c_3 \ m_2 \ m_3 \ m_4)$$

$$\Rightarrow \begin{cases} c_3 = m_2 \oplus m_3 \oplus m_4 \\ c_2 = m_1 \oplus m_3 \oplus m_4 \\ c_1 = m_1 \oplus m_2 \oplus m_4 \end{cases}$$

13.2-12

(a)

$M$				$C$				$w$	$M$				$C$				$w$
0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	4
0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	1	0	4
0	0	1	0	0	1	1	0	1	1	0	1	0	0	1	1	0	4
0	0	1	1	0	1	0	1	0	1	0	1	1	0	0	0	1	4
0	1	0	0	0	1	0	0	1	1	1	0	0	1	0	0	1	4
0	1	0	1	0	0	0	1	1	1	1	0	0	0	1	0	0	4
0	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	8

(b) Note from check-bit equations in Example 13.2-1 that

$$c_1 \oplus c_2 \oplus c_3 = m_2, \text{ so } m_1 \oplus m_2 \oplus m_3 \oplus m_4 \oplus c_1 \oplus c_2 \oplus c_3 = m_1 \oplus m_3 \oplus m_4$$

$$\text{Thus, } c_4 = m_1 \oplus m_3 \oplus m_4$$

(c) Form  $d = \sum_{i=1}^n y_i \pmod{2}$  so  $\begin{cases} d = 0 \Rightarrow \text{no errors or even number of errors} \\ d = 1 \Rightarrow \text{odd number of errors} \end{cases}$

13.2-12 continued

If  $S = (000)$  and  $d = 0$ , assume no errors so  $X = Y$ ,

If  $S \neq (000)$  and  $d = 1$ , assume single error so  $X = Y + \hat{E}$ ,

If  $S \neq (000)$  and  $d = 0$ , assume detected but uncorrectable errors.

13.2-13

$$\begin{array}{r} \overbrace{11101}^G \overline{)1010000} \quad \begin{array}{r} 111 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \underline{11101} \end{array}$$

$$10010$$

$$\underline{11101}$$

$$11110$$

$$\underline{11101}$$

$$0011$$

$$\Rightarrow Q_M(p) = p^2 + p + 1, \quad C(p) = 0 + 0 + p + 1 \Rightarrow X = (101 : 0011)$$

$$X' = (0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1) \quad Y(p) = 0 + p^5 + 0 + 0 + p^2 + p + 1$$

$$\begin{array}{r} \overbrace{11101}^G \overline{)0100111} \quad \begin{array}{r} 011 \\ \hline \end{array} \end{array}$$

$$\underline{11101}$$

$$11101$$

$$\underline{11101}$$

$$0000$$

$$\Rightarrow S(p) = 0$$

13.2-14

$$\begin{array}{r} \overbrace{10111}^G \overline{)1010000} \quad \begin{array}{r} 100 \\ \hline \end{array} \end{array}$$

$$\underline{10111}$$

$$1100$$

$$\Rightarrow Q_M(p) = p^2 + 0 + 0, \quad C(p) = p^3 + p^2 + 0 + 0 \Rightarrow X = (101 : 1100)$$

$$X' = (0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1) \quad Y(p) = 0 + p^5 + p^4 + p^3 + 0 + 0 + 1$$



$$\begin{array}{r} \phantom{0}011 \\ 10111 \overline{) 0111001} \\ \underline{\phantom{0}10111} \\ \phantom{0}10111 \\ \underline{\phantom{0}10111} \\ \phantom{0}0000 \end{array}$$

13.2-15

$$\begin{array}{r} \overbrace{1011}^G \overline{) 1000000} = p^6 \\ \underline{1011} \\ 1100 \\ \underline{1011} \\ 1110 \\ \underline{1011} \\ 101 = R_1 \end{array}$$

$$\begin{array}{r} 101 \\ 1011 \overline{) 0100000} = p^5 \\ \underline{1011} \\ 1100 \\ \underline{1011} \\ 111 = R_2 \end{array}$$

$$P = \begin{bmatrix} \frac{R_1}{R_2} \\ \frac{R_2}{R_3} \\ \frac{R_3}{R_4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \text{ which is the P matrix in Example 13.2-1.}$$

13.2-16

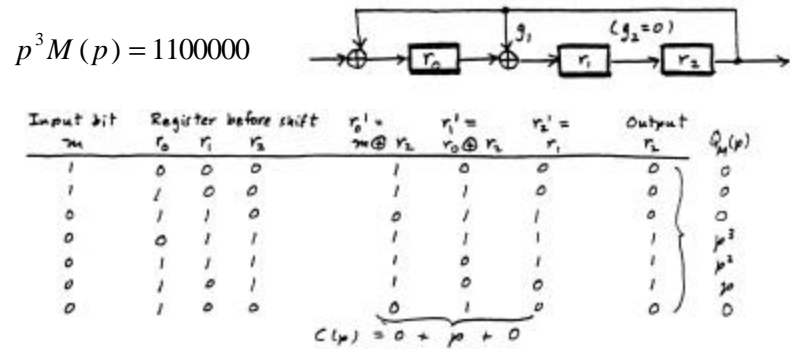
$$\begin{array}{r} \overbrace{1101}^G \overline{) 1110} \\ 1101 \overline{) 1000000} = p^6 \\ \underline{1101} \\ 1010 \\ \underline{1101} \\ 1110 \\ \underline{1101} \\ 110 = R_1 \end{array}$$

$$\begin{array}{r} \overline{) 0111} \\ 1101 \overline{) 0100000} = p^5 \\ \underline{1101} \\ 1010 \\ \underline{1101} \\ 1110 \\ \underline{1101} \\ 011 = R_2 \end{array}$$

Similarly,  $R_3 = 111$  and  $R_4 = 101$

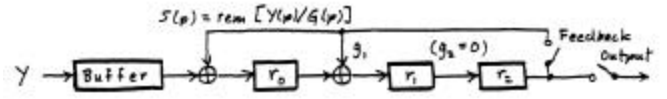
$$P = \begin{bmatrix} \overline{R_1} \\ \overline{R_2} \\ \overline{R_3} \\ \overline{R_4} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \text{ same rows as P matrix in Example 13.2-1, but different order.}$$

13.2-17



13.2-18

Initialize register to (00.....0), close feedback, open output switch, and shift y into register. After 7 shift cycles, open feedback switch and close output shift, shift out S(p) from register in 3 shift cycles.



Input bit	Register before shift			$r_0^1 =$	$r_1^1 =$	$r_2^1 =$
y	$r_0$	$r_1$	$r_2$	$y \oplus r_2$	$r_0 \oplus r_2$	$r_1$
1	0	0	0	1	0	0
1	1	0	0	1	1	0
0	1	1	0	0	1	1
0	0	1	1	1	1	1
0	1	1	1	1	0	1
1	1	0	1	1	0	1
0	0	0	0	0	0	0
$S(p) = 0 + 0 + 0$						

13.2-19

$$P_{be} = 1 \times 10^{-5} = Q(\sqrt{2\gamma_b}) \text{ and } d_{\min} = 3$$

$$\text{From Table T.6} \Rightarrow \sqrt{2\gamma_b} = 4.3 \Rightarrow \gamma_b = 9.2$$

$$(a) \text{ For } (7,4) \text{ code} \Rightarrow n = 7, k = 4, R_c = 4/7, t = 1$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \Rightarrow \binom{n-1}{t} = \frac{6!}{1!(6-1)!} = 6$$

$$P_{be} = \binom{n-1}{t} [Q(\sqrt{2R_c\gamma_b})]^{t+1} = 6[Q(\sqrt{8/7 \times 9.68})]^2 = 2.2 \times 10^{-6}$$

$$\alpha = Q(\sqrt{2R_c\gamma_b}) = Q(\sqrt{8/7 \times 9.2}) = 6 \times 10^{-4}$$

$$(b) (15,11) \text{ code} \Rightarrow d_{\min} = 3 \geq 2t + 1 \Rightarrow t = 1 \text{ and } R_c = 11/15$$

$$\binom{n-1}{t} = \frac{14!}{1!(14-1)!} = 14 \Rightarrow P_{be} = 14[Q(\sqrt{22/15 \times 9.2})]^2 = 3.2 \times 10^{-7}$$

$$\alpha = Q(\sqrt{2R_c\gamma_b}) = Q(\sqrt{22/15 \times 9.68}) = 1.5 \times 10^{-4}$$

13.2-19 continued

(c) (31,26) code  $\Rightarrow R_c = 26/31, t = 1$

$$\binom{n-1}{t} = \frac{30!}{1!(30-1)!} = 30 \Rightarrow P_{be} = 30Q(\sqrt{52/31 \times 9.2})^2 = 6.1 \times 10^{-8}$$

$$\alpha = Q(\sqrt{2R_c\gamma_b}) = Q(\sqrt{52/31 \times 9.2}) = 4.5 \times 10^{-5}$$

13.2-20

With  $\gamma_c = R_c \gamma_b$ ,  $\alpha = Q(\sqrt{2\gamma_c}) = Q(\sqrt{2R_c\gamma_b}) \Rightarrow$  if  $\gamma_c$  fixed  $\Rightarrow R_c \gamma_b$  fixed  $\Rightarrow \alpha$  fixed

Let's assume  $\alpha = 1 \times 10^{-2}$

With a (7,4) code and  $t = 1 \Rightarrow P_{be} = 6Q^2 = 6 \times 10^{-4}$

The percent redundancy of the (7,4) code is  $3/7 \times 100\% = 43\%$

With a (31,26) code and  $t = 1 \Rightarrow P_{be} = 30Q^2 = 3 \times 10^{-3}$

The percent redundancy of the (31,26) code is  $5/31 \times 100\% = 16\%$

$$\Rightarrow P_{be}(7,4) < P_{be}(31,26)$$

Because the (31,26) code has less redundancy than the (7,4) code, we would therefore expect  $P_{be(7,4)} < P_{be(31,26)}$ .

13.2-21

Uncoded system:  $P_{ube} = 1 \times 10^{-5} = Q(\sqrt{2\gamma_b}) \Rightarrow \gamma_b = 9.2$

If we transmit at 3 times data rate  $\Rightarrow R_c = 1/3 \Rightarrow P_e = Q(\sqrt{2 \times (1/3) \times 9.2}) = 7 \times 10^{-3}$

An error occurs if 2 or 3 transmissions are received in error  $\Rightarrow P_{be} = \sum_{j=2}^3 \binom{3}{j} p^j (1-p)^{3-j}$

with  $p = 7 \times 10^{-3}$

$$\binom{3}{j} = \frac{3!}{j!(3-j)!} \Rightarrow \binom{3}{j}_{j=2} = 3 \text{ and } \binom{3}{j}_{j=3} = 1$$

$$\Rightarrow P_e = 3(7 \times 10^{-3})^2 (1 - 7 \times 10^{-3})^1 + (7 \times 10^{-3})^3 (1 - 7 \times 10^{-3})^0$$

$$= 1.46 \times 10^{-4} = P_e \text{ (triple redundancy)}$$

versus the case from problem 13.2-19 where  $P_e(7,4) = 2.2 \times 10^{-6}$  and  $P_e(15,11) = 3.2 \times 10^{-7}$

13.2-22

$$Y = [0100101] \text{ and with a (7,4) code } H = \begin{bmatrix} 1110100 \\ 0111010 \\ 1101001 \end{bmatrix} \Rightarrow S = YH^T = [0100101] \begin{bmatrix} 101 \\ 111 \\ 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix} = [010]$$

Using Table 13.2-2  $S = [010] \Rightarrow$  error in 6th bit  $\Rightarrow X = [0100111]$

$$(b) Y = [0111111] \Rightarrow S = YH^T = [0111111] \begin{bmatrix} 101 \\ 111 \\ 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix} = [101] \Rightarrow X = [1111111]$$

$$(c) Y = [1010111] \Rightarrow S = YH^T = [1010111] \begin{bmatrix} 101 \\ 111 \\ 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix} = [100] \Rightarrow X = [1010011]$$

$$(d) Y = [1101000] \Rightarrow S = YH^T = [1101000] \begin{bmatrix} 101 \\ 111 \\ 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix} = [001] \Rightarrow X = [1101001]$$

13.2-23

$$G(p) = p^{12} + p^{11} + p^3 + p^2 + p + 1$$

$$\text{ASCII character "E"} = 1000101 \Rightarrow M(p) = p^6 + p^2 + 1 \Rightarrow p^{12}M(p) = p^{18} + p^{14} + p^{12}$$

$$C(p) = \text{rem} \left[ \frac{p^q M(p)}{G(p)} \right] \Rightarrow$$

$$\begin{array}{r}
 p^{12} + p^{11} + p^3 + p^2 + p + 1 \overline{) \begin{array}{l} p^{18} + p^{14} + p^{12} \\ p^{18} + p^{17} + p^9 + p^8 + p^7 + p^6 \\ \hline p^{17} + p^{14} + p^{12} + p^9 + p^8 + p^7 + p^6 \\ p^{17} + p^{16} + \phantom{p^{12}} + p^8 + p^7 + p^6 + p^5 \\ \hline p^{16} + p^{14} + p^{12} + p^9 + \phantom{p^8} + p^5 \\ p^{16} + p^{15} + \phantom{p^{12}} + p^7 + p^6 + p^5 + p^4 \\ \hline p^{15} + p^{14} + p^{12} + p^9 + p^7 + p^6 + \phantom{p^5} + p^4 \\ p^{15} + p^{14} + \phantom{p^{12}} + p^6 + p^5 + p^4 + p^3 \\ \hline p^{12} + p^9 + p^7 + p^5 + p^3 \\ p^{12} + p^{11} + \phantom{p^9} + p^3 + p^2 + p + 1 \\ \hline p^{11} + p^9 + p^7 + p^5 + p^2 + p + 1 = C(p) \end{array}}
 \end{array}$$

$$X(p) = p^{12}M(p) + C(p) = p^{18} + p^{14} + p^{12} + p^{11} + p^9 + p^7 + p^5 + p^2 + p + 1$$

$$X = [1000101101010100111]$$

13.2-24

$$X \text{ is transmitted} \Rightarrow X \Rightarrow Y.$$

$$\text{rem} \left[ \frac{Y(p)}{G(p)} \right] = 0 \Rightarrow \text{no errors}$$

$$\text{From problem 13.2-23, errors in received vector} \Rightarrow Y = 1000101101010100100$$

$$\Rightarrow Y(p) = p^{18} + p^{14} + p^{12} + p^{11} + p^9 + p^7 + p^5 + p^2$$

13.2-24 continued

$$Y(p)/G(p) =$$

$$\begin{array}{r}
 p^{12} + p^{11} + p^3 + p^2 + p + 1 \Bigg) \overline{\begin{array}{l} p^6 + p^5 + p^4 + p^3 + 1 \\ p^{18} + p^{14} + p^{12} + p^{11} + p^7 + p^5 + p^2 \\ p^{18} + p^{17} + p^9 + p^8 + p^7 + p^6 \end{array}} \\
 \hline
 \begin{array}{l} p^{17} + p^{14} + p^{12} + p^{11} + p^8 + p^6 + p^5 + p^2 \\ p^{17} + p^{16} + \phantom{p^{14} + p^{12} + p^{11} + p^8 + p^6 + p^5 + p^2} + p^8 + p^7 + p^6 + p^5 \end{array} \\
 \hline
 \begin{array}{l} p^{16} + p^{14} + p^{12} + p^{11} + p^7 + \phantom{p^8 + p^6 + p^5 + p^2} p^2 \\ p^{16} + p^{15} + \phantom{p^{14} + p^{12} + p^{11} + p^7 + p^8 + p^6 + p^5 + p^2} p^7 + p^6 + p^5 + p^4 \end{array} \\
 \hline
 \begin{array}{l} p^{15} + p^{14} + p^{12} + p^{11} + p^6 + p^5 + p^4 + p^2 \\ p^{15} + p^{14} + \phantom{p^{12} + p^{11} + p^6 + p^5 + p^4 + p^2} p^6 + p^5 + p^4 + p^3 \end{array} \\
 \hline
 \begin{array}{l} p^{12} + p^{11} + p^3 + p^2 \\ p^{12} + p^{11} + \phantom{p^3 + p^2} p^3 + p^2 + p + 1 \end{array} \\
 \hline
 p + 1 \Rightarrow
 \end{array}$$

$\Rightarrow$  remainder  $\neq 0 \Rightarrow$  an error has occurred.

13.2-25

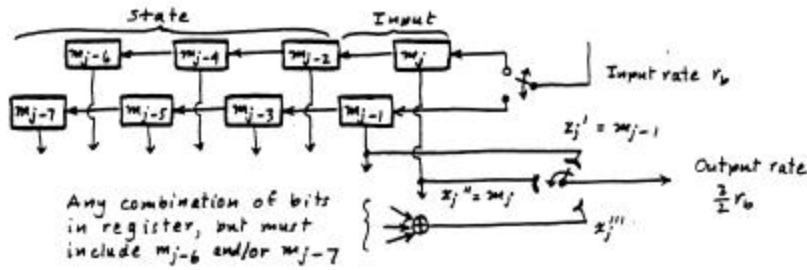
9600 bps  $\Rightarrow 1/9600 \Rightarrow 0.1$  ms/bit  $\Rightarrow 125$  ms noise burst  $\Rightarrow 125$  ms x 10 bit/ms = 1250 bits in error for each burst.

A (63,45) code can correct for 3 errors  $\Rightarrow$  distribute 1200 errors over 400 blocks  $\Rightarrow 3$  errors/block. We interleave the bits so we have 63+3=66 bits between errors,  $\Rightarrow$  each block=66 bits long

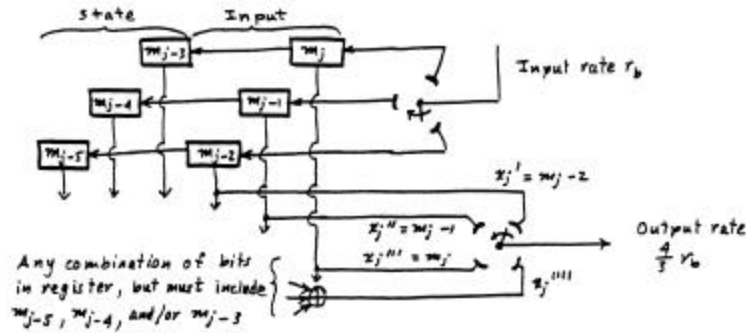
latency delay between interleaving and deinterleaving  $\Rightarrow$

$$2 \times (400 \text{ blocks} \times 66 \text{ bits/block} \times 0.1 \text{ ms/bit}) = 2 \times 2.64 \text{ seconds} = 5.28 \text{ seconds}$$

13.3-1

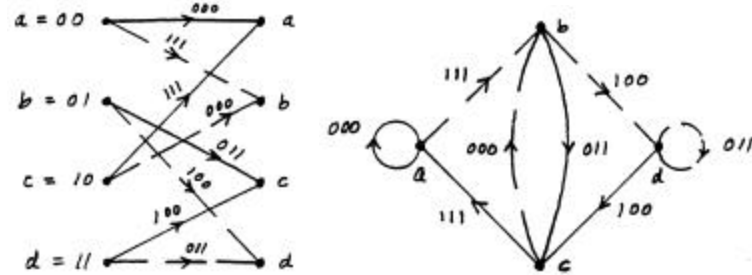


13.3-2



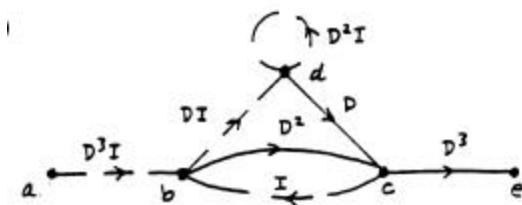
13.3-3

(a)



Input	1	0	1	1	0	0	1	1	1	1
State	a	b	c	b	d	c	a	b	d	d
Output	111	011	000	100	100	111	111	100	011	011

(b)



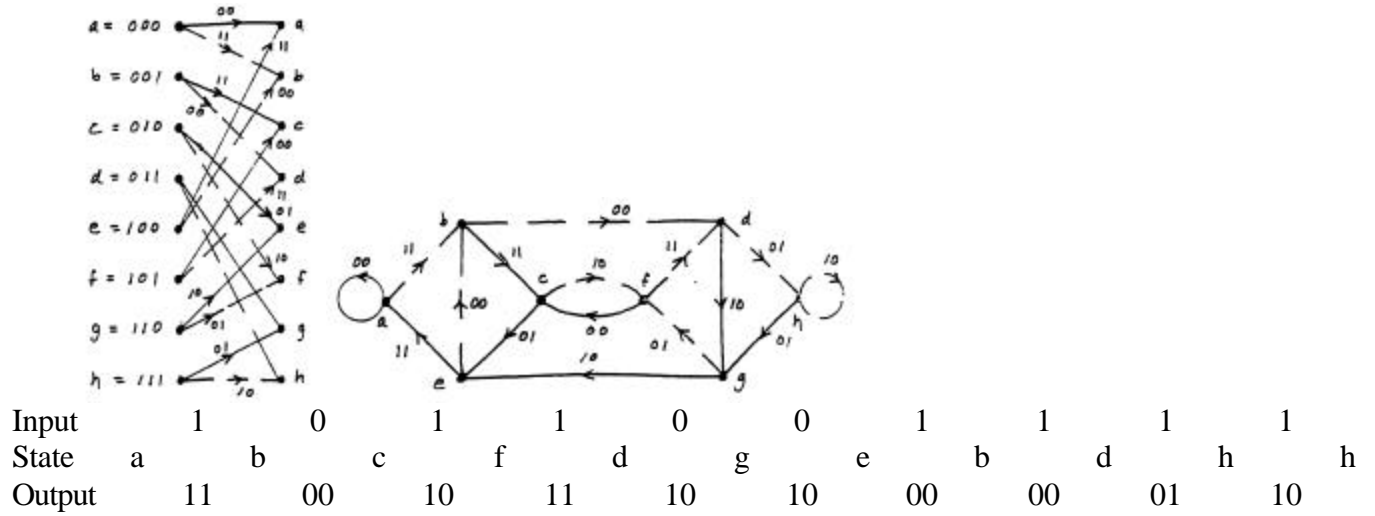
Minimum-weight paths:  $abce = D^8 I^1$  and  $abcde = D^8 I^2$

Thus,  $d_f = 8$ ,  $M(d_f) = 1 + 2 = 3$



13.3-4

(a)



(b)

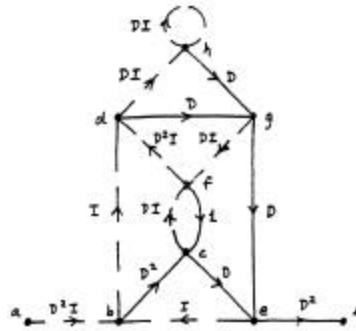
Minimum-weight paths

$$abdgei = D^6 I^2$$

$$abdce = D^8 I^2$$

Thus,

$$d_f = 6, M(d_f) = 2$$



13.3-5

$$M = 110101110111000000 \Rightarrow 1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}$$

$$G_1 = 1 + D + D^3 \text{ and } G_2 = D + D^2 \text{ and } G_3 = 1$$

$$X_j' = G_1 M = (1 + D + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11})$$

$$= 1 + D^2 + D^5 + D^6 + D^{10} + D^{13} + D^{14}$$

$$X_j' = G_2 M = (D + D^2)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11})$$

$$= D + D^3 + D^4 + D^5 + D^6 + D^8 + D^9 + D^{13}$$

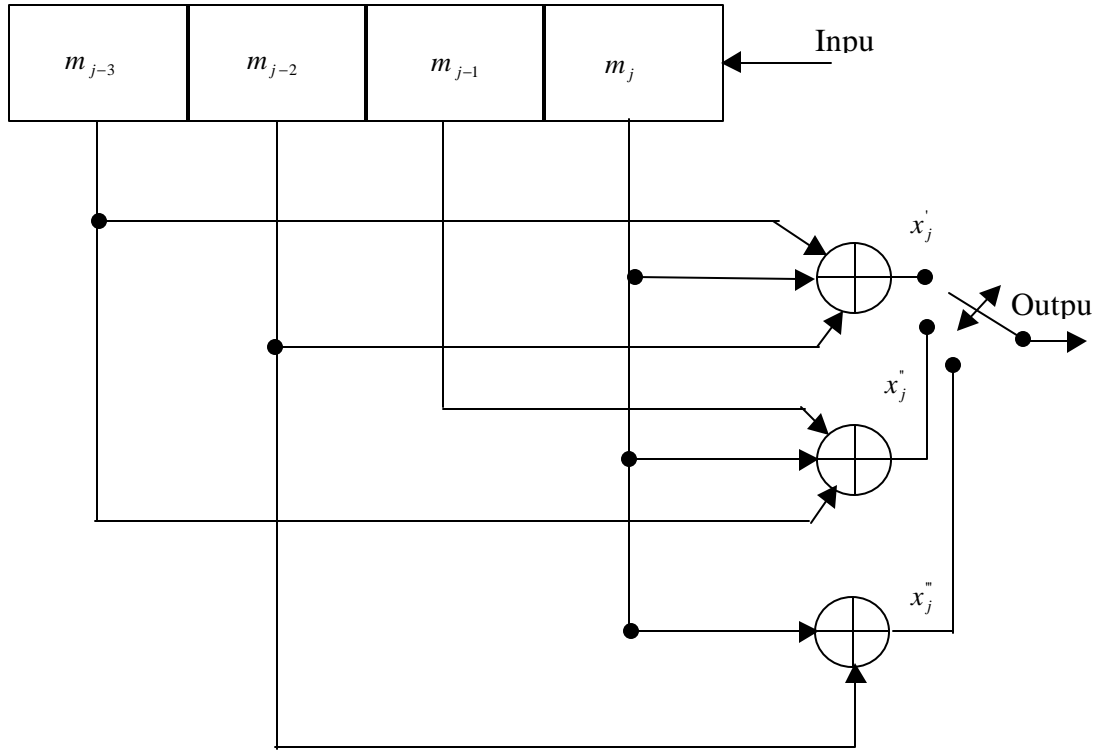
$$X_j''' = M = (1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11})$$

$$j = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16$$

$$x_j = 011 \ 101 \ 010 \ 101 \ 100 \ 111 \ 111 \ 001 \ 000 \ 101 \ 111 \ 001 \ 000 \ 110 \ 010 \ 000 \ 000$$

13.3-6

Redrawing Figure 13.3-5 to eliminate the input distributor, we have the following equivalent convolutional encoder:



With  $M = 110101110111000000 \Rightarrow 1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}$

and  $G_1 = 1 + D^2 + D^3$ ,  $G_2 = 1 + D + D^3$  and  $G_3 = 1 + D^2$

$$\begin{aligned} X'_j &= G_1 M = (1 + D^2 + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}) \\ &= 1 + D + D^2 + D^3 + D^4 + D^8 + D^9 + D^{12} + D^{14} + 0 \end{aligned}$$

Because we eliminated the input distributor  $\Rightarrow$  we will partition the output bits in groups of 2 and select the second bit for the output  $\Rightarrow$

$$\Rightarrow x'_j = \begin{matrix} & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$X''_j = G_2 M = (1 + D + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11})$$

$$= 1 + D^5 + D^6 + D^{10} + D^{12} + D^{13} + D^{14}$$

$$\Rightarrow \begin{matrix} 10 & 00 & 01 & 10 & 00 & 10 & 11 & 10 \end{matrix}$$

$$\Rightarrow x''_j = \begin{matrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{matrix}$$

### 13.3-6 continued

$$X_j''' = G_3 M = (1 + D^2)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$$

$$= 1 + D + D^2 + D^6 + D^8 + D^{10} + D^{12} + D^{13}$$

$$\Rightarrow \quad 11 \ 10 \ 00 \ 10 \ 10 \ 10 \ 11 \ 00$$

$$\Rightarrow x_j''' = \quad 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0$$

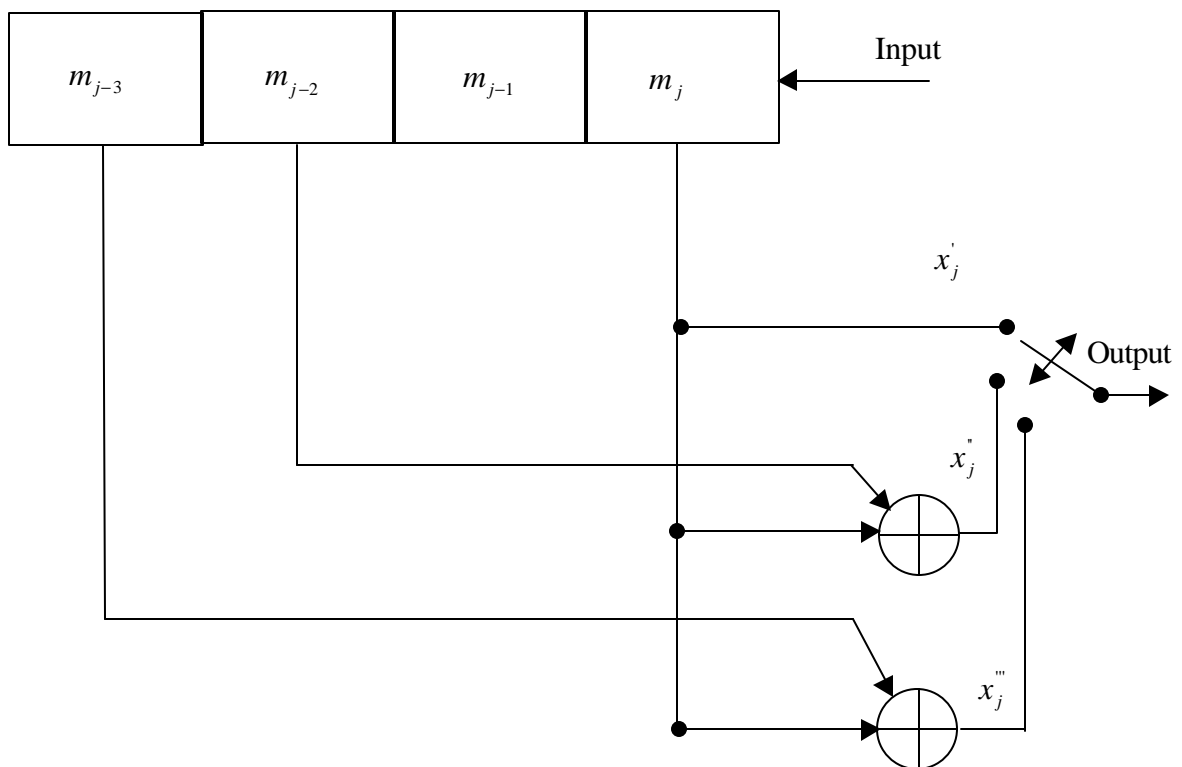
Interleaving  $x_j'$ ,  $x_j''$  and  $x_j'''$  we get

$$\text{Input message of} \quad 11 \ 01 \ 01 \ 11 \ 01 \ 11 \ 00 \ 00$$

$$\Rightarrow x_j = \quad 101 \ 100 \ 010 \ 000 \ 100 \ 000 \ 011 \ 000$$

### 13.3-7

Redrawing Figure 13.3-5 to eliminate the input distributor, we have the following equivalent convolutional encoder:



### 13.3-7 continued

With  $M = 110101110111000000 \Rightarrow 1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}$

and  $G_1 = 1$ ,  $G_2 = 1 + D^2$  and  $G_3 = D^2 + D^3$

$$\begin{aligned} X'_j &= G_1 M = (1)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}) \\ &= 1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11} + 0 + 0 \end{aligned}$$

Because we eliminated the input distributor  $\Rightarrow$  we will partition the output bits in groups of 2 and select the second bit for the output

$$\Rightarrow \quad 11 \ 01 \ 01 \ 11 \ 01 \ 11 \ 00 \ 00$$

$$\Rightarrow x'_j = \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0$$

$$\begin{aligned} X''_j &= G_2 M = (1 + D^2)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}) \\ &= 1 + D + D^2 + D^6 + D^8 + D^{10} + D^{12} + D^{13} \end{aligned}$$

$$\Rightarrow \quad 11 \ 10 \ 00 \ 10 \ 10 \ 10 \ 10 \ 11$$

$$\Rightarrow x''_j = \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1$$

$$\begin{aligned} X'''_j &= G_3 M = (D^2 + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^8 + D^{10} + D^{11}) \\ &= D^2 + D^4 + D^5 + D^6 + D^7 + D^{10} + D^{11} + D^{12} + D^{14} + 0 \end{aligned}$$

$$\Rightarrow \quad 00 \ 10 \ 11 \ 11 \ 00 \ 11 \ 10 \ 10$$

$$\Rightarrow x'''_j = \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0$$

Interleaving  $x'_j$ ,  $x''_j$  and  $x'''_j$  we get

Input message of  $11 \ 01 \ 01 \ 11 \ 01 \ 11 \ 00 \ 00$

$$\Rightarrow x_j = \quad 110 \ 100 \ 101 \ 101 \ 100 \ 101 \ 010 \ 000$$

### 13.3-8

(a) The state transition diagram does not have a transition with a nonzero input that has a zero output  $\Rightarrow$  noncatastrophic.

Alternatively, factoring  $G_2(D) = 1 + D^2 = (1 + D)(1 + D)$  and dividing

$G_1(D) = D^2 + D + 1$  by  $(D + 1)$  we get

13.3-8 continued

$$\begin{array}{r} D \\ D+1 \overline{) D^2 + D + 1} \\ \underline{D^2 + D} \phantom{+ 1} \\ 1 \end{array}$$

$\Rightarrow$  there are no common factors to  $G_1(D)$  and  $G_2(D) \Rightarrow$  noncatastrophic

(b) The state transition diagram of Fig 13.3-5 does not have a transition with a nonzero input that has a zero output  $\Rightarrow$  noncatastrophic.

Alternatively, with  $G_1(D) = D^3 + D^2 + 1$ ,  $G_2(D) = D^3 + D + 1$ , and  $G_3(D) = D^2 + 1 = (D+1)(D+1)$

Dividing  $G_1(D)$  and  $G_2(D)$  by  $D+1$  we get

$$\begin{array}{r} D^2 \\ D+1 \overline{) D^3 + D^2 + 1} \\ \underline{D^3 + D^2} \phantom{+ 1} \\ 1 \end{array} \quad \text{and} \quad \begin{array}{r} D^2 + D \\ D+1 \overline{) D^3 + D + 1} \\ \underline{D^3 + D^2} \phantom{+ 1} \\ D^2 + D + 1 \\ \underline{D^2 + D} \phantom{+ 1} \\ 1 \end{array}$$

$\Rightarrow$  there are no common factors to  $G_1(D)$ ,  $G_2(D)$  and  $G_3(D) \Rightarrow$  noncatastrophic

(c)  $G_1(D) = D + D^2 = D(D+1)$  and  $G_2(D) = D^2 + 1 = (D+1)(D+1)$

$\Rightarrow (D+1)$  is common to both  $G_1(D)$  and  $G_2(D) \Rightarrow$  catastrophic

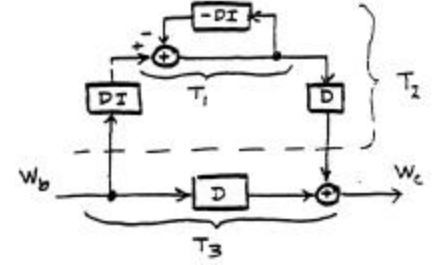
13.3-9

First, we consider  $W_b \rightarrow W_c$  section where

$$T_1 = \frac{1}{1-DI},$$

$$T_2 = DI \times T_1 \times D = \frac{D^2 I}{1-DI},$$

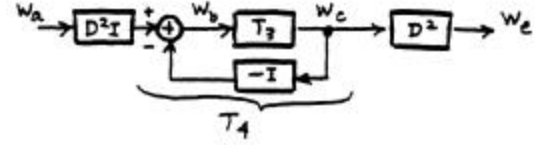
$$T_3 = D + T_2 = \frac{D}{1-DI}$$



Then,

$$T_4 = \frac{T_3}{1-IT_3} = \frac{D}{1-2DI},$$

$$\text{and } T(D,I) = D^2 I \times T_4 \times D^2 = \frac{D^5 I}{1-2DI}$$



13.3-10

$$\frac{\partial T(D,I)}{\partial I} = \sum_{d=d_f}^{\infty} \sum_{i=1}^{\infty} A(d,i) D^d I^{i-1} \Rightarrow \frac{\partial T(D,I)}{\partial I} \Big|_{I=1} = \sum_{d=d_f}^{\infty} \left[ \sum_{i=1}^{\infty} i A(d,i) \right] D^d$$

Thus,

$$P_{be} \leq \frac{1}{k} \sum_{d=d_f}^{\infty} M(d) [2\sqrt{a(1-a)}]^d \quad \text{where } M(d) = \sum_{i=1}^{\infty} i A(d,i)$$

$$\leq \frac{1}{k} \left\{ M(d_f) 2^{d_f} [a(1-a)]^{d_f/2} + \sum_{d=d_f+1}^{\infty} M(d) 2^d [a(1-a)]^{d/2} \right\}$$

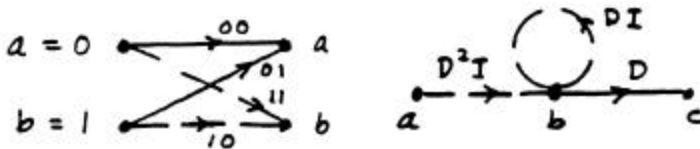
If  $\sqrt{a} \ll 1$ , then  $1-a \approx 1$  and  $M(d) 2^d [\alpha(1-\alpha)]^{d_f/2} \ll M(d_f) 2^{d_f} [\alpha(1-\alpha)]^{d_f/2}$

for  $d \geq d_f + 1$ , so

$$P_{be} \approx \frac{1}{K} M(d_f) 2^{d_f} a^{d_f/2}$$

13.3-11

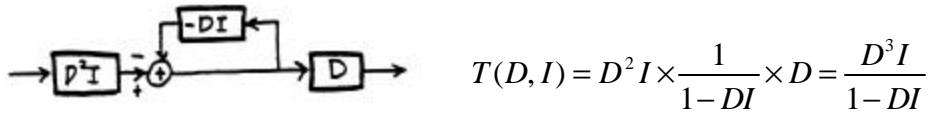
(a)



13.3-11 continued

Minimum-weight paths:  $abc = D^3 I^1$  so,  $d_f = 3$ ,  $M(d_f) = 1$

(b)



(c)

$$\frac{\partial T(D, I)}{\partial I} = \frac{D^3}{(1 - DI)^2}$$

Eq.(9) yields

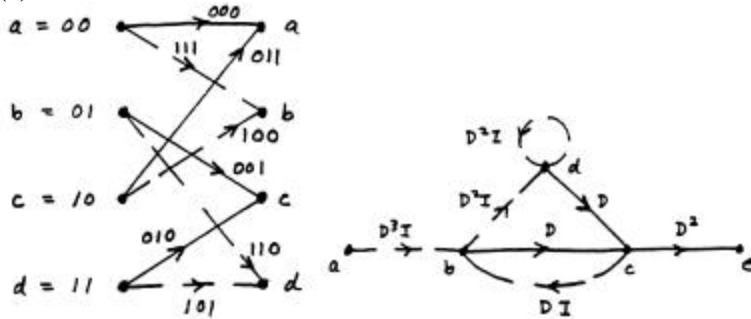
$$P_{be} \leq \frac{1}{1} \times \frac{2^3 [a(1-a)]^{3/2}}{[1 - 2\sqrt{a(1-a)}]^2} \approx 8a^{3/2}, a \ll 1$$

Eq.(10) yields

$$P_{be} \approx 2^3 a^{3/2} = 8a^{3/2}$$

13.3-12

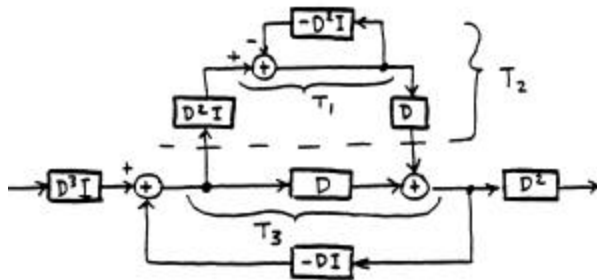
(a)



Minimum-weight paths:  $abce = D^6 I^1$  so,  $d_f = 6$ ,  $M(d_f) = 1$

13.3-12 continued

(b)



$$T_1 = \frac{1}{1 - D^2 I}$$

$$T_2 = D^2 I \times T_1 \times D = \frac{D^3 I}{1 - D^2 I}$$

$$T_3 = D + T_2 = \frac{D}{1 - D^2 I}$$

$$T(D, I) = D^3 I \times \frac{T_3}{1 - D I T_3} \times D^2 = \frac{D^6 I}{1 - 2D^2 I}$$

(c)

$$\frac{\partial T(D, I)}{\partial I} = \frac{D^6}{(1 - 2D^2 I)^2}$$

Eq.(9) yields

$$P_{be} \leq \frac{1}{1} \times \frac{4^3 a^3 (1 - a^3)}{[1 - 8a(1 - a)]^2} \approx 64a^3, a \ll 1$$

Eq.(10) yields

$$P_{be} \approx 2^6 a^{6/2} = 64a^3$$

13.3-13

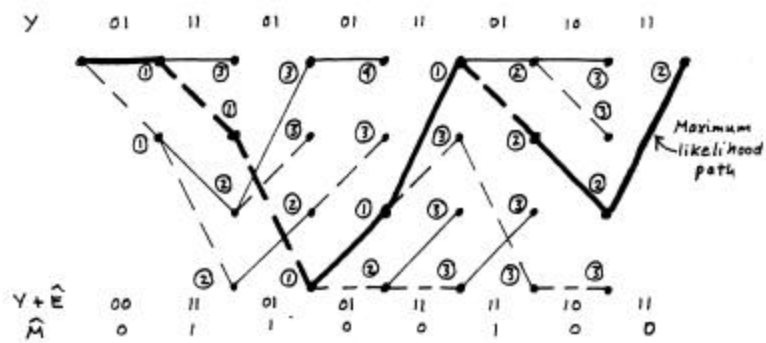




Figure 1 shows a trellis diagram for a sequence estimation problem. The vertical axis represents states (00, 01, 10, 11) and the horizontal axis represents time steps (0 to 11). Solid lines indicate the maximum likelihood path, while dashed lines show other possible paths. The final state is 00, and the path is labeled "Maximum likelihood path".

	$e'_{j-8}$	$e'_{j-7}$	$e'_{j-6}$	$e'_{j-5}$	$e'_{j-4}$	$e''_{j-4}$	$e'_{j-3}$	$e''_{j-3}$	$e'_{j-2}$	$e''_{j-2}$	$e'_{j-1}$	$e''_{j-1}$	$e'_j$	$e''_j$
$s_{j-4}$	✓													
$s_{j-3}$		✓												
$s_{j-2}$			✓											
$s_{j-1}$				✓						✓				
$s_j$					✓			✓				✓	✓	

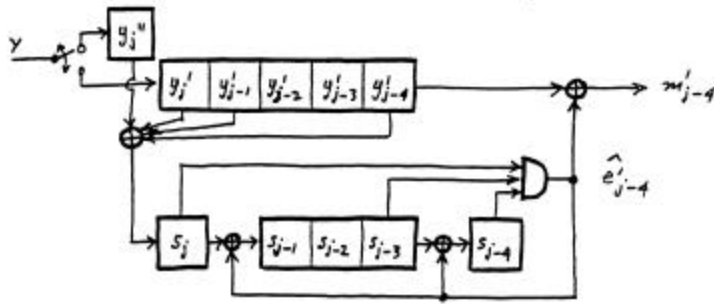
$$s_j = e'_{j-4} \oplus e'_{j-3} \oplus e'_j \oplus e''_j$$

	$e'_{j-8}$	$e'_{j-7}$	$e'_{j-6}$	$e'_{j-5}$	$e'_{j-4}$	$e''_{j-4}$	$e'_{j-3}$	$e''_{j-3}$	$e'_{j-2}$	$e''_{j-2}$	$e'_{j-1}$	$e''_{j-1}$	$e'_j$	$e''_j$
$s_{j-4}$	⊙			⊙	⊙	⊙								
$s_{j-3}$		⊙			⊙		⊙	⊙						
$s_{j-2}$			✓				✓			✓	✓			
$s_{j-1}$				✓					✓			✓		
$s_j$					⊙						⊙		⊙	⊙

$$s_j = e'_{j-4} \oplus e'_{j-1} \oplus e'_j \oplus e''_j$$

13-27

13.3-16 continued

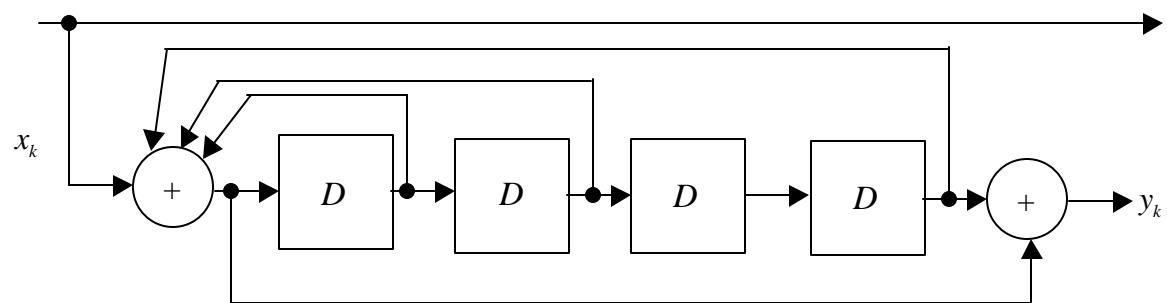


13.3-17

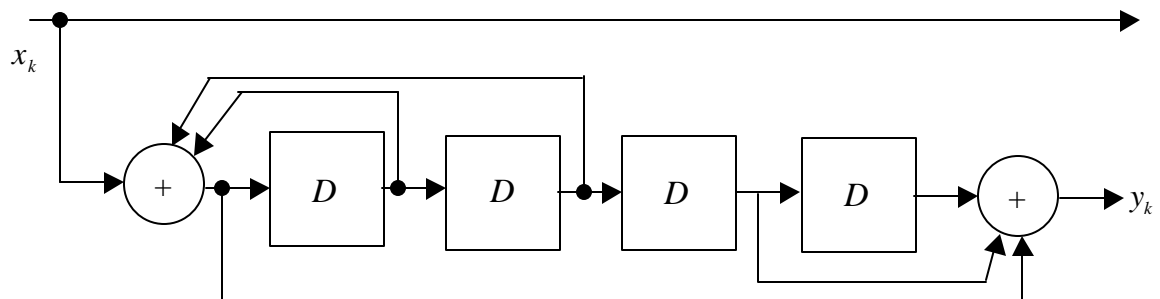
(a)  $(37,21) \Rightarrow (35_8, 21_8) \Rightarrow (11\ 101_2, 10\ 001_2)$  where the 1s and 0s define the feedback and output connections respectively.

(b)  $(34,23) \Rightarrow (34_8, 23_8) \Rightarrow (11\ 100_2, 10\ 011_2)$

The corresponding RSC block diagrams are shown as follows:



**(35,21) RSC encoder**



**(34,23) RSC encoder**

13.4-1

(a)  $p = 5, q = 11 \Rightarrow pq = 55$  and  $\phi(n) = 4 \times 10 = 40 = 2 \times 2 \times 2 \times 5 \Rightarrow$  pick  $e = 7$  (since 7 is relatively prime to 40).

$$7d = 40Q + 1 \Rightarrow \text{if } Q = 4 \Rightarrow 7d = 160 + 1 \Rightarrow d = 23$$

$$y = x^e \bmod n \text{ and } x = y^d \bmod n$$

$$x = 8 \Rightarrow y = 8^7 \bmod 55 = 2$$

$$\text{To check } \Rightarrow x = 2^{23} \bmod 55 = 8$$

$$x = 27 \Rightarrow y = 27^7 \bmod 55 = 3$$

$$\text{To check } \Rightarrow x = 3^{23} \bmod 55 = 27$$

$$x = 51 \Rightarrow y = 51^7 \bmod 55 = 6$$

$$\text{To check } \Rightarrow x = 6^{23} \bmod 55 = 51$$

(b)  $p = 11, q = 37 \Rightarrow pq = 407$  and  $\phi(n) = 10 \times 36 = 360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \Rightarrow$  pick  $e = 7$  (since 7 is relatively prime to 360)

$$7d = 360Q + 1 \Rightarrow \text{if } Q = 2 \Rightarrow 7d = 720 + 1 \Rightarrow d = 103$$

$$y = x^e \bmod n \text{ and } x = y^d \bmod n$$

$$x = 8 \Rightarrow y = 8^7 \bmod 407 = 288$$

$$\text{To check } \Rightarrow x = 288^{103} \bmod 407 = 8$$

$$x = 27 \Rightarrow y = 27^7 \bmod 407 = 212$$

$$\text{To check } \Rightarrow x = 212^{103} \bmod 407 = 27$$

$$x = 51 \Rightarrow y = 51^7 \bmod 407 = 171$$

$$\text{To check } \Rightarrow x = 171^{103} \bmod 407 = 51$$

(c)  $p = 13, q = 37 \Rightarrow pq = 481$  and  $\phi(n) = 12 \times 36 = 432 = 2^4 \times 3^3 \Rightarrow$  pick  $e = 5$  (since 5 is relatively prime to 432)

$$5d = 432Q + 1 \Rightarrow \text{if } Q = 2 \Rightarrow 5d = 864 + 1 \Rightarrow d = 173$$

$$y = x^e \bmod n \text{ and } x = y^d \bmod n$$

$$x = 8 \Rightarrow y = 8^5 \bmod 481 = 60$$

$$\text{To check } \Rightarrow x = 60^{173} \bmod 481 = 8$$

$$x = 27 \Rightarrow y = 27^5 \bmod 481 = 196$$

$$\text{To check } \Rightarrow x = 196^{173} \bmod 481 = 27$$

$$x = 51 \Rightarrow y = 51^5 \bmod 481 = 103$$

$$\text{To check } \Rightarrow x = 103^{173} \bmod 481$$

$$= \{(((103^2 \bmod 481)^6 \bmod 481)^{14} \bmod 481)[(103^5 \bmod 481) \bmod 481]\} = 51$$

13.4-2

$p = 11, q = 31 \Rightarrow n = 341$  and  $\phi(n) = 10 \times 30 = 300 = 2^2 \times 3 \times 5$

$\Rightarrow$  select  $d$  to be any prime number greater than 5

$de = Q\phi(n) + 1 \Rightarrow$  if  $e = 7$  and  $Q = 1 \Rightarrow d = 43$

There are as many values of  $e$  as there are prime numbers greater than 5.

if  $e = 7$  and  $Q = 8 \Rightarrow de = Q\phi(n) + 1 \Rightarrow d = 343$

$\Rightarrow$  pattern is such that for a given value of  $e$  and  $d$ ,

successive values of  $d$  can be found by adding  $\phi(n)$  to previous values of  $d$

It is also observed that the values of  $p$  and  $q$  must be such that the product of  $p, q$  must be greater than the number you are trying to encrypt.

## Chapter 14

14.1-1

$$\begin{aligned}
 \overline{x_c^2} &= \int_{-\infty}^{\infty} G_c(f) df = \frac{A_c^2}{4} \int_{-\infty}^{\infty} [G_{lp}(f - f_c) + G_{lp}(f + f_c)] df \\
 &= \frac{A_c^2}{4} 2 \int_{-\infty}^{\infty} G_{lp}(\lambda) d\lambda \quad \text{since } G_{lp}(f - f_c) \text{ and } G_{lp}(f + f_c) \text{ don't overlap if } f_c \gg r \\
 &= \frac{A_c^2}{2} \left[ \frac{M^2 - 1}{12r} \int_{-\infty}^{\infty} \text{sinc}^2(f/r) df + \frac{(M-1)^2}{4} \int_{-\infty}^{\infty} \delta(f) df \right] \\
 &= \frac{A_c^2}{2} \left[ \frac{M^2 - 1}{12r} r + \frac{(M-1)^2}{4} \right] = \frac{A_c^2}{12} (M-1)(2M-1) \\
 P_c &= 2 \times \frac{A_c^2}{4} \frac{(M-1)^2}{4}, \quad \frac{P_c}{\overline{x_c^2}} = \frac{3M-3}{4M-2} = \begin{cases} 1/2 & M=2 \\ 3/4 & M \geq 1 \end{cases}
 \end{aligned}$$

14.1-2

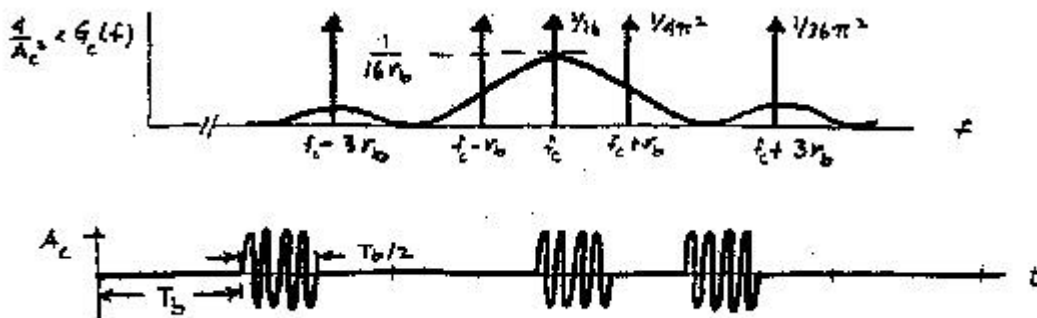
(a)  $x_i(t) = \sum_k a_k p(t)$  where  $p(t) = u(t) - u(t - T_b/2)$ , where  $x_q(t) = 0$

$a_k = 0, 1 \Rightarrow m_a = 1/2, \sigma_a^2 = 1/4$

$$|P(f)|^2 = \left( \frac{T_b}{2} \right)^2 \text{sinc}^2 \left( \frac{fT_b}{2} \right) = \frac{1}{4r_b^2} \text{sinc}^2 \left( \frac{f}{2r_b} \right)$$

$$\Rightarrow (m_a r_b)^2 |P(nr_b)|^2 = \begin{cases} 1/16 & n=0 \\ 0 & n=\pm 2, \pm 4, \dots \\ 1/(2\pi n)^2 & n \text{ odd} \end{cases}$$

$$G_{lp}(f) = G_i(f) = \frac{1}{16r_b} \text{sinc}^2 \frac{f}{2r_b} + \frac{1}{16} \delta(f) + \sum_{\pm n \text{ odd}} \frac{1}{(2\pi n)^2} \delta(f - nr_b)$$



14.1-3

$$m_a = 1/2, \quad \sigma_a^2 = 1/4, \quad r = r_b$$

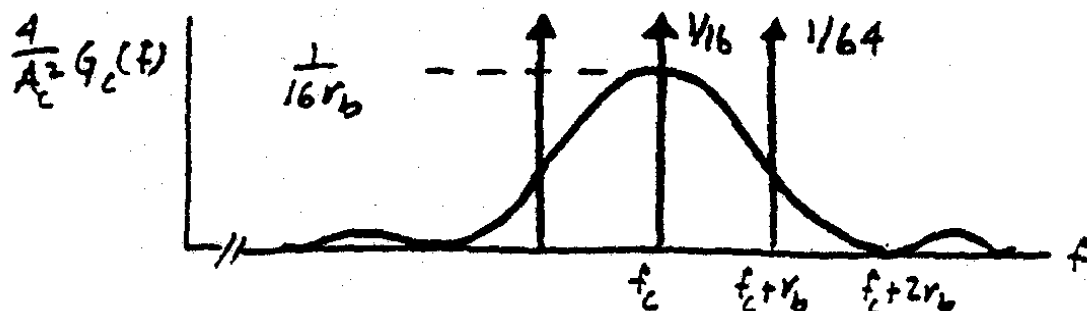
$$G_p(f) = \frac{r_b}{4} |P(f)|^2 + \frac{r_b^2}{4} \sum_n |P(nr_b)|^2 \delta(f - nr_b),$$

$$P(f) = \begin{cases} \tau & f = 0 \\ \tau/2 & f = \pm 1/2T \\ 0 & f = \pm \frac{m}{2\tau}, \quad m \geq 0 \end{cases}$$

(a)  $2\tau = T_b = 1/r_b$



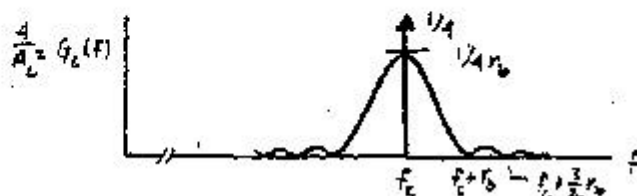
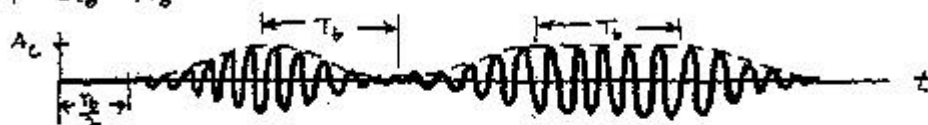
$$G_p(f) = \frac{1}{16r_b} \frac{\text{sinc}^2(f/r_b)}{|1 - (f/r_b)|^2} + \frac{1}{16} \delta(f) + \frac{1}{64} [\delta(f - r_b) + \delta(f + r_b)]$$



(b)  $2\tau = 2T_b = 2/r_b$

$$G_p(f) = \frac{1}{4r_b} \frac{\text{sinc}^2(2f/r_b)}{|1 - (2f/r_b)|^2} + \frac{1}{4} \delta(f)$$

$2\tau = 2T_b = 2/r_b$



14.1-4

(a) For  $kD < t < (k+1)D$ ,  $x_i(t) = a_{2k}$  and  $x_q(t) = a_{2k+1}$  with  $a_k = \pm 1$

Thus  $x_i^2(t) + x_q^2(t) = 2$  for all  $t$  so  $A(t) = \sqrt{2}A_c$  for all  $t$

whereas  $\phi(t) = \sum_k \arctan\left(\frac{a_{2k+1}}{a_{2k}}\right) p_D(t - kD)$

(b) For  $kD < t < (k+1)D$ ,  $x_i(t) = a_{2k} p(t - kD)$

and  $x_q(t) = a_{2k+1} p(t - kD)$ ,  $a_k = \pm 1$

Thus  $x_i^2(t) + x_q^2(t) = 2p^2(t - kD)$  so  $A(t) = \sqrt{2}A_c \sum_k |p(t - kD)|$

and  $\phi(t) = \sum_k \arctan\left[\frac{x_q(t)}{x_i(t)}\right] p_D(t - kD) = \sum_k \arctan\left(\frac{a_{2k+1}}{a_{2k}}\right) p_D(t - kD)$

14.1-5

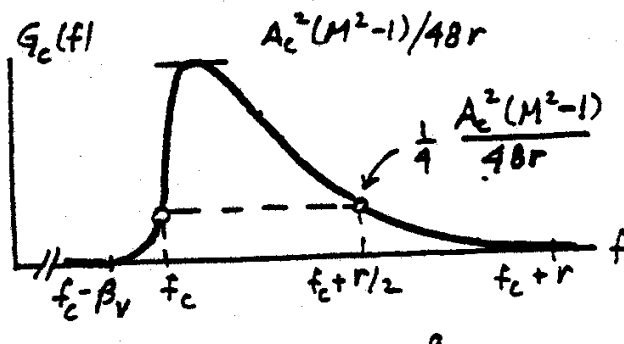
$m_a = \bar{a}_k = 0$ ,  $\sigma_a^2 = \overline{a_k^2} = (m^2 - 1)/12$  from Eq. (21), Sect 11.2, with  $A = 1$

$P(f) = \frac{1}{r} \cos^2\left(\frac{\pi f}{2r}\right) \Pi\left(\frac{f}{2r}\right)$  so  $P(f) = 0$  for  $|f| \geq r$

$G_p(f) = G_i(f) = \frac{M^2 - 1}{12r} \cos^4\left(\frac{\pi f}{2r}\right) \Pi\left(\frac{f}{2r}\right)$

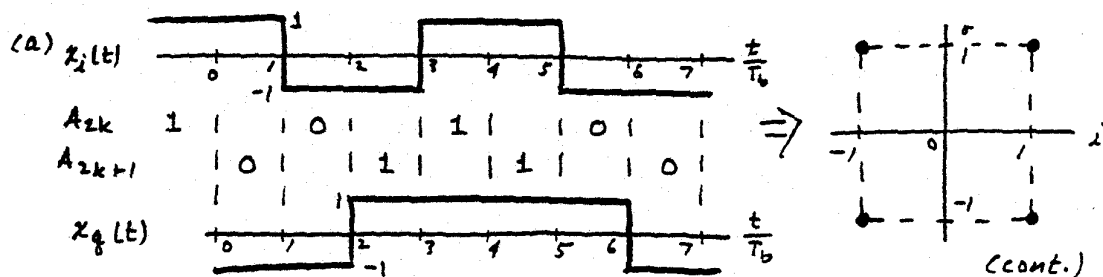
$G_c(f) = |H_{VSB}(f)|^2 \frac{A_c^2}{4} [G_p(f - f_c) + G_p(f + f_c)]$

where  $|H_{VSB}(f)| = \begin{cases} 1 & f > f_c + \beta_v \\ 1/2 & f = f_c \\ 0 & 0 < f < f_c - \beta_v \end{cases}$



14.1-6

(a)



Since  $x_q$  changes from  $\pm 1$  to  $\mp 1$  while  $x_i$  stays constant at  $\pm 1$ , and vice versa, it follows that  $\Delta\phi = \pm\pi/2$

(b)  $a_{2k}$  and  $a_{2k+1}$ , are independent sequences with  $m_a = 0$ ,  $\sigma_a^2 = 1$ ,  $r = r_b/2$ .

Time delay of  $x_q(t)$  affects only the phase of  $P(f)$ , so

$$G_q(f) = G_i(f) \text{ with } |P(f)| = |P_D(f)| \text{ where } D = 2T_b$$

Hence,  $G_{lp}(f) = 2 \times r |P_D(f)|^2 = \frac{4}{r_b} \text{sinc}^2(2f/r_b)$ , same as QPSK.

14.1-7

$A_k$	$B_k$	$C_k$	$I_k$	$Q_k$
0	0	0	$\alpha$	$\beta$
0	0	1	$\beta$	$\alpha$
0	1	0	$-\beta$	$\alpha$
0	1	1	$-\alpha$	$\beta$
1	0	0	$\alpha$	$-\beta$
1	0	1	$\beta$	$-\alpha$
1	1	0	$-\beta$	$-\alpha$
1	1	1	$-\alpha$	$-\beta$



14.1-7 continued

Let  $I_{ko} = (1 - C_k)\alpha + C_k\beta$  and  $Q_{ko} = C_k\alpha + (1 - C_k)\beta$   
and construct reduced table

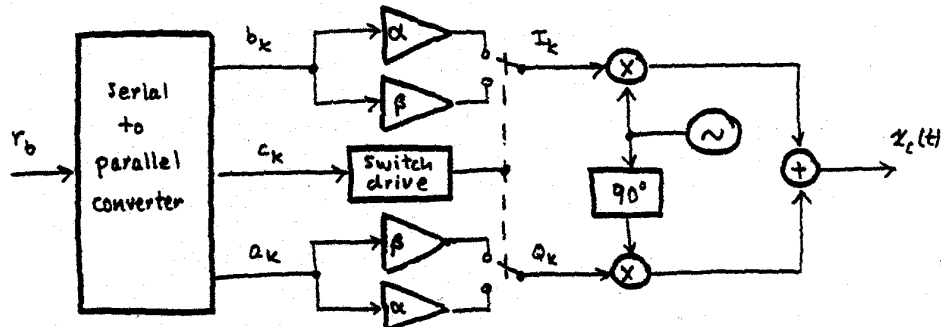
$A_k$	$B_k$	$I_k$	$Q_k$
0	0	$I_{k0}$	$Q_{k0}$
0	1	$-I_{k0}$	$Q_{k0}$
1	0	$I_{k0}$	$-Q_{k0}$
1	1	$-I_{k0}$	$-Q_{k0}$

Thus,  $I_k = (1 - 2B_k)I_{k0}$  and  $Q_k = (1 - 2A_k)Q_{k0}$

where  $(1 - 2A_k) = a_k$ ,  $(1 - 2B_k) = b_k$ , and  $C_k = (1 - c_k)/2$ , so

$$I_k = b_k \left[ \frac{1+c_k}{2}\alpha + \frac{1-c_k}{2}\beta \right] = \begin{cases} b_k\alpha & c_k = +1 \\ b_k\beta & c_k = -1 \end{cases}$$

$$Q_k = a_k \left[ \frac{1-c_k}{2}\alpha + \frac{1+c_k}{2}\beta \right] = \begin{cases} a_k\beta & c_k = +1 \\ a_k\alpha & c_k = -1 \end{cases}$$



14.1-8

$x_c(t) = A_c [x_1(t) + x_0(t)]$  where, with  $a_k = 0,1$

$$x_1(t) = \left[ \sum_k a_k p_{T_b}(t - kT_b) \right] \cos(\omega_1 t + \theta_1),$$

$$x_0(t) = \left[ \sum_k (1 - a_k) p_{T_b}(t - kT_b) \right] \cos(\omega_0 t + \theta_0)$$

14.1-8 continued

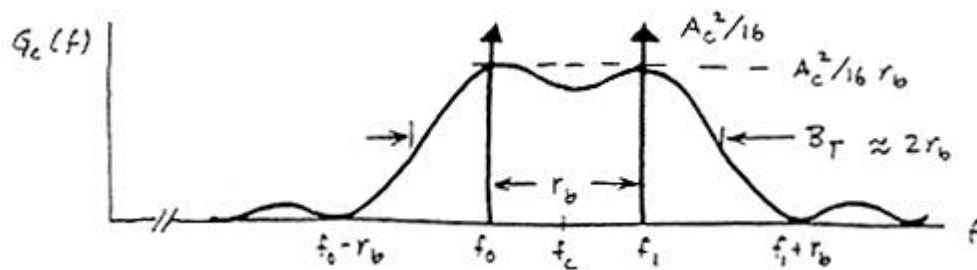
Then, from Eq. (7) with  $M = 2$  and  $r = r_b$ ,

$$G_{lp}(f) = G_{olp}(f) = \frac{1}{4r_b} \text{sinc}^2 \frac{f}{r_b} + \frac{1}{4} \delta(f)$$

$$\text{and } G_c(f) = \frac{A_c^2}{4} [G_{lp}(f - f_1) + G_{olp}(f - f_0) + G_{lp}(f + f_1) + G_{olp}(f + f_0)]$$

Since  $f_c \ll r_b$ , for  $f > 0$  we have

$$G_c(f) = \frac{A_c^2}{16} \left[ \frac{1}{r_b} \text{sinc}^2 \frac{f - f_1}{r_b} + \frac{1}{r_b} \text{sinc}^2 \frac{f - f_0}{r_b} + \delta(f - f_1) + \delta(f - f_0) \right]$$



14.1-9

$$p(t) = \cos\left(2\pi \frac{r_b}{2} t - \frac{\pi}{2}\right) \Pi\left(\frac{t - T_b}{T_b}\right) \text{ so modulation theorem yields}$$

$$P(f) = \frac{T_b}{2} \left[ \text{sinc}\left(f - \frac{r_b}{2}\right) T_b e^{-j\pi(f - r_b/2)T_b} e^{-j\pi/2} + \text{sinc}\left(f + \frac{r_b}{2}\right) T_b e^{-j\pi(f + r_b/2)T_b} e^{+j\pi/2} \right]$$

$$= \frac{1}{2r_b} \left[ \text{sinc}\left(\frac{(f - (r_b/2))}{r_b}\right) + \text{sinc}\left(\frac{(f + (r_b/2))}{r_b}\right) \right] e^{-j\pi f T_b}$$

$$\text{Thus, } |P(f)|^2 = \frac{1}{4r_b^2} \left[ \text{sinc}\left(\frac{(f - (r_b/2))}{r_b}\right) + \text{sinc}\left(\frac{(f + (r_b/2))}{r_b}\right) \right]^2$$

$$\text{But } \text{sinc}\left(\frac{f}{r_b} \pm \frac{1}{2}\right) = \frac{1}{\pi\left(\frac{f}{r_b} \pm \frac{1}{2}\right)} \sin\left(\frac{\pi f}{r_b} \pm \frac{\pi}{2}\right) = \pm \frac{2}{\pi} \frac{\cos(\pi f / r_b)}{(2f / r_b)^2 \pm 1}$$

$$\text{so } |P(f)|^2 = \frac{1}{4r_b^2} \left(\frac{2}{\pi}\right)^2 \left[ \frac{-\cos(\pi f / r_b)}{(2f / r_b) - 1} + \frac{\cos(\pi f / r_b)}{(2f / r_b) + 1} \right]^2 = \frac{4}{\pi^2 r_b^2} \left[ \frac{\cos(\pi f / r_b)}{(2f / r_b)^2 - 1} \right]^2$$

14.1-10

$$x_c(t) = A_c \sum_k [\cos(\omega_d a_k t) \cos(\omega_c t + \theta) - \sin(\omega_d a_k t) \sin(\omega_c t + \theta)] p_{T_b}(t - kT_b)$$

$$\text{with } a_k = \pm 1 \text{ and } \omega_d = \pi N / T_b = \pi N r_b$$

$$x_i(t) = \sum_k \cos(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k \cos \omega_d t p_{T_b}(t - kT_b)$$

$$= \cos \omega_d t \sum_k p_{T_b}(t - kT_b) = \cos \omega_d t = \cos 2\pi \frac{Nr_b}{2} t \text{ for all } t$$

$$\text{Thus, } G_i(f) = \frac{1}{4} [\delta(f - Nr_b/2) + \delta(f + Nr_b/2)]$$

$$x_q(t) = \sum_k \sin(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k a_k \sin(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k a_k \sin \omega_d t p_{T_b}(t - kT_b)$$

$$\begin{aligned} \text{where } \sin \omega_d a_k t &= \sin \left[ \frac{\pi N}{T_b} (t - kT_b) + \pi N k \right] \\ &= \cos \pi N k \sin \left[ \frac{\pi N}{T_b} (t - kT_b) \right] = (-1)^{Nk} \sin [\pi N r_b (t - kT_b)] \end{aligned}$$

$$\text{so } x_q(t) = \sum_k Q_k p(t - kT_b) \text{ with } Q_k = (-1)^{Nk} \text{ and}$$

$$p(t) = \sin \left( 2\pi \frac{Nr_b}{2} t \right) [u(t) - u(t - T_b)]$$

$$\overline{Q_k} = (-1)^{Nk} \overline{a_k} = 0 \text{ and } \overline{Q_k^2} = \overline{a_k^2} = 1 \text{ so } G_q(f) = r_b |P(f)|^2$$

$$\begin{aligned} P(f) &= \mathfrak{F}[p(t)] = \mathfrak{F} \left[ \cos \left( 2\pi \frac{Nr_b}{2} t - \frac{\pi}{2} \right) \Pi \left( \frac{t - T_b/2}{T_b} \right) \right] \\ &= \frac{T_b}{2} \left[ \text{sinc} \left( f - \frac{Nr_b}{2} \right) T_b e^{-j\pi(f - Nr_b/2)T_b} e^{-j\pi/2} + \text{sinc} \left( f + \frac{Nr_b}{2} \right) T_b e^{-j\pi(f + Nr_b/2)T_b} e^{+j\pi/2} \right] \end{aligned}$$

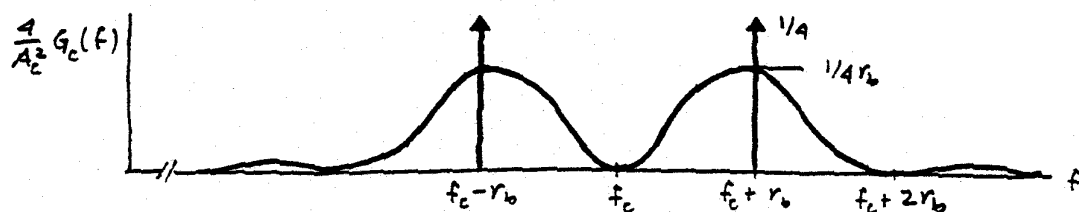
$$\text{But } e^{-j\pi(f \mp Nr_b/2)T_b} e^{\mp j\pi/2} = e^{-j\pi f T_b} e^{\pm j(N-1)\pi/2} = (\pm j)^{N-1} e^{-j\pi f T_b}$$

$$\text{Thus } G_q(f) \frac{1}{4r_b} \left| j^{N-1} \text{sinc} \left( \frac{f - Nr_b/2}{r_b} \right) + (-j)^{N-1} \text{sinc} \left( \frac{f + Nr_b/2}{r_b} \right) \right|^2$$

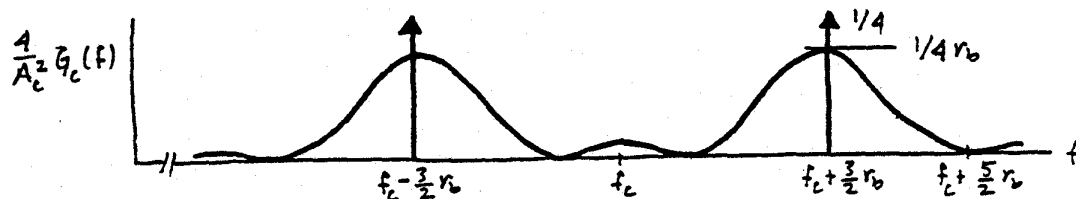
14.1-11

$$N = 2: G_i(f) = \frac{1}{4} [\delta(f - r_b) + \delta(f + r_b)], \quad G_q(f) = \frac{1}{4r_b} \left[ \text{sinc} \frac{f - r_b}{r_b} - \text{sinc} \frac{f + r_b}{r_b} \right]^2$$

14.1-11 continued

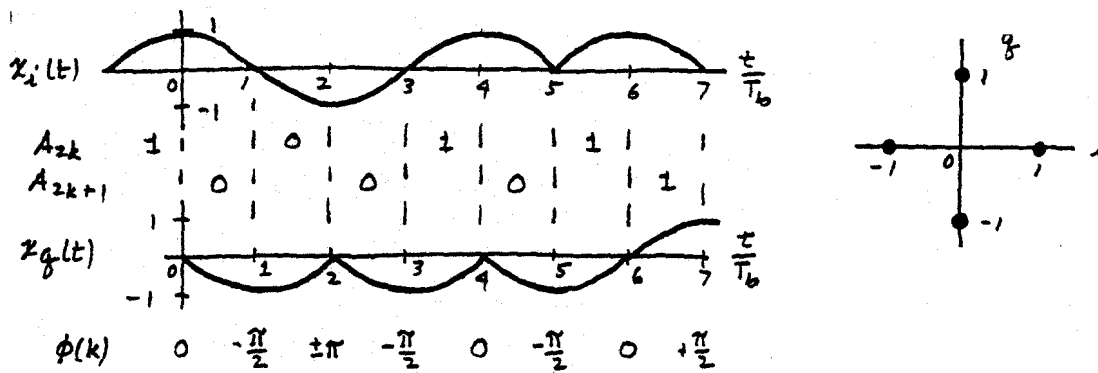


$$N = 3: G_i(f) = \frac{1}{4} \left[ \delta \left( f - \frac{3}{2} r_b \right) + \delta \left( f + \frac{3}{2} r_b \right) \right], \quad G_q(f) = \frac{1}{4 r_b} \left[ \text{sinc} \frac{f - 3 r_b / 2}{r_b} + \text{sinc} \frac{f + 3 r_b / 2}{r_b} \right]^2$$



14.1-12

(a)



(b) For  $2kT_b < t < (2k+1)T_b$ ,  $x_i(t) = a_{2k} \cos \left[ \frac{\pi}{2T_b} (t - 2kT_b) \right]$  and

$$x_q(t) = a_{2k+1} \cos \left[ \frac{\pi}{2T_b} (t - 2kT_b) - \frac{\pi}{2} \right] = a_{2k+1} \sin \left[ \frac{\pi}{2T_b} (t - 2kT_b) \right]$$

$$\text{so } x_i^2(t) + x_q^2(t) = a_{2k}^2 \cos^2 \left[ \frac{\pi}{2T_b} (t - 2kT_b) \right] + a_{2k+1}^2 \sin^2 \left[ \frac{\pi}{2T_b} (t - 2kT_b) \right] = 1 \quad \text{since } a_k^2 = 1$$

Thus,  $A(t) = A_c$  for all  $t$

#### 14.1-12 continued

(c) Let  $x_i(t) = \sum_{k \text{ even}} I_k p'(t - kT_b)$  and  $x_q(t) = \sum_{k \text{ odd}} Q_k p'(t - kT_b)$

where  $I_k = a_k$  for  $k$  even and  $Q_k = a_k$  for  $k$  odd so

$$\overline{I_k^2} = \overline{Q_k^2} = 1 \text{ and } \overline{I_k} = \overline{Q_k} = 0$$

$$p'(t - kT_b) = \cos\left[\frac{\pi}{2T_b}(t - 2kT_b)\right] \Pi\left(\frac{t - kT_b - T_b}{2T_b}\right)$$

$$p'(t) = \cos(\pi r_b t / 2) [u(t + T_b) - u(t - T_b)]$$

These expressions are identical to MSK, so  $G_{lp}(f)$  is as in Eq. (22).

#### 14.1-13

Consider  $(k-1)T_b < t < (k+1)T_b$  with  $k$  odd, so

$$x_q(t) = \sin(\phi_{k-1} + a_{k-1}c_{k-1})p_{T_b}[t - (k-1)T_b] + \sin(\phi_k + a_k c_k)p_{T_b}[t - kT_b]$$

since  $\cos \phi_k = 0$ ,  $\sin(\phi_k + a_k c_k) = \cos(a_k c_k) \sin \phi_k = \cos c_k \sin \phi_k$

also  $\sin \phi_{k-1} = 0$ ,  $\phi_{k-1} = \phi_k - a_{k-1}\pi/2$ , and  $c_{k-1} = c_k + \pi/2$  so

$$\begin{aligned} \sin(\phi_{k-1} + a_{k-1}c_{k-1}) &= \sin\left[a_{k-1}\left(c_k + \frac{\pi}{2}\right)\right] \cos\left(\phi_k - a_{k-1}\frac{\pi}{2}\right) \\ &= a_{k-1} \sin\left(c_k + \frac{\pi}{2}\right) \sin \phi_k \sin a_{k-1}\pi/2 = (a_{k-1} \cos c_k)(a_{k-1} \sin \phi_k) \\ &= \cos c_k \sin \phi_k \end{aligned}$$

Thus  $x_q(t) = \cos c_k \sin \phi_k$  for  $(k-1)T_b < t < (k+1)T_b$  with  $k$  odd,

and  $x_q(t) = \sum_{k \text{ odd}} Q_k p(t - kT_b)$  where  $Q_k = \sin \phi_k$  and

$$p(t - kT_b) = \cos\left[\frac{\pi r_b}{2}(t - kT_b)\right] \{u[t - (k-1)T_b] - u[t - (k+1)T_b]\}$$

$$\text{so } p(t) = \cos(\pi r_b t / 2) [u(t + T_b) - u(t - T_b)]$$

#### 14.1-14

$$G_{lp}(f) = \frac{1}{r} \text{sinc}^2(f/r) \text{ and } r = r_b \Rightarrow G_{lp \max} = G_{lp}(f=0)$$

Because we are at baseband  $\Rightarrow$  use  $f = B_T/2 \Rightarrow f = 1500$  and  $G_{lp}(f=0) = 1/r_b$

$$10 \log\left(\frac{G_{lp}(f=1500)}{G_{lp}(f=0)}\right) \leq -30 \text{ dB} \Rightarrow \log\left(\frac{G_{lp}(f=1500)}{G_{lp}(f=0)}\right) = \log(\text{sinc}^2(1500/r_b)) \leq -3$$

$$\Rightarrow \text{sinc}^2(1500/r_b) \leq 0.001 \Rightarrow \text{from sinc tables} \Rightarrow 3.9 = 1500/r_b \Rightarrow r_b < 385 \text{ bps.}$$

14.1-15

(a) Assume Sunde's FSK  $\Rightarrow f_d = r_b / 2$ ; and  $f = B_T / 2 = 1500$ .

Using Eq. 18 and neglecting impulses we have

$$G_p(f=0) = \frac{4}{\pi^2 r_b^2} = G_{lp_{\max}}$$

$$\Rightarrow 10 \log \left( \frac{G_{lp}(f=1500)}{G_{lp}(f=0)} \right) \leq -30 \text{ dB} \Rightarrow \log \left( \frac{\cos(1500\pi / r_b)}{(2 \times 1500 / r_b)^2 - 1} \right)^2 \leq -3$$

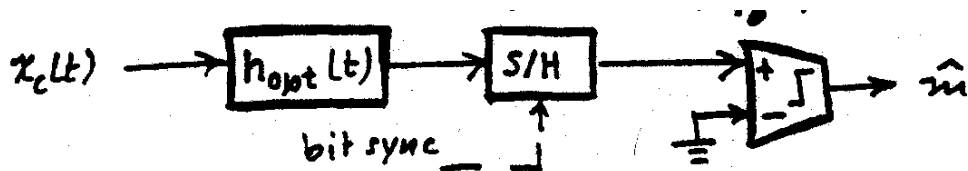
We get  $r_b \leq 656 \Rightarrow$  since  $f_d = r_b / 2 \Rightarrow f_d = 328 \text{ Hz}$ .

(b) Using Eqs. (21) and (22) in a similar way as (a), we get  $r_b = 1312 \text{ bps}$  and  $f_d = 328 \text{ Hz}$ .

14.2-1

Since  $-s_0(t) = s_1(t)$ ,  $V_{opt} = 0$  and  $h_{opt}(t) = 2KA_c p_{T_b}(T_b - t) \cos w_c(T_b - t)$

$$= 2KA_c \cos(w_c t - 2pN_c) p_{T_b}(t) = 2KA_c \cos(w_c t) \quad 0 < t < T_b$$

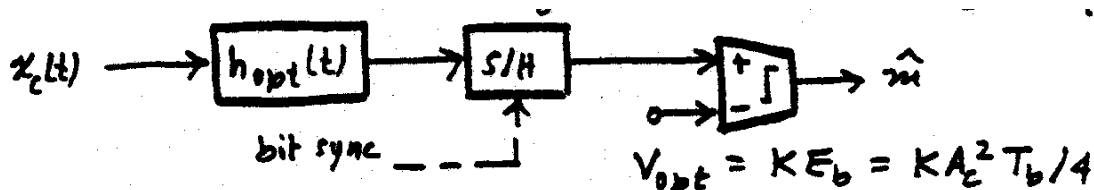


14.2-2

$$(a) h_{opt}(t) = KA_c \sin^2 \left[ \frac{p}{T_b}(T_b - t) \right] p_{T_b}(T_b - t) \cos w_c(T_b - t)$$

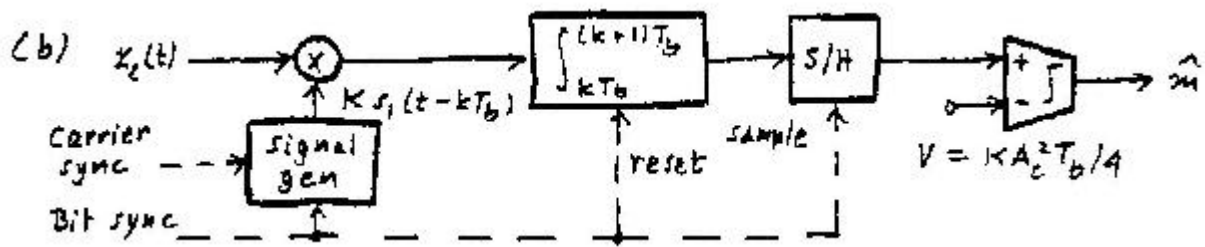
$$= KA_c \sin^2 \left( p - \frac{pt}{T_b} \right) p_{T_b}(t) \cos(w_c t - 2pN_c)$$

$$= KA_c \sin^2 \left( \frac{pt}{T_b} \right) \cos w_c t \quad 0 < t < T_b$$



14.2-2 continued

(b)



14.2-3

$$E_1 = \int_0^{T_b} A_c^2 \cos^2 2p(f_c + f_d)t dt = \frac{A_c^2 T_b}{2} \int_0^1 [1 + \cos 4p(f_c + f_d)t] dt$$

$$= \frac{A_c^2 T_b}{2} + \frac{A_c^2}{2} \frac{\sin 4p(f_c + f_d)T_b}{4p(f_c + f_d)} = \frac{A_c^2 T_b}{2} [1 + \text{sinc} 4(N_c + f_d T_b)]$$

$$\text{similarly, } E_0 = \int_0^{T_b} A_c^2 \cos^2 2p(f_c - f_d)t dt = \frac{A_c^2 T_b}{2} [1 + \text{sinc} 4(N_c - f_d T_b)]$$

$$\text{and } E_b = \frac{1}{2}(E_0 + E_1) = \frac{A_c^2 T_b}{2} [2 + \text{sinc} 4(N_c + f_d T_b) + \text{sinc} 4(N_c - f_d T_b)]$$

If  $N_c - f_d T_b \gg 1$  then  $N_c + f_d T_b \gg 1$  so  $|\text{sinc} 4(N_c \pm f_d T_b)| \ll 1$

$$\text{and } E_b \approx \frac{A_c^2 T_b}{2}$$

14.2-4

$$-\frac{E_{10}}{E_b} = -\text{sinc}(4f_d / r_b)$$

$$-\text{sinc} I |_{\max} \approx .216 \text{ at } I \approx 1.4$$

$$\text{so take } 4f_d / r_b \approx 0.216 \Rightarrow f_d \approx .35r_b$$

$$\text{Then } E_b - E_{10} = 1.216E_b \text{ and } P_e = Q(\sqrt{1.216g_b})$$

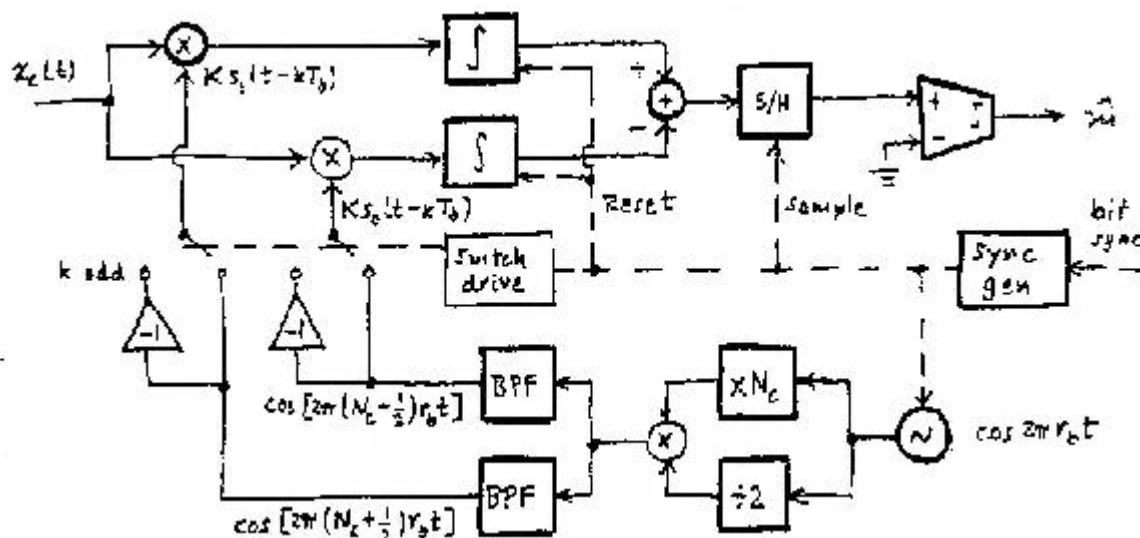
14.2-5

$$V_{opt} = 0$$

$$\text{Take } K = 1/A_c \text{ so}$$

14.2-5 continued

$$\begin{aligned}
 K_{S_{1,0}}(t - kT_b) &= \cos \left[ \frac{2p}{T_b} \left( N_c \pm \frac{1}{2} \right) (t - kT_b) \right] \quad kT_b < t < (k+1)T_b \\
 &= \cos[2p(N_c \pm 1/2)r_b t \mp pk] \\
 &= \begin{cases} \cos 2p(N_c r_b \pm r_b/2)t & k \text{ even} \\ -\cos 2p(N_c r_b \pm r_b/2)t & k \text{ odd} \end{cases}
 \end{aligned}$$

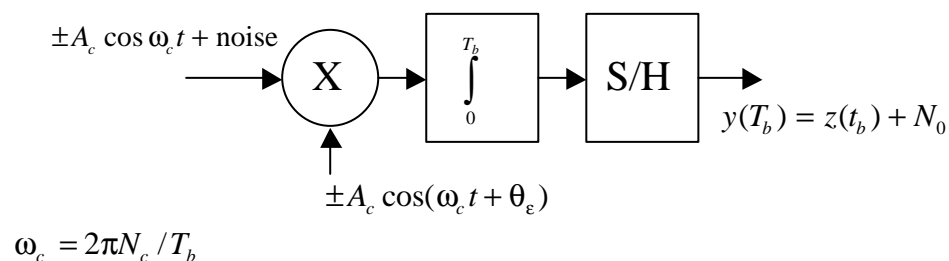


14.2-6

$$P_e = Q(\sqrt{2r_b}) = 10^{-5} \Rightarrow \sqrt{2r_b} = 4.27$$

$$Q(4.27 \cos q_e) < 10^{-4} \Rightarrow q_e < \arccos \frac{3.74}{4.27} \approx 29^\circ$$

14.2-7





14.2-7 continued

$$\begin{aligned}
 Z(t_b) &= \int_0^{T_b} K A_c \cos(\mathbf{w}_c t + \mathbf{q}_e) (\pm A_c \cos(\mathbf{w}_c t)) dt \\
 &= \pm K \frac{A_c^2}{2} \int_0^{T_b} [\cos(\mathbf{q}_e) + \cos(2\mathbf{w}_c t + \mathbf{q}_e)] dt \\
 &= \pm K \frac{A_c^2}{2} T_b \left\{ \cos \mathbf{q}_e + \frac{1}{2\mathbf{w}_c T_b} [\sin(4\mathbf{p} N_c + \mathbf{q}_e) - \sin \mathbf{q}_e] \right\} \\
 &= \pm K E_b \cos \mathbf{q}_e
 \end{aligned}$$

14.2-8

$$\begin{aligned}
 s(\mathbf{I}) &= A_c \cos \mathbf{w}_c \mathbf{I} \quad 0 < \mathbf{I} < T_b \\
 h(t - \mathbf{I}) &= K A_c \cos \mathbf{w}_c (t - \mathbf{I}) \quad t - T_b < \mathbf{I} < t \\
 \text{for } T_b < t < 0, \quad Z(t) &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{for } 0 < t < T_b \quad Z(t) &= K A_c^2 \int_0^t \cos \mathbf{w}_c \mathbf{I} \cos \mathbf{w}_c (t - \mathbf{I}) d\mathbf{I} \\
 &= \frac{K A_c^2}{2} \left[ t \cos \mathbf{w}_c t + \frac{2 \sin \mathbf{w}_c t}{2\mathbf{w}_c} \right] \\
 &= K E \frac{t}{T_b} [\cos \mathbf{w}_c t + \text{sinc}(2N_c t / T_b)]
 \end{aligned}$$

$$\begin{aligned}
 \text{for } T_b < t < 2T_b, \quad Z(t) &= K A_c^2 \int_{t-T_b}^t \cos \mathbf{w}_c \mathbf{I} \cos \mathbf{w}_c (t - \mathbf{I}) d\mathbf{I} \\
 &= \frac{K A_c^2}{2} \left[ (2T_b - t) \cos \mathbf{w}_c t + \frac{\sin \mathbf{w}_c (2T_b - t) - \sin \mathbf{w}_c (t - T_b)}{2\mathbf{w}_c} \right] \\
 &= K E \left[ \left( 2 - \frac{t}{T_b} \right) \cos \mathbf{w}_c t - \frac{2 \sin \mathbf{w}_c t}{2\mathbf{w}_c T_b} \right] \\
 &= K E \left\{ 2 \cos \mathbf{w}_c t - \frac{t}{T_b} \left[ \cos \mathbf{w}_c t + \text{sinc} \left( \frac{2N_c t}{T_b} \right) \right] \right\}
 \end{aligned}$$

14.2-8 continued

If  $N_c \gg 1$ , then  $|\text{sinc}(2N_c t / T_b)| \ll 1$  for  $t \neq 0$ , so

$$z(t) \approx \begin{cases} KE \frac{t}{T_b} \cos \mathbf{w}_c t & 0 < t < T_b \\ KE \left( 2 - \frac{t}{T_b} \right) \cos \mathbf{w}_c t & T_b < t < 2 T_b \end{cases}$$

$$\approx KE \Lambda \left( \frac{t - T_b}{T_b} \right) \cos \mathbf{w}_c t$$

14.2-9

$$E_1 = A_c^2 (1 + \mathbf{a})^2 \int_0^{T_b} \cos^2(\mathbf{w}_c t) dt = (1 + \mathbf{a})^2 A_c^2 T_b / 2 \quad \text{since } \mathbf{w}_c T_b = 2\mathbf{p} N_c$$

$$E_0 = A_c^2 (1 - \mathbf{a})^2 \int_0^{T_b} \cos^2(\mathbf{w}_c t) dt = (1 - \mathbf{a})^2 A_c^2 T_b / 2$$

$$E_{10} = -A_c^2 (1 - \mathbf{a})(1 + \mathbf{a}) \int_0^{T_b} \cos^2(\mathbf{w}_c t) dt = -(1 - \mathbf{a}^2) A_c^2 T_b / 2$$

$$E_b = \frac{(1 + \mathbf{a})^2 + (1 - \mathbf{a})^2}{2} \frac{A_c^2 T_b}{2} = (1 + \mathbf{a}^2) A_c^2 T_b / 2$$

$$E_b - E_{10} = \left[ (1 + \mathbf{a}^2) + (1 - \mathbf{a}^2) \right] A_c^2 T_b / 2 = 2E_b / (1 + \mathbf{a}^2)$$

$$\Rightarrow P_e = Q \left[ \sqrt{2\mathbf{g}_b / (1 + \mathbf{a}^2)} \right]$$

14.2-10

$\cos(\mathbf{w}_c t - \mathbf{p} / 2) = \sin \mathbf{w}_c t$  and  $\mathbf{w}_c T_b = 2\mathbf{p} N_c$  so

$$E_1 = A_c^2 \int_0^{T_b} (\cos^2 \mathbf{w}_c t + 2\mathbf{a} \sin \mathbf{w}_c t \cos \mathbf{w}_c t + \mathbf{a}^2 \sin^2 \mathbf{w}_c t) dt$$

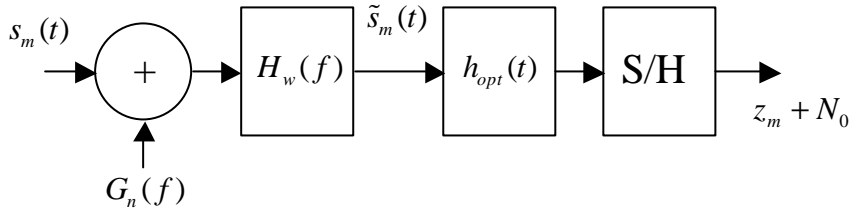
$$= (1 + \mathbf{a}^2) A_c^2 T_b / 2$$

$$\text{Similarly, } E_0 = E_1 \text{ and } E_b = \frac{1}{2} (E_0 + E_1) = (1 + \mathbf{a}^2) A_c^2 T_b / 2$$

$$E_{10} = -A_c^2 \int_0^{T_b} (\cos^2 \mathbf{w}_c t - \mathbf{a}^2 \sin^2 \mathbf{w}_c t) dt = -(1 - \mathbf{a}^2) A_c^2 T_b / 2$$

$$\text{Thus, } E_b - E_{10} = A_c^2 T_b / 2 = \frac{E_b}{1 + \mathbf{a}^2} \Rightarrow P_e = Q \left[ \sqrt{2\mathbf{g}_b / (1 + \mathbf{a}^2)} \right]$$

14.2-11



$$|H_w(f)|^2 = N_0 / 2G_n(f)$$

$$\bar{S}_m(f) = \mathfrak{I}[\tilde{s}_m(t)] = S_m(f)H_w(f) \text{ where } S_m(f) = \mathfrak{I}[s_m(t)], \quad m = 0, 1$$

If  $h_{opt}(t) = K[\tilde{s}_1(T_b - t) - \tilde{s}_0(T_b - t)]$  then

$$\left( \frac{z_1 - z_0}{2\sigma} \right)_{\max}^2 = \frac{1}{2N_0} (\tilde{E}_1 + \tilde{E}_0 - 2\tilde{E}_{10}) \text{ where}$$

$$\tilde{E}_m = \int_0^{T_b} \tilde{s}_m^2(t) dt = \int_{-\infty}^{\infty} \tilde{s}_m^2(t) dt = \int_{-\infty}^{\infty} |\tilde{S}_m(f)|^2 df = \int_{-\infty}^{\infty} |S_m(f)|^2 |H_w(f)|^2 df$$

$$\begin{aligned} \tilde{E}_{10} &= \int_0^{T_b} \tilde{s}_1(t)\tilde{s}_0(t) dt = \int_{-\infty}^{\infty} \tilde{s}_1(t)\tilde{s}_0(t) dt \\ &= \int_{-\infty}^{\infty} \tilde{S}_1(f)\tilde{S}_0^*(f) df = \int_{-\infty}^{\infty} S_1(f)S_0^*(f) |H_w(f)|^2 df \end{aligned}$$

$$\text{or } = \int_{-\infty}^{\infty} \tilde{S}_1^*(f)\tilde{S}_0(f) df = \int_{-\infty}^{\infty} S_1^*(f)S_0(f) |H_w(f)|^2 df$$

Thus, we can write

$$2\tilde{E}_{10} = \int_{-\infty}^{\infty} S_1^*(f)S_0(f) |H_w(f)|^2 df + \int_{-\infty}^{\infty} S_1(f)S_0^*(f) |H_w(f)|^2 df$$

$$\text{so } \tilde{E}_1 + \tilde{E}_0 - 2\tilde{E}_{10}$$

$$= \int_{-\infty}^{\infty} [ |S_1(f)|^2 + |S_0(f)|^2 - S_1(f)S_0^*(f) - S_1^*(f)S_0(f) ] |H_w(f)|^2 df$$

$$= \int_{-\infty}^{\infty} |S_1(f) - S_0(f)|^2 \frac{N_0}{2G_n(f)} df$$

$$\text{Hence, } \left( \frac{z_1 - z_0}{2\sigma} \right)_{\max}^2 = \int_{-\infty}^{\infty} \frac{|S_1(f) - S_0(f)|^2}{4G_n(f)} df$$

14.2-12

$$\begin{aligned}
 \int_0^{T_b} s_1(t) s_0(t) dt &= A_c^2 \int_0^{T_b} \cos[2\pi(f_c - f_d)t] \cos[2\pi(f_c + f_d)t] dt \\
 &= \frac{A_c^2}{2} \left\{ \int_0^{T_b} (\cos 4\pi f_c t) dt + \int_0^{T_b} (\cos 4\pi f_d t) dt \right\} = \frac{A_c^2}{2} \left[ \frac{1}{4\pi f_c} \sin 4\pi f_c T_b + \frac{1}{4\pi f_d} \sin 4\pi f_d T_b \right] \\
 \text{since } f_c T_b &\text{ is an integer } \Rightarrow \sin 4\pi f_c T_b = 0, \text{ and } r_b = 1/T_b \\
 \Rightarrow \int_0^{T_b} s_1(t) s_0(t) dt &= \frac{A_c^2}{2} \frac{T_b}{T_b} \frac{\sin 4\pi f_d T_b}{4\pi f_d} = \frac{A_c^2 T_b}{2} \text{sinc} 4 f_d / r_b \\
 \text{If } E_1 = E_0 = \frac{A_c^2 T_b}{2} &\Rightarrow \rho = \frac{1}{\sqrt{E_1 E_0}} \int_0^{T_b} s_1(t) s_0(t) dt = \text{sinc} 4 f_d / r_b
 \end{aligned}$$

14.2-13

$$\begin{aligned}
 \text{Eq. (9)} &\Rightarrow P_e = Q \left[ \sqrt{(E_b(1-\rho)/N_0)} \right] \Rightarrow \text{to minimize } P_e \Rightarrow \text{maximize } (1-\rho) \\
 &\Rightarrow \text{make } \rho \text{ as negative as possible. With } \rho = \text{sinc}(4 f_d / r_b) \\
 \text{From the sinc Table, the maximum negative value of the sinc function} &= -0.216 \\
 \Rightarrow \rho = \text{sinc} \lambda = -0.216 = \rho \\
 \text{with } \lambda &= 1.4
 \end{aligned}$$

14.2-14

$$\begin{aligned}
 \text{(a) Given } P_e = 10^{-5} \text{ and } N_0 = 10^{-11}, E_b &= \frac{A_c^2 T_b}{2} = \frac{A_c^2}{2 r_b}, P_e = Q \left( \sqrt{2 E_b / N_0} \right) \\
 \text{From Table, } P_e = 10^{-5} &\Rightarrow 2 E_b / N_0 = 18.3 \Rightarrow E_b = \frac{18.3}{2} \times 10^{-11} = 9.16 \times 10^{-11} \\
 \Rightarrow A_c^2 = 2 r_b E_b &= 2 \times 9600 \times 9.16 \times 10^{-11} = 1.76 \times 10^{-6} \\
 \Rightarrow A_c &= 0.00133 \text{ V.} \\
 \text{(b) } r_b = 28.8 \text{ kpbs} &\Rightarrow A_c^2 = 2 r_b E_b = 2 \times 28,800 \times 9.16 \times 10^{-11} = 5.28 \times 10^{-6} \\
 \Rightarrow A_c &= 0.0023 \text{ V}
 \end{aligned}$$

14.2-15

(a) Given  $P_e = 10^{-5}$  and  $N_0 = 10^{-11}$ ,  $E_b = \frac{A_c^2 T_b}{2} = \frac{A_c^2}{2r_b}$ ,

Assuming Sunde's FSK,  $\Rightarrow P_e = Q\left(\sqrt{E_b / N_0}\right)$

From Table,  $P_e = 10^{-5} \Rightarrow E_b / N_0 = 18.3 \Rightarrow E_b = 18.3 \times 1 \times 10^{-11} = 1.83 \times 10^{-10}$

$\Rightarrow A_c^2 = 2r_b E_b = 2 \times 9600 \times 1.83 \times 10^{-10} = 3.51 \times 10^{-6}$

$\Rightarrow A_c = 0.00187\text{V}$ .

(b)  $r_b = 28.8 \text{ kpbs} \Rightarrow A_c^2 = 2r_b E_b = 2 \times 28,800 \times 1.83 \times 10^{-10} = 1.05 \times 10^{-5}$

$\Rightarrow A_c = 0.00325 \text{ V}$

14.3-1

$$\frac{1}{2} \left[ e^{-r_b / 2} + Q\left(\sqrt{g_b}\right) \right] < 10^{-3} \Rightarrow g_b > 2 \ln \left( \frac{1}{2 \times 10^{-3}} \right) \approx 12.4 \approx 10.9 \text{ dB}$$

so  $P_{e1} \approx Q\left(\sqrt{g_b}\right) < 3 \times 10^{-4}$

14.3-2

$$\frac{1}{2} \left[ e^{-r_b / 2} + Q\left(\sqrt{g_b}\right) \right] < 10^{-5} \Rightarrow g_b > 2 \ln \left( \frac{1}{2 \times 10^{-5}} \right) \approx 21.6 \approx 13.4 \text{ dB}$$

so  $P_{e1} \approx Q\left(\sqrt{g_b}\right) < 2 \times 10^{-6}$

14.3-3

(a) Given  $P_e = 10^{-5}$  and  $N_0 = 10^{-11}$ ,  $E_b = \frac{A_c^2 T_b}{2} = \frac{A_c^2}{2r_b}$ ,

Noncoherent FSK,  $\Rightarrow P_e = \frac{1}{2} e^{-E_b / 2 N_0} = \frac{1}{2} e^{-\gamma_b / 2} = 1 \times 10^{-5}$

$\ln(2 \times 10^{-5}) = -\gamma_b / 2 \Rightarrow \gamma_b = 21.6 = E_b / N_0 \Rightarrow E_b = 21.6 \times 1 \times 10^{-11} = 2.16 \times 10^{-10}$

$\Rightarrow A_c^2 = 2r_b E_b = 2 \times 9600 \times 2.16 \times 10^{-10} = 4.14 \times 10^{-6}$

$\Rightarrow A_c = 0.00204\text{V}$ .

(b)  $r_b = 28.8 \text{ kpbs} \Rightarrow A_c^2 = 2r_b E_b = 2 \times 28,800 \times 2.16 \times 10^{-10} = 1.24 \times 10^{-5}$

$\Rightarrow A_c = 0.00353 \text{ V}$

14.3-4

(a) Sunde's FSK  $\Rightarrow f_d = r_b / 2$ ,  $B_T \approx r_b$ , and  $E_{10} = 0$

$\Rightarrow f_d = 14,400/2 = 7200$ ,  $S/N = 12 \text{ dB} = 15.9$

$$\gamma_b = E_b / N_0 = \left( \frac{S}{N} \right) \left( \frac{B_T}{r_b} \right) = 15.9 \times 1$$

$$\text{Coherent FSK} \Rightarrow P_e = Q\left(\sqrt{E_b / N_0}\right) = Q\left(\sqrt{15.9}\right) = 3.4 \times 10^{-5}$$

$$(b) \text{ Noncoherent FSK} \Rightarrow P_e = \frac{1}{2} e^{-\gamma_b/2} = \frac{1}{2} e^{-15.9/2} = 1.76 \times 10^{-4}$$

14.3-5

$$KA_c = A_c^2 / E_1 = 2 / T_b$$

$$\begin{aligned} \mathbf{s}^2 &= \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df = \frac{N_0}{2} \int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{N_0}{2} \frac{2}{T_b} \int_0^{T_b} \cos^2(\mathbf{w}_c t) dt \\ &= \frac{N_0}{T_b} \left[ 1 + \text{sinc}\left(\frac{4f_c}{r_b}\right) \right] \approx \frac{N_0}{T_b} \text{ since } f_c \gg r_b \end{aligned}$$

$$\text{Thus } A_c^2 / \mathbf{s}^2 = A_c^2 T_b / N_0 = 4E_b / N_0 \quad \text{where } E_b = A_c^2 T_b / 4$$

14.3-6

$$\mathbf{s}^2 = (N_0 / 2) * 2 \int_0^{\infty} |H(f)|^2 df = N_0 \int_0^{\infty} \frac{df}{\left(1 + \frac{4(f - f_c)^2}{B^2}\right)}$$

$$\approx (N_0 B / 2) \mathbf{p} \text{ since } f_c / B \gg 1 \Rightarrow \arctan(-2f_c / B) \approx -\mathbf{p} / 2$$

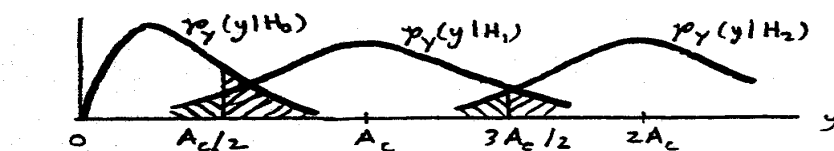
$$P_e \approx 1/2 P_{e0} = 1/2 e^{-A_c^2 / 8\mathbf{s}^2} \text{ where } A_c^2 / 8\mathbf{s}^2 = A_c^2 / 4\mathbf{p}BN_0 \text{ and}$$

$$A_c^2 = 4E_b r_b \text{ so } A_c^2 / 8\mathbf{s}^2 = 4E_b r_b / 4\mathbf{p}BN_0 = (2r_b / \mathbf{p}B) r_b / 2$$

$$\Rightarrow \text{increase } r_b \text{ by } \mathbf{p}B / 2r_b \geq \mathbf{p} \approx 5\text{dB}$$

14.3-7

Take  $K = A_c / E_1$  and thresholds at  $A_c / 2$  and  $3A_c / 2$



14.3-7 continued

$$\text{Then } P_{e0} = e^{-A_c^2/8s^2}, \quad P_{e1} \approx 2Q\left(\frac{A_c}{2s}\right), \quad P_{e2} \approx Q\left(\frac{A_c}{2s}\right)$$

$$E_0 = 0, \quad E_1 \approx A_c^2 D/2, \quad E_2 \approx (2A_c)^2 D/2 \quad \text{where } D = 1/r$$

$$\text{so } E = \frac{1}{3}(E_0 + E_1 + E_2) = 5A_c^2 D/6 \Rightarrow A_c^2 D = 6E/5$$

$$\text{and } \frac{A_c^2}{s^2} = \frac{4}{N_0} \frac{A_c^2 D}{4} = \frac{6E}{5N_0} \quad \text{from Eq.(9) with } E_b = \frac{A_c^2 D}{4}$$

$$\text{Thus, } P_e = \frac{1}{3}(P_{e0} + P_{e1} + P_{e2}) \approx \frac{1}{3}e^{-3E/20N_0} + Q\left(\sqrt{3E/10N_0}\right)$$

14.3-8

$$\frac{S_T}{L} = S_R = E_b r_b = N_0 g_b r_b \Rightarrow r_b = \frac{S_T}{LN_0 g_b} = \frac{2 \times 10^5}{g_b}$$

$$(a) \quad \frac{1}{2}e^{-g_b/2} \leq 10^{-4} \Rightarrow g_b \geq 2\ln \frac{1}{2 \times 10^{-4}} \approx 17 \quad \text{so } r_b \leq 2 \times 10^5 / 17 = 11.8 \text{ kbps}$$

$$(b) \quad \frac{1}{2}e^{-g_b} \leq 10^{-4} \Rightarrow g_b \geq \ln \frac{1}{2 \times 10^{-4}} \approx 8.5 \quad \text{so } r_b \leq 2 \times 10^5 / 8.5 = 23.5 \text{ kbps}$$

$$(c) \quad Q\left(\sqrt{2g_b}\right) \leq 10^{-4} \Rightarrow g_b \geq \frac{1}{2} \times 3.75^2 \approx 7 \quad \text{so } r_b \leq 2 \times 10^5 / 7 = 28.4 \text{ kbps}$$

14.3-9

$$\frac{S_T}{L} = S_R = E_b r_b = N_0 g_b r_b \Rightarrow r_b = \frac{S_T}{LN_0 g_b} = \frac{2 \times 10^5}{g_b}$$

$$(a) \quad \frac{1}{2}e^{-g_b/2} \leq 10^{-5} \Rightarrow g_b \geq 2\ln \frac{1}{2 \times 10^{-5}} \approx 21.6 \quad \text{so } r_b \leq 2 \times 10^5 / 21.6 = 9.2 \text{ kbps}$$

$$(b) \quad \frac{1}{2}e^{-g_b} \leq 10^{-5} \Rightarrow g_b \geq 2\ln \frac{1}{2 \times 10^{-5}} \approx 10.8 \quad \text{so } r_b \leq 2 \times 10^5 / 10.8 = 18.4 \text{ kbps}$$

$$(c) \quad Q\left(\sqrt{2g_b}\right) \leq 10^{-5} \Rightarrow g_b \geq \frac{1}{2} \times 4.27^2 \approx 9.1 \quad \text{so } r_b \leq 2 \times 10^5 / 9.1 = 21.9 \text{ kbps}$$

14.3-10

$$\text{DPSK} : \frac{1}{2} e^{-g_b} \leq 10^{-4} \Rightarrow g_b \geq \ln\left(\frac{1}{2 \times 10^{-4}}\right) = 8.52$$

$$\text{BPSK} : Q\left(\sqrt{2g_b \cos^2 q_e}\right) \leq 10^{-4} \Rightarrow g_b \geq \frac{1}{2} \left(\frac{3.75}{\cos q_e}\right)^2 = \frac{7.03}{\cos^2 q_e}$$

BPSK requires less energy if  $|q_e| < \arccos \sqrt{7.03/8.52} \approx 25^\circ$

14.3-11

$$\text{DPSK} : \frac{1}{2} e^{-g_b} \leq 10^{-6} \Rightarrow g_b \geq \ln\left(\frac{1}{2 \times 10^{-6}}\right) = 13.12$$

$$\text{BPSK} : Q\left(\sqrt{2g_b \cos^2 q_e}\right) \leq 10^{-6} \Rightarrow g_b \geq \frac{1}{2} \left(\frac{4.75}{\cos q_e}\right)^2 = \frac{11.28}{\cos^2 q_e}$$

BPSK requires less energy if  $|q_e| < \arccos \sqrt{11.28/13.12} \approx 22^\circ$

14.3-12

$$p_y(y) = p_{nq}(y) = \frac{1}{\sqrt{2ps^2}} e^{-y^2/2s^2}, \quad p_x(x) = p_{n_i}(x - A_c) = \frac{1}{\sqrt{2ps^2}} e^{-(x-A_c)^2/2s^2}$$

$$p_{xy}(x, y) = p_x(x)p_y(y) = \frac{1}{\sqrt{2ps^2}} e^{-[(x-A_c)^2+y^2]/2s^2}$$

For polar transformation:  $x = A \cos f$ ,  $y = A \sin f$ ,  $dx dy = A dA df$

so  $p_{Af}(A, f) dA df = p_{xy}(x, y) dx dy = p_{xy}(A \cos f, A \sin f) A dA df$

where  $(x - A_c)^2 + y^2 = A^2 \cos^2 f - 2AA_c \cos f + A_c^2 + A^2 \sin^2 f = A^2 - 2AA_c \cos f + A_c^2$

$$\text{Thus } p_{Af}(A, f) = A p_{xy}(A \cos f, A \sin f) = \frac{A}{2ps^2} e^{-(A^2 - 2AA_c \cos f + A_c^2)/2s^2}$$

14.4-1

$r_b / B_T \geq 1.25 \Rightarrow$  Modulation types with  $r_b / B_T = 2$

(a) QAM/QPSK:  $Q(\sqrt{2\gamma_b}) \leq 10^{-6} \Rightarrow \gamma_b \geq 1/2 \times 4.75^2 = 10.5 \text{ dB}$

(b) DPSK with  $M = 4$  so  $K = 2$ :  $\frac{2}{2} Q(\sqrt{4 \times 2\gamma_b \sin^2 \pi/8}) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{8} \left(\frac{4.75}{0.383}\right)^2 = 12.8 \text{ dB}$$



## 14.4-2

$r_b / B_T \geq 2.5 \Rightarrow$  modulation types with  $r_b / B_T = 3$

(a) PSK with  $M = 8$  so  $K = 3$ :  $\frac{2}{3} Q\left(\sqrt{2 \times 3 \gamma_b \sin^2 \pi/8}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{6} \left( \frac{4.7}{0.383} \right)^2 = 14.0 \text{ dB}$$

(b) DPSK with  $M = 8$  so  $K = 3$ :  $\frac{2}{3} Q\left(\sqrt{4 \times 3 \gamma_b \sin^2 (\pi/16)}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{12} \left( \frac{4.7}{0.195} \right)^2 = 16.8 \text{ dB}$$

## 14.4-3

$r_b / B_T \geq 3.2 \Rightarrow$  modulation types with  $r_b / B_T = 4$

(a) QAM with  $M = 16$  so  $K = 4$ :  $\frac{4}{4} \left(1 - \frac{1}{4}\right) Q\left(\sqrt{\frac{3 \times 4}{15} \gamma_b}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{15}{12} 4.7^2 = 14.4 \text{ dB}$$

(b) PSK with  $M = 16$  so  $K = 4$ :  $\frac{2}{4} Q\left(\sqrt{2 \times 4 \gamma_b \sin^2 \pi/16}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{8} \left( \frac{4.6}{0.195} \right)^2 = 18.4 \text{ dB}$$

## 14.4-4

$r_b / B_T \geq 4.8 \Rightarrow$  modulation types with  $r_b / B_T = 5$  or 6

(a) QAM with  $M = 64$  so  $K = 6$ :  $\frac{4}{6} \left(1 - \frac{1}{8}\right) Q\left(\sqrt{\frac{3 \times 6}{63} \gamma_b}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{63}{18} 4.65^2 = 18.8 \text{ dB}$$

(b) PSK with  $M = 32$  so  $K = 5$ :  $\frac{2}{5} Q\left(\sqrt{2 \times 5 \gamma_b \sin^2 (\pi/32)}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{10} \left( \frac{4.55}{0.098} \right)^2 = 23.3 \text{ dB}$$

14.4-5

$$x_c(t) = A_c \cos(\omega_c t + \phi_k)$$

$$\begin{aligned} \text{Upper delay output} &= x_c(t-D)2\cos[\omega_c(t-D) + \theta_\epsilon] \\ &= A_c [\cos(\theta_\epsilon - \phi_k) + \cos(2\omega_c t - 2\omega_c D + \theta_\epsilon + \phi_k)] \end{aligned}$$

$$\begin{aligned} \text{Lower delay output} &= x_c(t-D)\{-2\sin[\omega_c(t-D) + \theta_\epsilon]\} \\ &= -A_c [\sin(\theta_\epsilon - \phi_k) + \sin(2\omega_c t - 2\omega_c D + \theta_\epsilon + \phi_k)] \end{aligned}$$

$$\text{LPF input} = A_c \left[ \sin \hat{\phi}_k \cos(\theta_\epsilon - \phi_k) - (-\cos \hat{\phi}_k) \sin(\theta_\epsilon - \phi_k) + \text{high frequency terms} \right]$$

$$\begin{aligned} \text{Thus, } v(t) &= A_c \left[ \frac{1}{2} \sin(\hat{\phi}_k - \theta_\epsilon + \phi_k) + \frac{1}{2} \sin(\hat{\phi}_k + \theta_\epsilon - \phi_k) \right. \\ &\quad \left. + \frac{1}{2} \sin(\theta_\epsilon - \phi_k - \hat{\phi}_k) + \frac{1}{2} \sin(\theta_\epsilon - \phi_k + \hat{\phi}_k) \right] \\ &= A_c \sin(\theta_\epsilon + \hat{\phi}_k - \phi_k) = A_c \sin(\theta_\epsilon) \quad \text{when } \hat{\phi}_k = \phi_k \end{aligned}$$

14.4-6

$$\text{QAM: } P_e \approx 3Q \left[ \sqrt{\frac{3E}{15N_0}} \right] \quad \text{DPSK: } P_e \approx 2Q \left[ \sqrt{\frac{4E}{N_0} \sin^2(\pi/32)} \right]$$

Since magnitude of  $P_e$  is dominated by argument of  $Q$ , we want

$$\frac{4E_{DPSK}}{N_0} \times (0.098)^2 \approx \frac{4E_{QAM}}{15N_0} \Rightarrow \frac{E_{DPSK}}{E_{QAM}} \approx \frac{3}{15 \times 4 \times (0.098)^2} = 5.2$$

14.4-7

$$\text{PSK: } P_e \approx 2Q \left[ \sqrt{\frac{2E}{N_o} \times \left( \frac{\pi}{M} \right)^2} \right] \quad \text{QAM: } P_e \approx 4Q \left[ \sqrt{\frac{3E}{MN_0} \gamma_b} \right]$$

$$\text{since } \sin \pi/M \approx \pi/M \quad \text{since } 1 - \frac{1}{\sqrt{M}} \approx 1, \quad M-1 \approx M$$

Magnitude of  $P_e$  is dominated by argument of  $Q$ , so we want

$$\frac{3E_{QAM}}{MN_0} \approx \frac{2E_{PSK}}{N_o} \left( \frac{\pi}{M} \right)^2 \Rightarrow \frac{E_{QAM}}{E_{PSK}} \approx \frac{2\pi^2}{3M}$$

14.4-8

$$p_{\mathbf{f}}(\mathbf{f}) = \int_0^{\infty} p_{A\mathbf{f}}(A, \mathbf{f}) dA = \frac{e^{-A_c^2/2\mathbf{s}^2}}{2p\mathbf{s}^2} \int_0^{\infty} A e^{-(A^2 - 2AA_c \cos \mathbf{f})/2\mathbf{s}^2} dA$$

$$\text{let } \mathbf{I} = (A - A_c \cos \mathbf{f})/\mathbf{s} \text{ and } \mathbf{I}_0 = (A_c \cos \mathbf{f})/\mathbf{s}$$

$$\text{so } A = \mathbf{s}(\mathbf{I} + \mathbf{I}_0) \text{ and } (A^2 - 2AA_c \cos \mathbf{f})/2\mathbf{s}^2 = (\mathbf{I}^2/2) - (\mathbf{I}_0^2/2)$$

$$\begin{aligned} \text{Then } p_{\mathbf{f}}(\mathbf{f}) &= \frac{e^{-A_c^2/2\mathbf{s}^2}}{2p\mathbf{s}^2} e^{\mathbf{I}_0^2/2} \int_{-\mathbf{I}_0}^{\infty} \mathbf{s}(\mathbf{I} + \mathbf{I}_0) e^{-\mathbf{I}^2/2} \mathbf{s} d\mathbf{I} \\ &= \frac{1}{2p} e^{-A_c^2/2\mathbf{s}^2} e^{-\mathbf{I}_0^2/2} \left[ \int_{-\mathbf{I}_0}^{\infty} \mathbf{I} e^{-\mathbf{I}^2/2} d\mathbf{I} + \mathbf{I}_0 \int_{\mathbf{I}_0}^{\infty} e^{-\mathbf{I}^2/2} d\mathbf{I} \right] \end{aligned}$$

$$\text{where } \int_{-\mathbf{I}_0}^{\infty} \mathbf{I} e^{-\mathbf{I}^2/2} d\mathbf{I} = e^{-\mathbf{I}_0^2/2}$$

$$\int_{-\mathbf{I}_0}^{\infty} e^{-\mathbf{I}^2/2} d\mathbf{I} = \int_{-\infty}^{\infty} e^{-\mathbf{I}^2/2} d\mathbf{I} - \int_{-\infty}^{-\mathbf{I}_0} e^{-\mathbf{I}^2/2} d\mathbf{I} = \sqrt{2p} [1 - Q(\mathbf{I}_0)]$$

$$\text{and } A_c^2/2\mathbf{s}^2 - \mathbf{I}_0^2/2 = A_c^2(1 - \cos^2 \mathbf{f})/2\mathbf{s}^2 = A_c^2 \sin^2 \mathbf{f}/2\mathbf{s}^2$$

$$\begin{aligned} \text{Thus } p_{\mathbf{f}}(\mathbf{f}) &= \frac{1}{2p} e^{-A_c^2/2\mathbf{s}^2} e^{\mathbf{I}_0^2/2} \left\{ e^{-\mathbf{I}_0^2/2} + \mathbf{I}_0 \sqrt{2p} [1 - Q(\mathbf{I}_0)] \right\} \\ &= \frac{1}{2p} e^{-A_c^2/2\mathbf{s}^2} + \frac{A_c \cos \mathbf{f}}{\sqrt{2p}\mathbf{s}^2} e^{-A_c^2 \sin^2 \mathbf{f}/2\mathbf{s}^2} \left[ 1 - Q\left(\frac{A_c \cos \mathbf{f}}{\mathbf{s}}\right) \right] \end{aligned}$$

14.4-9

Use the design of Fig. 14.4-2 and (1) change the 4th law device to a second law device, (2) change the  $4f_c$  BPF to a  $2f_c$  BPF, (3) change the  $\div 4$  block to a  $\div 2$  block, (4) eliminate the  $+90$  deg block.  $\Rightarrow$  The output of the  $\div 2$  block is the reference signal and is  $\cos(2\pi f_c t + \pi N)$ . The  $\pi N$  term is a phase ambiguity that depends on the lock-in transient and have to be accounted for. This could be done using a known preamble at the beginning of the transmission.

14.4-10

Use the design of Fig. 14.4-2 and (1) change the 4th law device to a  $M$ th-law device, (2) change the  $4f_c$  BPF to a  $Mf_c$  BPF. The output of the PLL is  $\cos[2M\pi t + M\phi_k + 2\pi N]$ . The  $2\pi N$  term is a phase ambiguity that depends on the lock-in transient and will have to be accounted for. This could be done using a known preamble at the beginning of the transmission (3) At the output of the PLL, change the  $\div 4$  block to a  $\div M$  block, giving an output of  $\cos[2\pi t + \phi_k + 2\pi N/M]$ . (4) Replace  $+90$  deg block with an  $M$  output phase network.

14.4-11

$$\gamma_b = 13 \text{ dB} = 20, P_e = P_{be} K \text{ and } K = \log_2 M$$

$$(a) \text{ FSK} \Rightarrow P_e = \frac{1}{2} e^{-\gamma_b/2} = \frac{1}{2} e^{-20/2} = 2.3 \times 10^{-5}$$

$$(b) \text{ BPSK} \Rightarrow P_e = Q(\sqrt{2\gamma_b}) = Q(\sqrt{2 \times 20}) = 1.8 \times 10^{-10}$$

$$(c) \text{ 64-PSK} \Rightarrow P_{be} = \frac{2}{K} Q\left(\sqrt{2K\gamma_b \sin^2 \frac{\pi}{M}}\right) = \frac{2}{7} Q\left(\sqrt{2 \times 7 \times \gamma_b \times \sin^2 \frac{\pi}{64}}\right)$$

$$= 0.0571 \Rightarrow P_e = 0.031 \times 7 = 0.40$$

$$(d) \text{ 64-QAM} \Rightarrow P_e = \frac{4}{K} \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3K\gamma_b}{M-1}}\right)$$

$$K = 7, M = 64 \Rightarrow P_{be} = \frac{4}{7} \left(1 - \frac{1}{\sqrt{64}}\right) Q\left(\sqrt{\frac{3 \times 7 \times 20}{63}}\right) = 2.4 \times 10^{-3}$$

$$\Rightarrow P_e = P_{be} \times K = 2.4 \times 10^{-3} \times 7 = 0.017$$

14.5-1

$$\text{Eq. (6)} \Rightarrow P_e = N_{\min} Q\left(\sqrt{d_{\min}^2 / 2N_0}\right) = 1 \times 10^{-5}$$

$$\text{For uncoded QPSK} \Rightarrow N_{\min} = 2 \text{ and } (d_{\min})_{\text{uncoded}} = \sqrt{2}$$

$$\Rightarrow 1 \times 10^{-5} = 2Q\left(\sqrt{2/2N_0}\right) \Rightarrow \text{solving gives } N_0 = 0.052$$

$$\text{TCM with 8 states, } m = \tilde{m} = 2 \Rightarrow N_{\min} = 2 \text{ and } g = 3.6 \text{ dB} = 2.3$$

$$\text{From Eq. (5)} \Rightarrow g = \frac{(d_{\min}^2)_{\text{coded}}}{(d_{\min}^2)_{\text{uncoded}}} = \frac{(d_{\min}^2)_{\text{uncoded}}}{2} = 2.3 \Rightarrow (d_{\min}^2)_{\text{coded}} = 4.6$$

$$\Rightarrow P_e = 2Q\left(\sqrt{\frac{4.6}{2 \times 0.052}}\right) = 4.0 \times 10^{-11}$$

### 14.5-2

From Ungerbroeck (1982), we have

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \overline{d_0} = 1$$

$$\begin{array}{ccc} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} d_1 = \sqrt{2}$$

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$$

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$$

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \overline{d_2} = 2$$

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$$

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$$

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} d_2 = 2\sqrt{2}$$

### 14.5-3

Input:  $x_2 x_1$  00  $\rightarrow$  01  $\rightarrow$  10  $\rightarrow$  01  $\rightarrow$  11  $\rightarrow$  00

Output:  $y_3 y_2 y_1$  000  $\rightarrow$  100  $\rightarrow$  011  $\rightarrow$  010  $\rightarrow$  111  $\rightarrow$  111

State: a b g b h e

### 14.5-4

What is the distance between paths (0,2,4,2) and (6,1,3,0)?

Using Figs. 14.5-4 and 14.5-7 we have:

$$\begin{aligned} \sqrt{d_{0 \rightarrow 6}^2 + d_{2 \rightarrow 1}^2 + d_{4 \rightarrow 3}^2 + d_{2 \rightarrow 0}^2} &= \sqrt{d_1^2 + d_0^2 + d_0^2 + d_1^2} \\ &= \sqrt{(\sqrt{2})^2 + (2 \sin \pi/8)^2 + (2 \sin \pi/8)^2 + (\sqrt{2})^2} = \sqrt{5.2} = 2.3 \end{aligned}$$

### 14.5-5

See error event Trellis diagram of Fig. P14.5-5

$$\begin{aligned} d_{\min}^2 &= d_{0 \rightarrow 6}^2 + d_{0 \rightarrow 1}^2 + d_{0 \rightarrow 7}^2 + d_{0 \rightarrow 2}^2 = d_1^2 + d_0^2 + d_0^2 + d_1^2 \\ &= 2 + 4 \sin^2 \pi/8 + 4 \sin^2 \pi/8 + 2 = 5.17 \\ \Rightarrow \text{coding gain} &= 10 \log \left( \frac{5.17}{2} \right) = 4.13 \text{ dB} \end{aligned}$$

## Chapter 15

### 15.1-1

(a) With DSS  $(S/N)_D = (S/N)_R = \frac{S_R}{N_R} = \frac{E_b r_b}{N_0 r_b} = \frac{E_b}{N_0} = 60 \text{ dB} = 10^6$

with  $N_0 = 10^{-21} \Rightarrow E_b = 10^{-15}$

$S_R = E_b \times r_b = 10^{-15} \times 3000 = 3 \times 10^{-12} \Rightarrow J = 15 \times 10^{-12}$

Since  $J \ll N_0$  we can neglect noise and use Eq. (13) and  $J/S_R = 5$  giving

$$P_e = 10^{-7} = Q\left(\sqrt{\frac{2P_g}{5}}\right) \Rightarrow 5.2^2 = 2P_g/5 \Rightarrow P_g = 67.6$$

Eq. (9) with  $W_x = 3000$   $W_c = 67.6 \times 3000 = 203 \text{ kcps}$

(b)  $B_T = 2 \times W_c = 2 \times 203 \times 10^3 = 0.406 \text{ MHz}$

### 15.1-2

$P_g = 30 \text{ dB} = 1000$  With Eq. (13) we have  $P_e = Q\left(\sqrt{\frac{2 \times 1000}{J/S_R}}\right) = 10^{-7}$

$$\Rightarrow 5.2^2 = \frac{2000}{J/S_R} \Rightarrow J/S_R = 74.0 \Rightarrow \text{jamming margin} = 10\log(74) = 18.7 \text{ dB}$$

$$\begin{aligned} \text{Jamming margin} &= 10 \times \log(J/S_r) \\ &= 10 \times \log(P_g) - 10 \times \log(E_b/N_J) \\ &= 10 \times \log(1000) - 10 \times \log(1352) = 18.69 \end{aligned}$$

### 15.1-3

(a)  $(S/N)_D = 20 \text{ dB} = 100 = E_b/N_0$

$W_c = 10 \times 10^6$  and  $r_b = 6000 = W_x \Rightarrow P_g = 10 \times 10^6 / 6000 = 1.67 \times 10^3$

With Eq. (19) we have  $P_e = 10^{-7} = Q\left(\frac{1}{\sqrt{M/(3 \times 1.67 \times 10^3) + N_0/2E_b}}\right)$

$$\Rightarrow 5.2^2 = \frac{1}{\frac{M}{5000} + \frac{1}{200}} \Rightarrow M = 159 \text{ additional users for a total of 160 users.}$$

15.1-3 continued

(b) If each user reduces their power by 6 dB  $\Rightarrow 6 \text{ dB} = 4 \Rightarrow E_b / N_0 = 100/4 = 25$

Using the results of part (a) we have

$$5.2^2 = \frac{1}{\frac{M-1}{5000} + \frac{1}{50}} \Rightarrow M-1 = 85 \text{ additional users for a total of 86 users.}$$

15.1-4

Let  $d$  = distance between the transmitter and authorized receiver then

$d_m = d + 500$  = distance of the multipath,  $T_m$  multipath travel time and  $c$  speed of light.

With  $d_m = c \times T_m \Rightarrow T_m = d_m / c = 500 / 3 \times 10^8 = 1.67 \mu s$

To avoid multipath interference  $T_c < T_m \Rightarrow W_c > 600 \text{ kcps}$

15.1-5

Using Eq. (2) with  $M-1 = 9$  additional users (10 total users), we have  $P_e = Q\left(\sqrt{\frac{3P_g}{9}}\right) = 10^{-7}$

$$9 \times 5.2^2 / 3 = P_g = 81.1 \Rightarrow W_c = P_g \times r_b = 81.6 \times 6000 = 487 \text{ kcps} = W_c$$

15.1-6

$$\text{With } P_e = 10^{-9} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \Rightarrow \frac{2E_b}{N_0} = 6^2 = 36$$

$$\text{Eq. (19), } M-1 = 9 \text{ additional users and } P_e = 10^{-7} = Q\left(\sqrt{\frac{1}{\frac{9}{3P_g} + \frac{1}{36}}}\right)$$

$$\Rightarrow 5.2^2 = \frac{1}{\frac{9}{3P_g} + \frac{1}{36}} \Rightarrow P_g = 326 = \frac{W_c}{6000} \Rightarrow W_c = 1.96 \text{ Mcps}$$

15.1-7

$r_b = 9 \text{ kbps}$ ,  $J/S_R = 30 \text{ dB} = 1000$  and  $P_e < 10^{-7}$ ;

$$\text{Eq. (13) } 10^{-7} = Q\left(\sqrt{\frac{2P_g}{1000}}\right) \Rightarrow 5.2^2 = 2P_g / 1000 \Rightarrow P_g = 13,520$$

15.1-8

$P_g = 30 \text{ dB} = 1000$  and assuming negligible noise,

$$\text{Using Eq. (20) } P_e = 10^{-7} = Q \left( \sqrt{\frac{3P_g}{M-1}} \right) \Rightarrow 3 \times 1000 / (M-1) = 5.2^2 \Rightarrow M-1 = 111 \\ \Rightarrow 112 \text{ total users}$$

15.1-9

$$r_b = 6 \text{ kbps}, W_c = 10 \text{ Mcps} \Rightarrow P_g = 10 \times 10^6 / 6000 = 1667$$

$$\text{For single user } P_e = 10^{-10}, \text{ then Eq. (6) } \Rightarrow 10^{-10} = Q \left( \sqrt{\frac{2E_b}{N_0}} \right) \Rightarrow 6.4^2 = 41 = \frac{2E_b}{N_0}$$

$$\text{If each user reduces their power by 3 dB} = 2 \Rightarrow E_b \rightarrow E_b / 2 \Rightarrow \frac{2E_b}{N_0} \rightarrow 41/2 = 20.5$$

$$\text{Eq. (19) we have } 10^{-5} = Q \left( \frac{1}{\sqrt{\frac{M-1}{3 \times 1667} + \frac{1}{20.5}}} \right) \Rightarrow 4.3^2 = \frac{1}{\frac{M-1}{3 \times 1667} + \frac{1}{20.5}} \Rightarrow$$

$$M-1 = 26 \text{ additional users} \Rightarrow M = 27 \text{ total users}$$

15.2-1

$$(a) E_b/N_0 = 60 \text{ dB} = 10^6 \text{ and if } N_0 = 10^{-21} \Rightarrow E_b = 10^6 / 10^{-21} = 10^{-15}$$

$$S_R = E_b \times r_b = 10^{-15} \times 3000 = 3 \times 10^{-12}$$

$$J = 5 \times S_r = 15 \times 10^{-12}$$

$$P_e = \frac{1}{2} e^{-\frac{E_b}{2(N_0 + N_J)}} \Rightarrow 10^{-7} = \frac{1}{2} e^{-\frac{10^{-15}}{2(10^{-21} + 3.24 \times 10^{-17})}} \Rightarrow N_J = 3.24 \times 10^{-17}$$

$$N_J = \frac{J}{W_c} \Rightarrow W_c = \frac{15 \times 10^{-12}}{3.2424 \times 10^{-17}} = 4.62 \times 10^5$$

$$P_g = \frac{W_c}{W_x} = \frac{4.62 \times 10^5}{3 \times 10^3} = 154$$

$$\text{If } k = 7 \Rightarrow P_g = 2^7 = 128$$

$$\text{If } k = 8 \Rightarrow P_g = 2^8 = 256$$

$$\Rightarrow \text{if } P_g = 256 \Rightarrow W_c = 256 \times 3103 = 768 \text{ kHz}$$

$$(b) B_T = W_c = 768 \text{ kHz}$$



15.2-2

10 users  $\Rightarrow M = 10$

$$P_e = \frac{(1/2)(K-1)}{P_g} + (1/2) \times e^{-Eb/(2 \times N_0)} \left[ 1 - \frac{(K-1)}{P_g} \right]$$

With  $P_e = 10^{-10}$  for one user  $\Rightarrow$  first term in above Eq. dominates.

$$2 \times 10^{-5} = \frac{9}{P_g} + 10 - 10 \times \left( 1 - \frac{9}{P_g} \right) \Rightarrow P_g = 450000$$

But  $P_g = 2^k \geq 450,000 \Rightarrow k = 19 \Rightarrow P_g = 2^{19} = 524,288$

$$P_g = \frac{W_c}{W_x} \Rightarrow W_c = 524,288 \times 3000 = 1.57 \text{ GHz}$$

15.2-3

From problem 15.2.1  $S_R = 3 \times 10^{-12}$ ,  $J = 1.5 \times 10^{-11}$ ,  $W_x = 3000$

$$\text{Using Sect 15.1, Eq. (13) we have } 10^{-7} = Q\left(\sqrt{\frac{2P_g}{J/S_R}}\right) = Q\left(\sqrt{\frac{2W_c/3000}{5}}\right)$$

$$\Rightarrow \frac{2W_c/3000}{5} = 5.2^2 \Rightarrow W_c = 2 \times 10^5$$

$$P_g = W_c/W_x = 2 \times 10^5/3000 = 67$$

$$B_T = 2 \times W_c = 400 \text{ kHz}$$

15.2-4

With  $k = 10$ ,  $\Rightarrow P_g = 2^{10} = 1024$

and with  $r_b = 6000 \Rightarrow W_c = P_g W_x = 1024 \times 6000 = 6144 \text{ kbps}$

$$N_J = J/W_c = 6 \times 10^{-3}/1024 = 9.76 \times 10^{-10}$$

$$\text{From Eq. (4) } P_e = \frac{1-0.1}{2} e^{-2 \times 10^{-11}/2 \times 10^{-12}} + \frac{0.1}{2} e^{\frac{-2 \times 10^{-11}}{2 \times 10^{-12} + 9.766 \times 10^{-10}/0.1}}$$

$$P_e = 2.04 \times 10^{-5} + 4.99 \times 10^{-2} = 0.05$$

15.2-5

$$d = 5 \text{ miles} \Rightarrow d = 5 \times 1610 = 8050 \text{ meters}$$

$$\Delta d = 2 \times 8050 = 16100$$

$$\Delta d = v \times t \Rightarrow t = \Delta d / v = 16100 / (2.99 \times 10^8) = 5.38 \times 10^{-9} \text{ s}$$

$$f = 1/t = 1 / (5.38 \times 10^{-9}) = 18.6 \text{ kHz}$$

15.3-1

- (a) A shift register with [4,1] configuration, and initial state of 0100 has the following contents after each clock pulse:

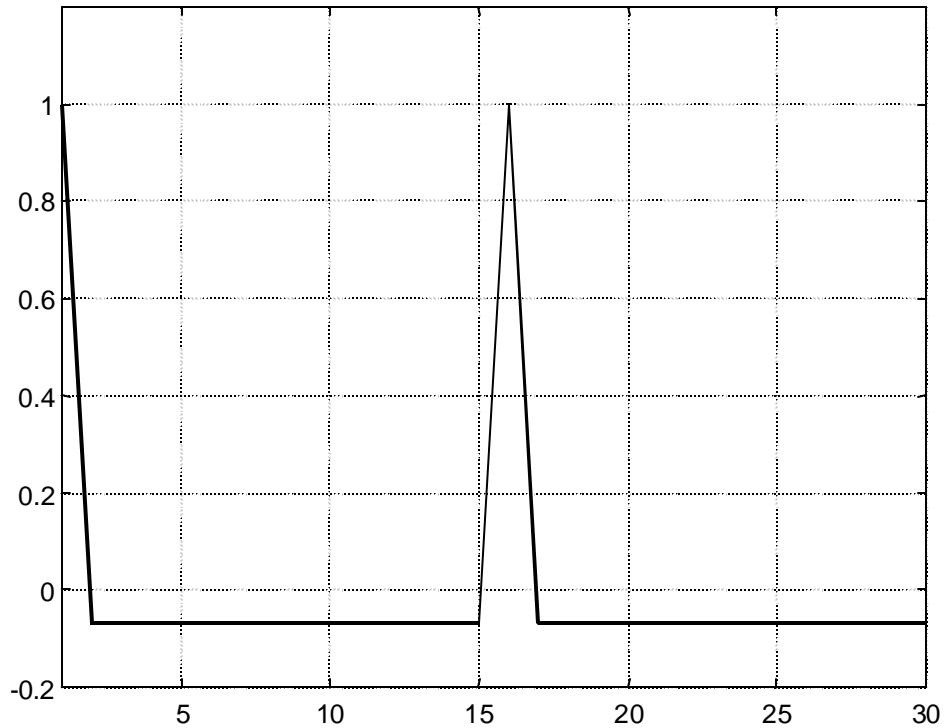
<u>Clock shift</u>	<u>Register contents</u>	<u>Clock shift</u>	<u>Register contents</u>
0	0100	8	1011
1	0010	9	0101
2	0001	10	1010
3	1000	11	1101
4	1100	12	0110
5	1110	13	0011
6	1111	14	1001
7	0111	15	0100

Thus, the output sequence= 001000111101011...

- (b) PN sequence length =  $N = 15$

- (c)  $f_c = 10 \text{ MHz} \Rightarrow T_c = 10^{-7} \text{ s}$ . With  $T_{PN \text{ sequence}} = NT_c = 15 \times 10^{-7} = 1500 \text{ ns}$

- (d) Autocorrelation function,  $R_{[4,1],[4,1]}(\tau)$  versus  $\tau$



15.3-2

- (a) Shift register with [4,2] configuration, and initial state of 0100 has the following contents after each clock pulse:

<u>Clock shift</u>	<u>Register contents</u>
0	0100
1	1010
2	0101
3	0010
4	0001
5	1000
6	0100

The output sequence: 001010...

- (b) PN sequence length =  $N = 6$

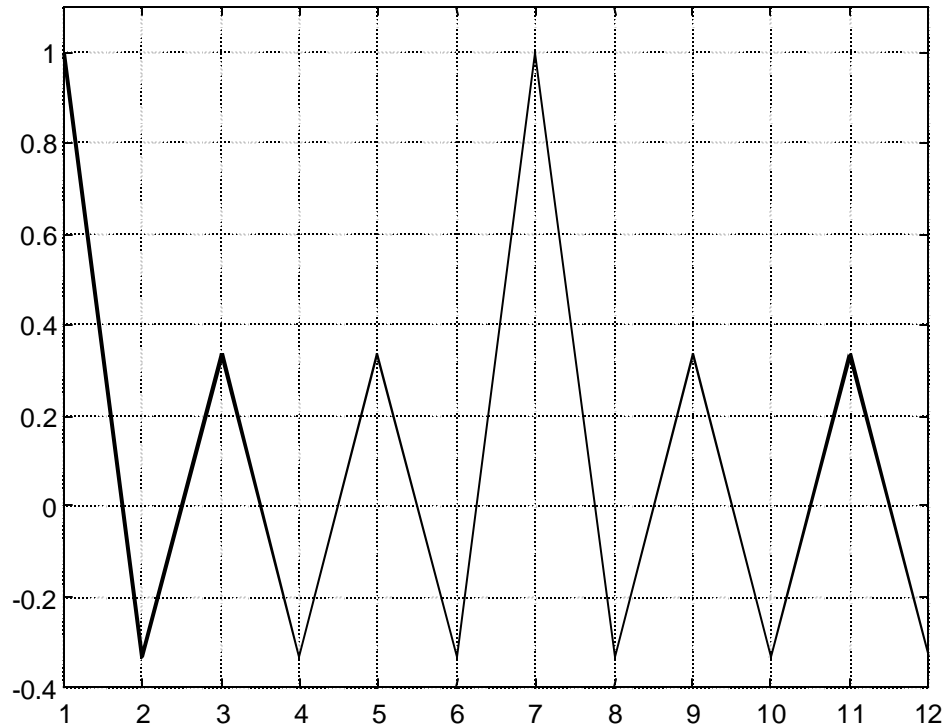
- (c)  $f_c = 10 \text{ MHz} \Rightarrow T_c = 10^{-7} \text{ s}$ . With  $T_{PN \text{ sequence}} = NT_c = 6 \times 10^{-7} = 600 \text{ ns}$

- (d) To calculate the autocorrelation function, we use the method of Example 11.4-1 to get:

$\tau$	original/shifted	$v(\tau)$	$R_{[4,2][4,2]}(\tau) = v(\tau) / N$
0	001010 001010	6-0=6	6/6=6
1	001010 000101	2-4=-2	-2/6=-0.33
2	001010 100010	4-2=2	0.33
3	001010 010001	2-4=-2	-0.33
4	001010 101000	4-2=2	0.33
5	001010 010100	2-4=-2	-0.33
6	001010 001010	6-0=6	1

### 15.3-2 continued

The plot of  $R_{[4,2][4,2]}(\tau)$  versus  $\tau$



### 15.3-3

Given the results of Problem 15.3-1,

- (1) Number of 1s = 8, number of 0s = 7  $\Rightarrow$  satisfies balance property.
  - (2) Length of single type of digit: 4/8 of length 1, 2/8 of length 2, 1/8 of length 3, 1/8 of length 4  $\Rightarrow$  satisfies run property.
  - (3) Single autocorrelation peak  $\Rightarrow$  satisfies autocorrelation property.
  - (4) Mod 2 addition of the output with a shifted version results in another shifted version
  - (5) All 15 states exist.
- $\Rightarrow$  A [4,1] register produces a ml sequence.

### 15.3-4

A shift register with [5,4] configuration, and initial state of 11111 has the following contents after each clock pulse:

### 15.3-4 continued

<u>Clock pulse</u>	<u>register contents</u>	<u>clock pulse</u>	<u>register contents</u>
0	11111	10	11000
1	01111	11	01100
2	00111	12	00110
3	00011	13	10011
4	00001	14	01001
5	10000	15	10100
6	01000	16	01010
7	00100	17	10101
8	00010	18	11010
9	10001	19	11101
10	11000	20	11110
		21	11111

The output sequence is thus: 1111100001000110010101  $\Rightarrow N = 21$

To be a ml sequence, the PN length should be  $N = 2^n - 1 = 2^5 - 1 = 31$

$21 \leq 31 \Rightarrow$  the shift register does not produce a ml sequence

### 15.3-5

- (a) From Ex. 11.4-1 and Sect. 11.4 we get the output sequences from [5,4,3,2] and [5,2] configurations to obtain  $[5,2] \oplus [5,4,3,2] =$

$$\begin{array}{r}
 1111100110100100001010111011000 \\
 \oplus \quad 1111100100110000101101010001110 \\
 \hline
 0000000010010100100111101010110 = \text{output sequence}
 \end{array}$$

$$(b) \quad |R_{st}| = \left( \frac{2^{\frac{n+1}{2}} + 1}{N} \right) = (2^3 - 1) / 31 = 0.29$$

### 15.3-6

0.01 miles  $\times$  1610 meters/mile = 16.1 meters

From Eq. (7) we have  $T_c = \Delta d / c = 16.1 / 3 \times 10^8 = 53.7 \text{ ns}$

$\Rightarrow f_c = 1/T_c = 1/53.8 \times 10^{-9} = 18.6 \text{ MHz}$

15.4-1

With 5 Hz/hour drift  $\Rightarrow$  chip uncertainty/day = 5 chips/hour x 24 hour/day = 120 chips

15.4-2

$f_c = 900$  MHz,  $f_{clock} = 10$  MHz, and  $v = 500$  Mph, and light speed =  $c = 3 \times 10^8$  M/s

$$\begin{aligned} \text{(a) Doppler shift} = \Delta f_c &= \pm \frac{vf_c}{c} = \frac{500 \text{ Mph} \times 1610 \text{ M/mile} \times 1 \text{ hour}/3600 \text{ s} \times 900 \times 10^6 \text{ s}^{-1}}{3 \times 10^8 \text{ M/s}} \\ &= 671 \text{ Hz} \end{aligned}$$

$$\begin{aligned} \text{(b) Doppler shift} = \Delta f_{clock} &= \pm \frac{vf_{clock}}{c} = \frac{500 \text{ Mph} \times 1610 \text{ M/mile} \times 1 \text{ hour}/3600 \text{ s} \times 10 \times 10^6 \text{ s}^{-1}}{3 \times 10^8 \text{ M/s}} \\ &= 7.45 \text{ chips} \end{aligned}$$

15.4-3

(a) Using Eq. (2) with a preamble of  $l = 2047$ ,  $T_c = 1/f_{clock} = 1 \times 10^{-7}$  s  $\alpha=100$ ,  $P_D = 0.9$ ,  $P_{FA} = 0.01$ , and assuming that the average phase error is 2048/2 chips we have

$$\begin{aligned} \overline{T_{acq}} &= \frac{2-P_D}{P_D} (1 + \alpha P_{FA}) N_c l T_c = \frac{2-0.9}{0.9} (1+100 \times 0.01) \times 1024 \times 2047 \times 10^{-7} \\ &= 0.51 \end{aligned}$$

(b) Using Eq. (3) we have

$$\sigma_{T_{acq}}^2 = (2 \times 1024 \times 2047 \times 10^{-7})^2 \times (1+100 \times 0.01)^2 \left( \frac{1}{12} + \frac{1}{0.9^2} - \frac{1}{0.9} \right) = 0.15$$

$$\sigma_{T_{acq}} = 0.38$$

15.4-4

(a) 12 stage shift register  $\Rightarrow l = 4095$ , then using Eq. (2) with  $T_c = 1/f_{clock} = 2 \times 10^{-8}$  s  $\alpha=10$ ,  $P_D = 0.9$ ,  $P_{FA} = 0.001$ , and assuming that the average phase error is 4096/2 chips we have

$$\begin{aligned} \overline{T_{acq}} &= \frac{2-P_D}{P_D} (1 + \alpha P_{FA}) N_c l T_c = \frac{2-0.9}{0.9} (1+10 \times 0.001) \times 2048 \times 4097 \times 2 \times 10^{-8} \\ &= 0.21 \end{aligned}$$

(b) Using Eq. (3) we have

$$\sigma_{T_{acq}}^2 = (2 \times 2048 \times 4095 \times 2 \times 10^{-8})^2 \times (1+10 \times 0.001)^2 \left( \frac{1}{12} + \frac{1}{0.9^2} - \frac{1}{0.9} \right) = 0.024$$

$$\sigma_{T_{acq}} = 0.15$$

## Chapter 16

### 16.1-1

$P(\text{not } F) = 4/5 \Rightarrow I = \log 5/4 = 0.322 \text{ bits}$ ,  $P(\text{specific grade}) = 1/5 \Rightarrow I = \log 5 = 2.322 \text{ bits}$

so  $I_{\text{needed}} = 2.322 - 0.322 = 2 \text{ bits}$

### 16.1-2

(a)  $P(\text{heart}) = 13/52 = 1/4 \Rightarrow I = \log 4 = 2 \text{ bits}$ ,  $P(\text{face card}) = 12/52 = 3/13 \Rightarrow$

$I = \log 13/3 = 2.12 \text{ bits}$ ,  $I_{\text{heart face card}} = 2 + 2.12 = 4.12 \text{ bits}$

(b)  $P(\text{red face card}) = 6/52 \Rightarrow I_{\text{given}} = \log 52/6 = 3.12 \text{ bits}$ ,  $P(\text{specific card}) = 1/52 \Rightarrow$

$I = \log 52$ ,  $I_{\text{needed}} = \log 52 - 3.12 = 2.58 \text{ bits}$

### 16.1-3

Including the direction of the first turn, the number of different combinations is

$2 \times 10^2 \times 10^2 \times 10^2$ , assumed to be equally likely. Thus,  $I = \log (2 \times 10^6) = 20.9 \text{ bits}$

### 16.1-4

$H(X) = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{\ln 2} \left( 2 \times \frac{1}{20} \ln 20 + \frac{1}{40} \ln 40 \right) = 1.94 \text{ bits}$ ,  $6H(X) = 11.64$

$P(ABABBA) = \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{4}\right)^3 = \frac{1}{2^9} \Rightarrow I = \log 2^9 = 9 \text{ bits} < 6H(X)$

$P(FDDFDF) = \left(\frac{1}{40}\right)^3 \times \left(\frac{1}{20}\right)^3 = \frac{1}{512 \times 10^6} \Rightarrow I = \log (512 \times 10^6) = 28.93 \text{ bits} > 6H(X)$

### 16.1-5

$H(X) = \frac{1}{\ln 2} \left( 0.4 \ln \frac{1}{0.4} + 0.2 \ln \frac{1}{0.2} + 0.12 \ln \frac{1}{0.12} + 2 \times 0.1 \ln \frac{1}{0.1} + 0.08 \ln \frac{1}{0.08} \right) = 2.32 \text{ bits}$ ,

$6H(X) = 13.90 \text{ bits}$

$P(ABABBA) = (0.4)^3 (0.2)^3 = 5.12 \times 10^{-4} \Rightarrow I = \log \frac{1}{5.12 \times 10^{-4}} = 10.93 \text{ bits} < 6H(X)$

$P(FDDFDF) = (0.1)^3 (0.08)^3 = 5.12 \times 10^{-7} \Rightarrow I = \log \frac{1}{5.12 \times 10^{-7}} = 20.90 \text{ bits} > 6H(X)$

16.1-6

Since the first symbol is always the same, there are  $M = 8 \times 8 = 64$  different blocks, and  $H(X) = \log 64 = 6$  bits/block. Thus,  $R = 1000 \text{ blocks/sec} \times 6 \text{ bits/block} = 6000 \text{ bits/sec}$

16.1-7

There are  $M = 16^{15}$  different blocks, and  $H(X) = \log 16^{15} = 60$  bits/block at the rate  $r = 1/(15 + 5)\text{ms} = 50 \text{ blocks/sec}$ , so  $R = 50 \times 60 = 3000 \text{ bits/sec}$

16.1-8

$P_{dot} + P_{dash} = \frac{3}{2}P_{dot} = 1 \Rightarrow P_{dot} = \frac{2}{3}, P_{dash} = \frac{1}{3}$  so  $H(X) = \frac{2}{3}\log \frac{3}{2} + \frac{1}{3}\log 3 = 0.920 \text{ bits/symbol}$

$T_{dash} = 2T_{dot} = 0.4, \frac{1}{r} = \bar{T} = \frac{2}{3} \times 0.2 + \frac{1}{3} \times 0.4 = \frac{0.8}{3} \text{ sec/symbol}$

Thus,  $R = (3/0.8) \times 0.920 = 3.45 \text{ bits/sec}$

16.1-9

$P_3 = 1 - (P_1 + P_2) = \frac{2}{3} - p \Rightarrow H(X) = \frac{1}{3}\log 3 + p\log \frac{1}{p} + (\frac{2}{3} - p)\log \frac{1}{(\frac{2}{3} - p)}$

When  $p = 0$  or  $2/3$ ,  $H(X) = \frac{1}{3}\log 3 + \frac{2}{3}\log \frac{3}{2} = 0.918 \text{ bits}$

16.1-10

Let  $P_1 = \alpha$  so  $P_i = (1 - \alpha)/(M - 1)$  for  $i = 2, 3, \dots, M$

$$\begin{aligned} H(X) &= \alpha \log \frac{1}{\alpha} + (M - 1) \frac{1 - \alpha}{M - 1} \log \frac{M - 1}{1 - \alpha} = \alpha \log \frac{1}{\alpha} + (1 - \alpha) [\log(M - 1) - \log(1 - \alpha)] \\ &= \log(M - 1) + \alpha \left[ \log \frac{1}{\alpha} - \log(M - 1) \right] - (1 - \alpha) \log(1 - \alpha) \end{aligned}$$

But  $\log(1 - \alpha) = \frac{1}{\ln 2} \ln(1 - \alpha) = \frac{1}{\ln 2} \left( -\alpha - \frac{1}{2}\alpha^2 - \frac{1}{3}\alpha^3 - \dots \right)$  so

$-(1 - \alpha) \log(1 - \alpha) \approx \frac{(1 - \alpha)\alpha}{\ln 2} \approx \frac{\alpha}{\ln 2} = \alpha \frac{\ln e}{\ln 2} = \alpha \log e$ . Thus,

$$\begin{aligned} H(X) &\approx \log(M - 1) + \alpha \left[ \log \frac{1}{\alpha} - \log(M - 1) + \log e \right] \\ &\approx \log(M - 1) + \alpha \log \frac{1}{\alpha} \text{ if } \frac{1}{\alpha} \square M - 1 \text{ and } \frac{1}{\alpha} \square e \end{aligned}$$



16.1-11

$$-\log P_i \odot N_i < -\log P_i + 1 \Rightarrow 2^{\log P_i} \geq 2^{-N_i} > 2^{\log P_i} 2^{-1} \text{ so } P_i \geq 2^{-N_i} > \frac{1}{2} P_i$$

$$\text{Thus, } \sum_{i=1}^M P_i \geq \sum_{i=1}^M 2^{-N_i} > \frac{1}{2} \sum_{i=1}^M P_i \text{ where } \sum_{i=1}^M P_i = 1. \text{ Hence, } 1 \geq \sum_{i=1}^M 2^{-N_i} > \frac{1}{2} \Rightarrow \frac{1}{2} < K \leq 1$$

16.1-12

$x_i$	$P_i$	1	2	3	4	5	Codewords	$P_i N_i$
A	1/2	0					0	0.5
B	1/4	1	0				10	0.5
C	1/8	1	1	0			110	0.375
D	1/20	1	1	1	0		1110	0.2
E	1/40	1	1	1	1	0	11110	0.25
F	1/40	1	1	1	1	1	11111	0.125

$H(X) = 1.940$  bits from Prob. 16.1-4, and  $\bar{N} = 1.950$ , so  $H(X)/\bar{N} = 1.940/1.950 = 99.5\%$

16.1-13

Since  $P_A = 0.4$  and  $P_A + P_B = 0.6$ , the dividing line at the first coding step can be between A and B or between B and C.

$x_i$	$P_i$					Code I	$P_i N_i$				Code II	$P_i N_i$
A	0.4	0				0	0.4	0	0		00	0.8
B	0.2	1	0	0		100	0.6	0	1		01	0.4
C	0.12	1	0	1		101	0.36	1	0	0	100	0.36
D	0.1	1	1	0		110	0.3	1	0	1	101	0.3
E	0.1	1	1	1	0	1110	0.4	1	1	0	110	0.3
F	0.08	1	1	1	1	1111	0.32	1	1	1	111	0.24

$H(X) = 2.32$  bits from Prob. 16.1-5, and  $\bar{N} \approx 2.4$ , so  $H(X)/\bar{N} \approx 2.32/2.4 \approx 97\%$

16.1-14

$$H(X) = 0.5 + \frac{1}{\ln 2} \left( 0.4 \ln \frac{1}{0.4} + 0.1 \ln \frac{1}{0.1} \right) = 1.36 \text{ bits}$$

(a)  $\bar{N} = 1.5$  so  $H(X)/\bar{N} = 1.36/1.5 = 90.5\%$

$x_i$	$P_i$			Code	$P_i N_i$
A	0.5	0		0	0.5
B	0.4	1	0	10	0.8

C	0.1	1	1	11	0.2
---	-----	---	---	----	-----

(b)  $2\bar{N} = 2.78$  so  $H(X)/\bar{N} = 1.36/1.39 \approx 97.8\%$

(cont.)

$x_{ij}$	$P_{ij}$							Codewords	$P_{ij}N_{ij}$
AA	0.25	0	0					00	0.5
AB	0.2	0	1					01	0.4
BA	0.2	1	0					10	0.4
BB	0.16	1	1	0				110	0.48
AC	0.05	1	1	1	0	0		11100	0.25
CA	0.05	1	1	1	0	1		11101	0.25
BC	0.04	1	1	1	1	0		11110	0.20
CB	0.04	1	1	1	1	1	0	111110	0.24
CC	0.01	1	1	1	1	1	1	111111	0.06

16.1-15

$$H(X) = \frac{1}{\ln 2} \left( 0.8 \ln \frac{1}{0.8} + 0.2 \ln \frac{1}{0.2} \right) = 0.7219 \text{ bits}$$

(a)  $2\bar{N} = 1.56$  so  $H(X)/\bar{N} = 0.7219/0.78 = 92.6\%$

$x_{ij}$	$P_{ij}$				Codeword	$P_{ij}N_{ij}$
AA	0.64	0			0	0.64
AB	0.16	1	0		10	0.32
BA	0.16	1	1	0	110	0.48
BB	0.04	1	1	1	111	0.12

(b)  $3\bar{N} = 2.184$  so  $H(X)/\bar{N} = 0.7219/0.728 = 99.2\%$

$x_{ijk}$	$P_{ijk}$						Codeword	$P_{ijk}N_{ijk}$
AAA	0.512	0					0	0.512
AAB	0.128	1	0	0			100	0.384
ABA	0.128	1	0	1			101	0.384
BAA	0.128	1	1	0			110	0.384

<i>ABB</i>	0.032	1	1	1	0	0	11100	0.160
<i>BAB</i>	0.032	1	1	1	0	1	11101	0.160
<i>BBA</i>	0.032	1	1	1	1	0	11110	0.160
<i>BBB</i>	0.008	1	1	1	1	1	11111	0.040

16.1-16

$$H(X) = P_0 H(X|0) + P_1 H(X|1) = \frac{1}{2} [H(X|0) + H(X|1)]$$

$$P_{01} = P(0|1) = 3/4 \Rightarrow P_{11} = 1 - P(0|1) = 1/4$$

$$P_{10} = P(1|0) = 3/4 \Rightarrow P_{00} = 1 - P(1|0) = 1/4$$

Thus,  $H(X|0) = H(X|1) = \frac{3}{4} \log \frac{4}{3} + \frac{1}{4} \log 4 = 0.811$  and  $H(X) = 2 \times \frac{1}{2} 0.811 = 0.811$  bits

16.2-1

$$P(x_i y_j) = P(x_i | y_j) P(y_j) \text{ and } \sum_x P(x_i y_j) = P(y_j) \text{ so } H(X, Y) = \sum_{x,y} P(x_i y_j) \log \frac{1}{P(x_i | y_j) P(y_j)}$$

$$= \sum_y P(y_j) \log \frac{1}{P(y_j)} + \sum_{x,y} P(x_i y_j) \log \frac{1}{P(x_i | y_j)} = H(Y) + H(X|Y)$$

16.2-2

$$P(x_i | y_j) = P(x_i y_j) / P(y_j) \text{ so}$$

$$I(X; Y) = \sum_{x,y} P(x_i y_j) \log \frac{P(x_i y_j)}{P(x_i) P(y_j)} = \sum_{x,y} P(x_i y_j) \left[ \log \frac{1}{P(x_i)} + \log \frac{1}{P(y_j)} - \log \frac{1}{P(x_i y_j)} \right]$$

$$\text{where } \sum_{x,y} P(x_i y_j) \log \frac{1}{P(x_i)} = \sum_x \underbrace{\left[ \sum_y P(x_i y_j) \right]}_{P(x_i)} \log \frac{1}{P(x_i)} = H(X)$$

$$\text{and likewise } \sum_{x,y} P(x_i y_j) \log \frac{1}{P(y_j)} = H(Y)$$

$$\text{Thus, } I(X; Y) = H(X) + H(Y) - \sum_{x,y} P(x_i y_j) \log \frac{1}{P(x_i y_j)} = H(X) + H(Y) - H(X, Y)$$

16.2-3

$$P(x_i y_j) = P(y_j | x_i) P(x_i) = \begin{cases} P(x_i) & j=i \\ 0 & j \neq i \end{cases} \Rightarrow P(y_j) = \sum_{i=1}^M P(x_i y_j) = P(x_j).$$

(cont.)

$$\text{Thus, } H(Y|X) = \sum_i \sum_j P(x_i, y_j) \log \frac{1}{P(y_j|x_i)} = \sum_i P(x_i) \log \frac{1}{P(y_i|x_i)} = \sum_i P(x_i) \log 1 = 0$$

$$I(X;Y) = H(Y) - H(Y|X) = H(X) - 0 = H(X)$$

16.2-4

$$P(y_1) = p(1 - \alpha) + (1 - p)\beta = \beta + (1 - \alpha - \beta)p, \quad P(y_2) = (1 - p)(1 - \beta) + p\alpha = 1 - \beta - (1 - \alpha - \beta)p$$

$$= 1 - P(y_1) \text{ so } H(Y) = P(y_1) \log \frac{1}{P(y_1)} + P(y_2) \log \frac{1}{P(y_2)} = \Omega[\beta + (1 - \alpha - \beta)p]$$

$$H(Y|X) = p \left[ (1 - \alpha) \log \frac{1}{1 - \alpha} + \alpha \log \frac{1}{\alpha} \right] + (1 - p) \left[ \beta \log \frac{1}{\beta} + (1 - \beta) \log \frac{1}{1 - \beta} \right]$$

$$= p\Omega(\alpha) + (1 - p)\Omega(\beta) \text{ so}$$

$$I(X;Y) = H(Y) - H(Y|X) = \Omega[\beta + (1 - \alpha - \beta)p] - p\Omega(\alpha) - (1 - p)\Omega(\beta)$$

16.2-5

If  $\beta = 1 - \alpha$ , then  $P(y_1|x_1) = P(y_1|x_2)$  and  $P(y_2|x_1) = P(y_2|x_2)$  so the occurrence of  $y_1$  or  $y_2$  gives no information about the source. Analytically, if  $\beta = 1 - \alpha$ , then  $\Omega(\beta) = \Omega(1 - \alpha)$  and  $\Omega[\beta + (1 - \alpha - \beta)p] = \Omega(1 - \alpha)$ . Thus,  $I(X;Y) = \Omega(\alpha) - p\Omega(\alpha) - (1 - p)\Omega(\alpha) = 0$  so no information is transferred.

16.2-6

$$\Omega(a + bp) = (a + bp) \log \frac{1}{a + bp} + (1 - a - bp) \log \frac{1}{1 - a - bp}$$

$$= -[(a + bp) \log(a + bp) + (1 - a - bp) \log(1 - a - bp)] \text{ and } \frac{d}{dp} [\log_2 x(p)] = (\log_2 e) \frac{1}{x} \frac{dx}{dp}$$

$$\text{so } \frac{d}{dp} \Omega(a + bp) = -b \log(a + bp) - (a + bp)(\log e) \frac{b}{a + bp} - (-b) \log(1 - a - bp)$$

$$- (1 - a - bp)(\log e) \frac{-b}{1 - a - bp}$$

$$= -b [\log(a + bp) - \log(1 - a - bp)] = b \log \frac{1 - a - bp}{a + bp} \quad (\text{cont.})$$

$$\frac{d}{dp} I(X; Y) = \frac{d}{dp} \Omega[\alpha + (1-2\alpha)p] + 0 = (1-2\alpha) \log \frac{1-\alpha-(1-2\alpha)p}{\alpha+(1-2\alpha)p} = 0 \text{ so}$$

$$\frac{1-\alpha-(1-2\alpha)p}{\alpha+(1-2\alpha)p} = 1 \Rightarrow 1-2\alpha = 2(1-2\alpha)p \Rightarrow p = \frac{1}{2}$$

16.2-7

$$I(X; Y) = \Omega(p/2) - p\Omega(1/2) - (1-p)\Omega(0) = \Omega(p/2) - p \text{ since } \Omega(1/2)=1 \text{ and } \Omega(0)=0$$

$$\text{and } \frac{d}{dp} \Omega\left(0 + \frac{1}{2}p\right) = \frac{1}{2} \log \frac{1-\frac{1}{2}p}{\frac{1}{2}p} = \frac{1}{2} \log \frac{2-p}{p} \text{ so}$$

$$\frac{d}{dp} I(X; Y) = \frac{1}{2} \log \frac{2-p}{p} - 1 = 0 \Rightarrow \frac{2-p}{p} = 2^2 \Rightarrow p = \frac{2}{5}$$

$$\text{Thus, } C_s = \Omega\left(\frac{1}{5}\right) - \frac{2}{5} = \frac{1}{5} \log 5 + \frac{4}{5} \log \frac{5}{4} - \frac{2}{5} = 0.322 \text{ bits/symbol}$$

16.2-8

$$I(X; Y) = \Omega(3p/4) - p\Omega(1/4) - (1-p)\Omega(0) = \Omega(3p/4) - 0.811p \text{ since } \Omega(0)=0$$

$$\text{and } \frac{d}{dp} \Omega\left(0 + \frac{3}{4}p\right) = \frac{3}{4} \log \frac{1-\frac{3}{4}p}{\frac{3}{4}p} = \frac{3}{4} \log \frac{4-3p}{3p} \text{ so}$$

$$\frac{d}{dp} I(X; Y) = \frac{3}{4} \log \frac{4-3p}{3p} - 0.811 = 0 \Rightarrow \frac{4-3p}{3p} = 2^{(4/3 \cdot 0.811)} = 2.12 \Rightarrow p = 0.428$$

$$\text{Thus, } C_s = \Omega(0.321) - 0.811 \times 0.428 = 0.557 \text{ bits/symbol}$$

16.2-9

$$P(0) = P(1) = \frac{1}{2}(1-\alpha), P(E) = \frac{1}{2}\alpha + \frac{1}{2}\alpha = \alpha$$

$$H(Y) = 2 \frac{1-\alpha}{2} \log \frac{2}{1-\alpha} + \alpha \log \frac{1}{\alpha} = (1-\alpha) \log 2 + (1-\alpha) \log \frac{1}{1-\alpha} + \alpha \log \frac{1}{\alpha} = 1 - \alpha + \Omega(\alpha)$$

$$H(Y|X) = 2 \times \frac{1}{2} \left[ (1-\alpha) \log \frac{1}{1-\alpha} + \alpha \log \frac{1}{\alpha} + 0 \log \frac{1}{0} \right] = \Omega(\alpha)$$

$$\text{Thus, } C_s = \max I(X; Y) = H(Y) - H(Y|X) = 1 - \alpha$$

16.2-10

$$P(|n - \bar{n}| \geq k\sigma) = P(n - \bar{n} \geq k\sigma) + P(n - \bar{n} \leq -k\sigma) \leq 1/k^2 \text{ so}$$

$$P(n \geq \bar{n} + k\sigma) \leq \frac{1}{k^2} - P(n - \bar{n} \leq -k\sigma) \leq 1/k^2$$

Let  $d = \bar{n} + k\sigma \Rightarrow k = \frac{d - \bar{n}}{\sigma}$  where  $d = N\beta, \bar{n} = N\alpha, \sigma^2 = N\alpha(1-\alpha)$

$$\text{Then } P(n \geq d) \leq \left( \frac{\sigma}{d - \bar{n}} \right)^2 = \frac{N\alpha(1-\alpha)}{(N\beta - N\alpha)^2} = \frac{\alpha(1-\alpha)}{N(\beta - \alpha)^2}$$

16.3-1

$$p(x) = 1/2a \quad \text{for } |x| \leq a, \quad S = \int_{-\infty}^{\infty} x^2 p(x) dx = a^2/3$$

$$H(X) = \int_{-a}^a \frac{1}{2a} \log 2a dx = \log 2a = \log \sqrt{12S} = \frac{1}{2} \log 12S < \frac{1}{2} \log 2\pi eS \quad \text{since } 2\pi e = 17.08 > 12$$

16.3-2

$$p(x) = \frac{\alpha}{2} e^{-\alpha|x|} \quad \text{and } S = \frac{2}{\alpha^2}. \quad \text{For } x \rightarrow 0, \log \frac{1}{p(x)} = \log \frac{2}{\alpha} + \alpha x \log e, \text{ so}$$

$$\begin{aligned} H(X) &= 2 \left[ \int_0^{\infty} \frac{\alpha}{2} e^{-\alpha x} \log \frac{2}{\alpha} dx + \int_0^{\infty} \frac{\alpha}{2} e^{-\alpha x} \alpha x \log e dx \right] = \alpha \left( \log \frac{2}{\alpha} \right) \frac{1}{\alpha} + \alpha^2 (\log e) \frac{1}{\alpha^2} \\ &= \log \frac{2}{\alpha} + \log e = \log \frac{2e}{\alpha} = \log \sqrt{2e^2 S} = \frac{1}{2} \log 2e^2 S < \frac{1}{2} \log 2\pi eS \quad \text{since } \pi > e \end{aligned}$$

16.3-3

$$S = \int_0^{\infty} ax^2 e^{-ax} dx = \frac{2}{a^2}, \quad \log \frac{1}{p(x)} = ax \log e - \log a \quad \text{for } x \rightarrow 0$$

$$\begin{aligned} H(X) &= \int_0^{\infty} ae^{-ax} (ax \log e) dx - \int_0^{\infty} ae^{-ax} (\log a) dx = a^2 (\log e) \frac{1}{a^2} - \log a = \log \frac{e}{a} \\ &= \log \sqrt{e^2 S / 2} = \frac{1}{2} \log e^2 S / 2 < \frac{1}{2} \log 2\pi eS \quad \text{since } 2\pi > e/2 \end{aligned}$$

16.3-4

$$p_Z(z) = \frac{1}{|a|} p_X \left( \frac{z-b}{a} \right) \quad (\text{cont.})$$

$$\begin{aligned}
H(Z) &= \int_{-\infty}^{\infty} \frac{1}{|a|} p_X \left( \frac{z-b}{a} \right) \left[ \log |a| + \log \frac{1}{p_X \left( \frac{z-b}{a} \right)} \right] dz \\
&= \log |a| \int_{-\infty}^{\infty} p_X \left( \frac{z-b}{a} \right) \frac{dz}{|a|} + \int_{-\infty}^{\infty} p_X \left( \frac{z-b}{a} \right) \log \frac{1}{p_X \left( \frac{z-b}{a} \right)} \frac{dz}{|a|} \\
&= \log |a| \int_{-\infty}^{\infty} p_X(\lambda) d\lambda + \int_{-\infty}^{\infty} p_X(\lambda) \log \frac{1}{p_X(\lambda)} d\lambda = \log |a| + H(X)
\end{aligned}$$

16.3-5

$$\int_0^{\infty} p(x) dx = 1 \Rightarrow F_1 = p, c_1 = 1, \frac{\partial F_1}{\partial p} = 1 \text{ and } \int_0^{\infty} xp(x) dx = m \Rightarrow F_2 = xp, c_2 = m, \frac{\partial F_2}{\partial p} = x$$

$$\text{Thus, } -\frac{\ln p + 1}{\ln 2} + \lambda_1 + \lambda_2 x = 0 \Rightarrow p = e^{(\lambda_1 \ln 2 - 1)} e^{(\lambda_2 \ln 2)x} = Ke^{-ax} \quad x \geq 0$$

$$\int_0^{\infty} Ke^{-ax} dx = K/a = 1, \int_0^{\infty} xKe^{-ax} dx = K/a^2 = m \Rightarrow K = a = 1/m$$

$$\text{Hence, } p(x) = \frac{1}{m} e^{-x/m} u(x) \text{ and } H(X) = \int_0^{\infty} p(x) \left[ \log m + \frac{x}{m} \log e \right] dx = \log m + \log e = \log em$$

16.3-6

$$\int_0^{\infty} p(x) dx = 1 \Rightarrow F_1 = p, c_1 = 1, \frac{\partial F_1}{\partial p} = 1 \text{ and } \int_0^{\infty} x^2 p(x) dx = S \Rightarrow F_2 = x^2 p, c_2 = S, \frac{\partial F_2}{\partial p} = x^2$$

$$\text{Thus, } -\frac{\ln p + 1}{\ln 2} + \lambda_1 + \lambda_2 x^2 = 0 \Rightarrow p = e^{(\lambda_1 \ln 2 - 1)} e^{(\lambda_2 \ln 2)x^2} = Ke^{-ax^2} \quad x \geq 0$$

$$\int_0^{\infty} Ke^{-ax^2} dx = \frac{K}{2} \sqrt{\frac{\pi}{a}} = 1, \int_0^{\infty} x^2 Ke^{-ax^2} dx = \frac{K}{a\sqrt{a}} \frac{\sqrt{\pi}}{4} = S \Rightarrow K = \frac{2}{\sqrt{2\pi S}}, a = \frac{1}{2S}$$

$$\text{Hence, } p(x) = \frac{2}{\sqrt{2\pi S}} e^{-x^2/2S} u(x) \text{ and}$$

$$H(X) = \int_0^{\infty} p(x) \left[ \log \sqrt{\frac{\pi S}{2}} + \frac{x^2}{2S} \log e \right] dx = \log \sqrt{\frac{\pi S}{2}} + \frac{\log e}{2S} S = \frac{1}{2} \log \frac{\pi S}{2} + \frac{1}{2} \log e = \frac{1}{2} \log \frac{\pi e S}{2}$$

16.3-7

$$I(X; Y) = - \int \int_{-\infty}^{\infty} p_{XY}(x, y) \log \frac{p_X(x)p_Y(y)}{p_{XY}(x, y)} dx dy \text{ and}$$

$$\log \frac{p_X(x)p_Y(y)}{p_{XY}(x, y)} = \frac{1}{\ln 2} \ln \frac{p_X(x)p_Y(y)}{p_{XY}(x, y)} \leq \frac{1}{\ln 2} \left[ \frac{p_X(x)p_Y(y)}{p_{XY}(x, y)} - 1 \right]$$

$$\begin{aligned} \text{Thus, } I(X; Y) &\geq \frac{1}{\ln 2} \int \int_{-\infty}^{\infty} p_{XY}(x, y) \left[ 1 - \frac{p_X(x)p_Y(y)}{p_{XY}(x, y)} \right] dx dy \\ &= \frac{1}{\ln 2} \left[ \int \int_{-\infty}^{\infty} p_{XY}(x, y) dx dy - \int \int_{-\infty}^{\infty} p_X(x)p_Y(y) dx dy \right] \end{aligned}$$

$$\text{where } \int \int_{-\infty}^{\infty} p_{XY}(x, y) dx dy = 1, \int \int_{-\infty}^{\infty} p_X(x)p_Y(y) dx dy = \int_{-\infty}^{\infty} p_X(x) dx \int_{-\infty}^{\infty} p_Y(y) dy = 1$$

Hence,  $I(X; Y) \geq 0$

16.3-8

$$R \leq C = B \log \left( 1 + \frac{S}{N_0 B} \right) = \frac{B}{\ln 2} \ln \left( 1 + \frac{10^4}{B} \right)$$

$$B = 10^3 \Rightarrow R \leq \frac{10^3}{\ln 2} \ln 1.1 = 3459 \text{ bits/sec}$$

$$B = 10^4 \Rightarrow R \leq 10^4 \log_2 2 = 10,000 \text{ bits/sec}$$

$$B = 10^5 \Rightarrow R \leq \frac{10^5}{\ln 2} \ln 1.1 = 13,750 \text{ bits/sec}$$

16.3-9

$$R \leq C = 3000 \log(1 + S/N) \Rightarrow S/N \geq 2^{R/3000} - 1$$

$$R = 2400 \Rightarrow S/N \geq 2^{0.8} - 1 = 0.741 \approx -1.3 \text{ dB}$$

$$R = 4800 \Rightarrow S/N \geq 2^{1.6} - 1 = 2.03 \approx 3.1 \text{ dB}$$

$$R = 9600 \Rightarrow S/N \geq 2^{3.2} - 1 = 8.19 \approx 9.1 \text{ dB}$$



16.3-10

$$\frac{S}{N_0 R} \geq \frac{B}{R} (2^{R/B} - 1), \quad N_0 B = 10^{-6} \times 10^3 = 10^{-3}, \quad S \geq 10^{-3} (2^{R/1000} - 1)$$

$$R = 100 \Rightarrow S \geq 10^{-3} (2^{0.1} - 1) = 0.072 \text{ mW}$$

$$R = 1000 \Rightarrow S \geq 10^{-3} (2 - 1) = 1 \text{ mW}$$

$$R = 10,000 \Rightarrow S \geq 10^{-3} (2^{10} - 1) = 1023 \text{ mW}$$

16.3-11

For an ideal system with the same parameters,

$$\left( \frac{S}{N} \right)_D = \left( 1 + \frac{S_R}{N_0 B_T} \right)^{B_T/W} - 1 = (1 + 3)^{10} - 1 = 60.2 \text{ dB}$$

Since the claimed performance approaches an ideal system, the claim is highly doubtful.

16.3-12

$$b = 4: \left( \frac{S}{N} \right)_D = \left( 1 + \frac{\gamma}{4} \right)^4 - 1 = 10^4 \Rightarrow \gamma \approx 36$$

$$b = 3 \times 4: \left( \frac{S}{N} \right)_D = \left( 1 + \frac{36}{12} \right)^{12} - 1 \approx 4^{12} = 72.2 \text{ dB}$$

16.3-13

$$b = 4: \left( \frac{S}{N} \right)_D = \left( 1 + \frac{\gamma}{4} \right)^4 - 1 = 10^4 \Rightarrow \gamma \approx 36$$

$$b = \frac{1}{2}: \left( \frac{S}{N} \right)_D = \left( 1 + \frac{36}{1/2} \right)^{1/2} - 1 = 7.54 = 8.8 \text{ dB}$$

16.3-14

$$b = 4, \quad L N_0 = 10^6 \times 100 \times 10^{-12} = 10^{-4}$$

$$\left( \frac{S}{N} \right)_D = \left( 1 + \frac{\gamma}{4} \right)^4 - 1 = 10^3 \Rightarrow \gamma = 4(1001^{1/4} - 1) = 18.5 \Rightarrow S_T = \gamma L N_0 W = 5.55 \text{ W}$$

16.3-15

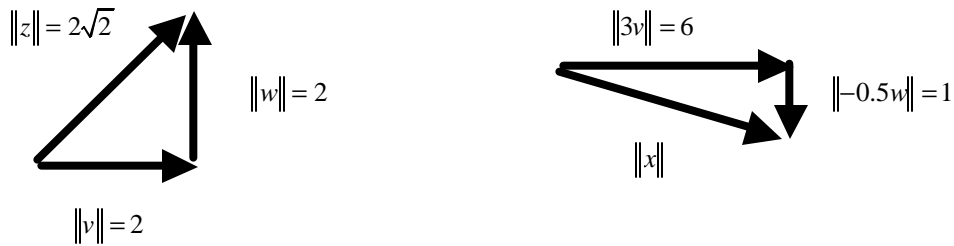
$$b = 2, \quad LN_0 = 10^6 \times 100 \times 10^{-12} = 10^{-4}$$

$$\left(\frac{S}{N}\right)_D = \left(1 + \frac{\gamma}{2}\right)^2 - 1 = 10^3 \Rightarrow \gamma = 2(\sqrt{1001} - 1) = 61.3 \Rightarrow S_T = \gamma LN_0 W = 36.8 \text{ W}$$

16.4-1

$$\|v\|^2 = E_v = 4, \quad \|w\|^2 = E_w = 4, \quad \|z\|^2 = \|v + w\|^2 = E_z = 8$$

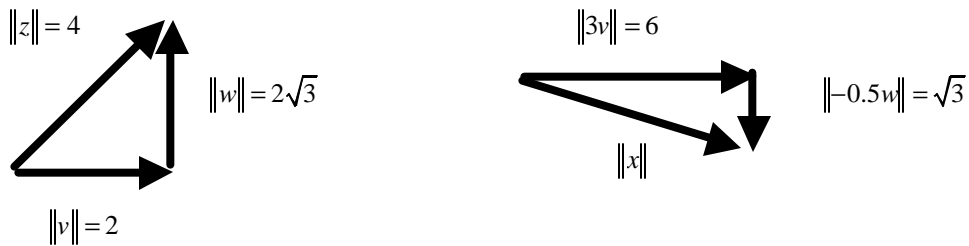
$$\|v + w\|^2 = \|v\|^2 + \|w\|^2 \Rightarrow (v, w) = 0, \quad E_x = \|x\|^2 = 6^2 + 1^2 = 37$$



16.4-2

$$\|v\|^2 = E_v = 4, \quad \|w\|^2 = E_w = 12, \quad \|z\|^2 = \|v + w\|^2 = E_z = 16$$

$$\|v + w\|^2 = \|v\|^2 + \|w\|^2 \Rightarrow (v, w) = 0, \quad E_x = \|x\|^2 = 6^2 + (\sqrt{3})^2 = 39$$



16.4-3

$$\|v + w\|^2 = \|v\|^2 + 2(v, w) + \|w\|^2 \text{ and } |(v, w)| \leq \|v\|\|w\| \text{ so}$$

$$2(v, w) \leq 2\|v\|\|w\| \Rightarrow \|v + w\|^2 \leq \|v\|^2 + 2\|v\|\|w\| + \|w\|^2 = (\|v\| + \|w\|)^2. \text{ Thus, } \|v + w\| \leq \|v\| + \|w\|$$

16.4-4

With  $v_w = \alpha w$ ,  $(v - v_w, w) = (v - \alpha w, w) = (v, w) - \alpha(w, w)$  so  $(v - v_w, w) = 0$

$$\alpha = (v, w)/(w, w) = (v, w)/\|w\|^2$$

$$\text{Hence, } v_w = \frac{(v, w)}{\|w\|^2} w \text{ and } \|v_w\|^2 = (\alpha w, \alpha w) = \alpha^2 \|w\|^2 = \left[ \frac{(v, w)}{\|w\|^2} \right]^2 \|w\|^2 = \frac{(v, w)^2}{\|w\|^2}$$

$$\text{But } (v, w)^2 \leq \|v\|^2 \|w\|^2 \text{ so } \|v_w\| \leq \left[ \|v\|^2 \|w\|^2 / \|w\|^2 \right]^{1/2} = \|v\|$$

16.4-5

$$\|s_1\|^2 = \int_{-1}^1 1 dt = 2 \Rightarrow \phi_1 = s_1 / \|s_1\| = 1/\sqrt{2} \quad |t| \leq 1$$

$$(s_2, \phi_1) = \int_{-1}^1 \frac{t}{\sqrt{2}} dt = 0 \Rightarrow g_2 = s_2 \Rightarrow \|g_2\|^2 = \int_{-1}^1 t^2 dt = \frac{2}{3} \Rightarrow \phi_2 = \frac{s_2}{\sqrt{2/3}} = \sqrt{\frac{3}{2}} t \quad |t| \leq 1$$

$$(s_3, \phi_1) = \int_{-1}^1 \frac{t^2}{\sqrt{2}} dt = \frac{\sqrt{2}}{3}, \quad (s_3, \phi_2) = \int_{-1}^1 t^2 \sqrt{\frac{3}{2}} t dt = 0 \text{ so}$$

$$g_3 = s_3 - \frac{\sqrt{2}}{3} \phi_1 = t^2 - \frac{1}{3} \text{ and } \|g_3\|^2 = \int_{-1}^1 \left(t^2 - \frac{1}{3}\right)^2 dt = \frac{8}{45} \Rightarrow \phi_3 = \sqrt{\frac{45}{8}} \left(t^2 - \frac{1}{3}\right)$$

$$\text{Thus, } s_1 = \sqrt{2} \phi_1, \quad s_2 = \sqrt{\frac{2}{3}} \phi_2, \quad s_3 = \sqrt{\frac{8}{45}} \phi_3 + \frac{1}{3} = \frac{\sqrt{2}}{3} \phi_1 + \sqrt{\frac{8}{45}} \phi_3$$

16.4-6

$$\|s_1\|^2 = \int_{-1}^1 1 dt = 2 \Rightarrow \phi_1 = s_1 / \|s_1\| = 1/\sqrt{2} \quad |t| \leq 1$$

$$(s_2, \phi_1) = \int_{-1}^1 \cos \frac{\pi t}{2} \frac{1}{\sqrt{2}} dt = \frac{2\sqrt{2}}{\pi} \Rightarrow g_2 = s_2 - \frac{2\sqrt{2}}{\pi} \frac{1}{\sqrt{2}}$$

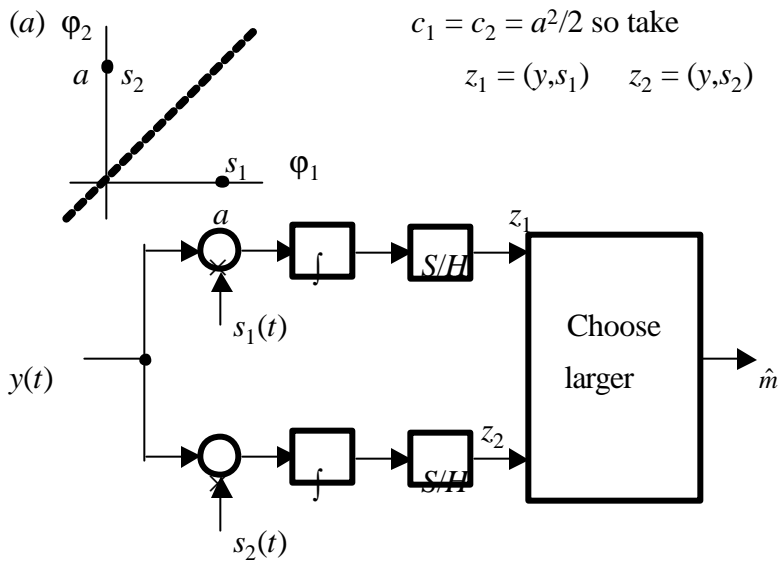
$$\|g_2\|^2 = \int_{-1}^1 \left( \cos^2 \frac{\pi t}{2} - \frac{4}{\pi} \cos \frac{\pi t}{2} + \frac{4}{\pi^2} \right) dt = 1 - \frac{8}{\pi^2} \text{ so } \phi_2 = \frac{(s_2 - \frac{2}{\pi})}{\|g_2\|} = \frac{\pi}{\sqrt{\pi^2 - 8}} \left( \cos \frac{\pi t}{2} - \frac{2}{\pi} \right) \quad |t| \leq 1$$

$$(s_3, \phi_1) = \int_{-1}^1 \sin \pi t dt = 0, \quad (s_3, \phi_2) = \int_{-1}^1 \sin \pi t \frac{\pi}{\sqrt{\pi^2 - 8}} \left( \cos \frac{\pi t}{2} - \frac{2}{\pi} \right) dt = 0, \text{ so}$$

$$g_3 = s_3, \quad \|g_3\|^2 = \int_{-1}^1 \sin^2 \pi t dt = 1 \Rightarrow \phi_3 = s_3 = \sin \pi t \quad |t| \leq 1. \text{ Thus,}$$

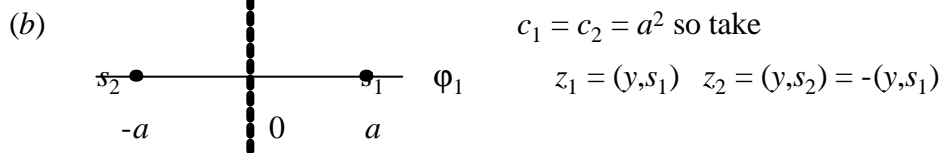
$$s_1 = \sqrt{2} \phi_1, \quad s_2 = \frac{\sqrt{\pi^2 - 8}}{\pi} \phi_2 + \frac{2}{\pi} = \frac{2\sqrt{2}}{\pi} \phi_1 + \frac{\sqrt{\pi^2 - 8}}{\pi} \phi_2, \quad s_3 = \phi_3$$

16.5-1



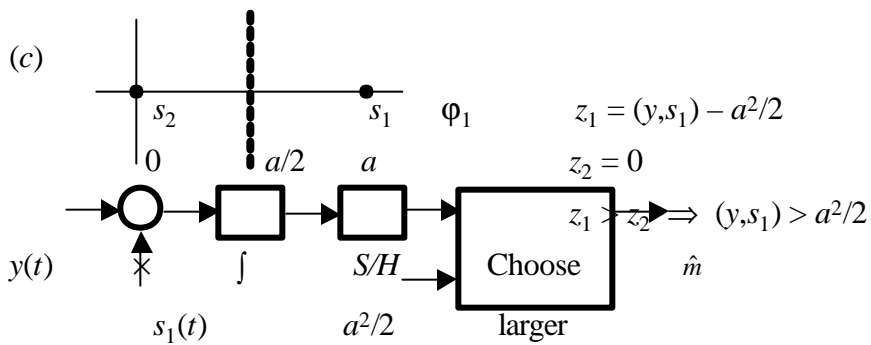
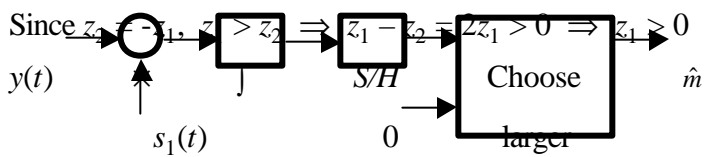
$$c_1 = c_2 = a^2/2 \text{ so take}$$

$$z_1 = (y, s_1) \quad z_2 = (y, s_2)$$



$$c_1 = c_2 = a^2 \text{ so take}$$

$$z_1 = (y, s_1) \quad z_2 = (y, s_2) = -(y, s_1)$$

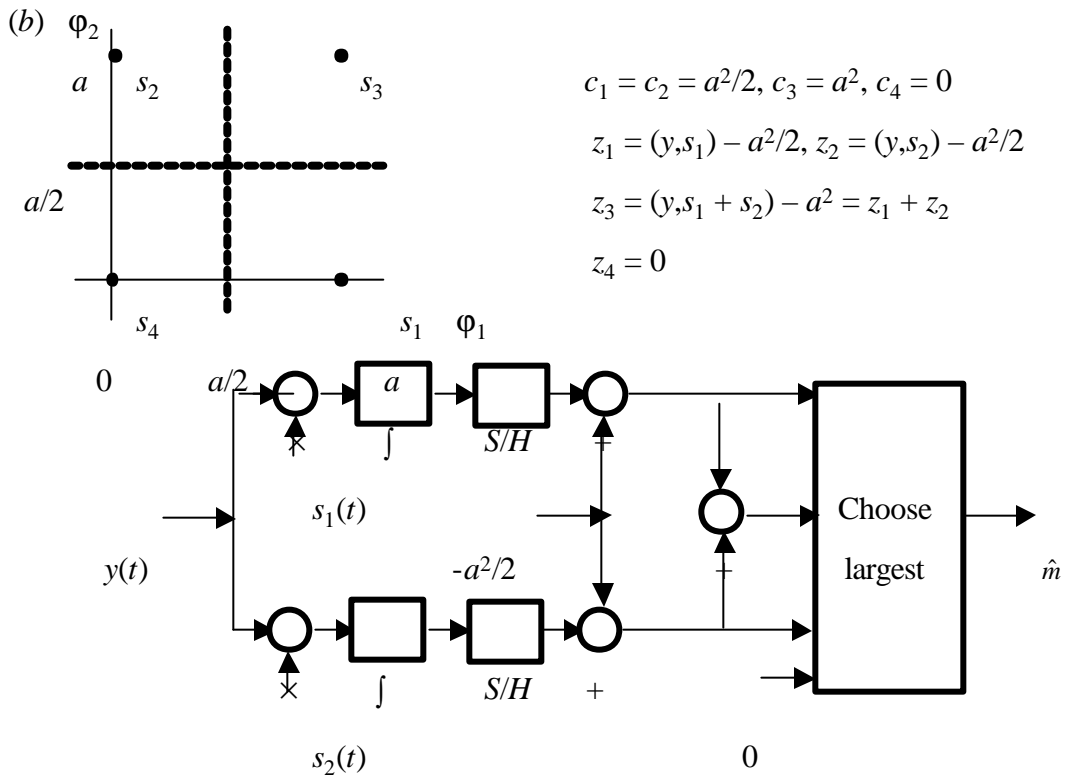
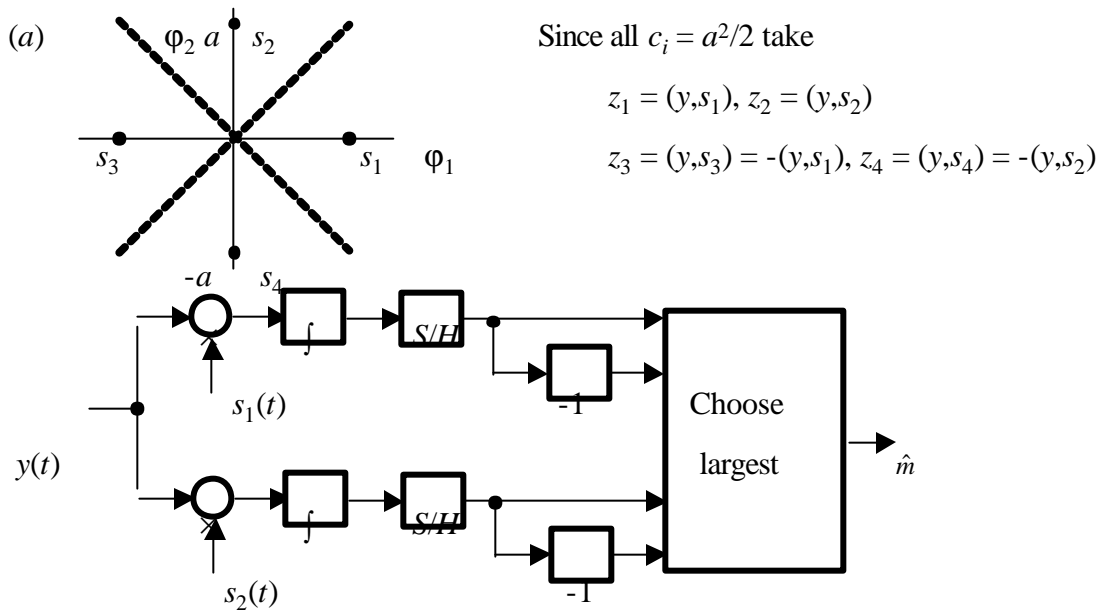


$$z_1 = (y, s_1) - a^2/2$$

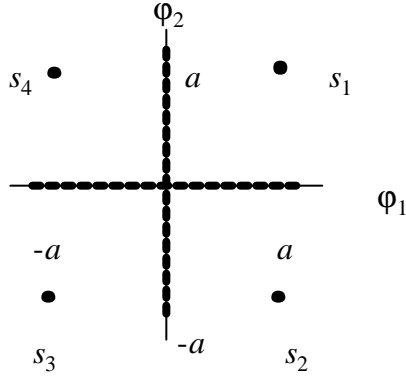
$$z_2 = 0$$

$$z_1 > z_2 \Rightarrow (y, s_1) > a^2/2$$

16.5-2



16.5-3

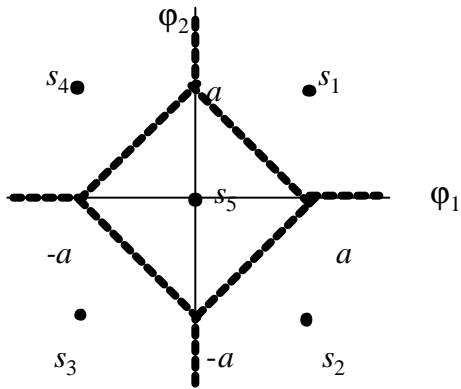


$$E_i = (\sqrt{2}a)^2 \text{ for all } i \text{ so } E = \frac{1}{4} \times 4 (\sqrt{2}a)^2 = 2a^2$$

$$\text{For all } i, P(c | m_i) = \int_{-a}^{\infty} p_{\beta}(\beta_1) d\beta_1 \int_{-a}^{\infty} p_{\beta}(\beta_2) d\beta_2 = \left[ 1 - Q\left(\frac{a}{\sqrt{N_0/2}}\right) \right]^2 \text{ where } a = \sqrt{E/2}$$

$$\text{Thus, } P_e = 1 - \frac{1}{4} \times 4 \left[ 1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \right]^2 = 2Q\left(\sqrt{\frac{E}{N_0}}\right) - Q^2\left(\sqrt{\frac{E}{N_0}}\right)$$

16.5-4



$$E_i = (\sqrt{2}a)^2 \quad i = 1, 2, 3, 4$$

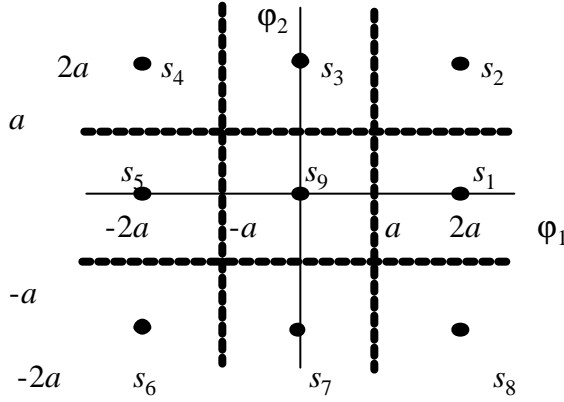
$$E_5 = 0$$

$$E = \frac{1}{5} \times 4 (\sqrt{2}a)^2 = \frac{8}{5} a^2$$

Since  $s_5$  is nearest neighbor to all other  $s_i$ ,  $d_j = \sqrt{2}a = \sqrt{5E/4} \quad j = 1, 2, \dots, 5$

$$\text{Thus, } P_e \leq \frac{4}{5} \times 5Q\left(\frac{\sqrt{2}a}{\sqrt{2N_0}}\right) = 4Q\left(\sqrt{\frac{5E}{8N_0}}\right)$$

16.5-5



$$\begin{aligned} E_i &= (2a)^2 & i = 1, 3, 5, 7 \\ &= 2(2a)^2 & i = 2, 4, 6, 8 \\ &= 0 & i = 9 \end{aligned}$$

$$E = \frac{1}{9} [4 \times (2a)^2 + 4 \times 2(2a)^2] = \frac{48}{9} a^2$$

$$\text{Let } q = Q\left(\frac{a}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{9E}{24N_0}}\right)$$

$$P(c|m_9) = \int_{-a}^a p_{\beta}(\beta_1) d\beta_1 \int_{-a}^a p_{\beta}(\beta_2) d\beta_2 = (1-2q)^2$$

$$\text{For } i = 1, 3, 5, 7 \quad P(c|m_i) = \int_{-a}^{\infty} p_{\beta}(\beta_1) d\beta_1 \int_{-a}^a p_{\beta}(\beta_2) d\beta_2 = (1-q)(1-2q)$$

$$\text{For } i = 2, 4, 6, 8 \quad P(c|m_i) = \int_{-a}^{\infty} p_b(\mathbf{b}_1) d\mathbf{b}_1 \int_{-a}^{\infty} p_b(\mathbf{b}_2) d\mathbf{b}_2 = (1-q)^2$$

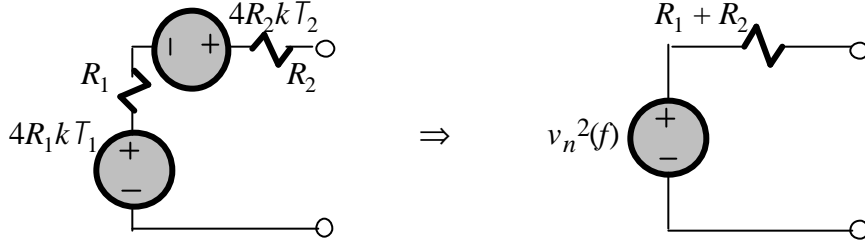
$$\text{Thus, } P_c = \frac{1}{9} [4(1-q)(1-2q) + 4(1-q)^2 + (1-2q)^2] = \frac{1}{9} (9 - 24q + 16q^2) \text{ and}$$

$$P_e = 1 - P_c = \frac{24}{9} q - \frac{16}{9} q^2 = \frac{24}{9} Q\left(\sqrt{\frac{9E}{24N_0}}\right) - \frac{16}{9} Q^2\left(\sqrt{\frac{9E}{24N_0}}\right)$$

$$\text{For union bound note that } d_j = 2a, j = 1, 2, \dots, 9 \text{ so } P_e \leq \frac{8}{9} \times 9Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 8Q\left(\sqrt{\frac{9E}{24N_0}}\right)$$

## Appendix

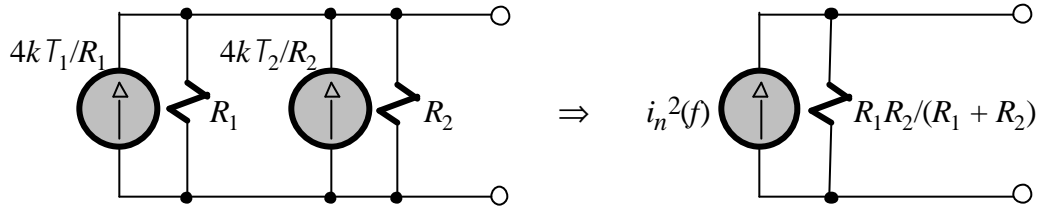
A-1



$$v_n^2(f) = 4R_1 k T_1 + 4R_2 k T_2, \quad i_n^2(f) = \frac{v_n^2(f)}{|Z(f)|^2} = \frac{4k(R_1 T_1 + R_2 T_2)}{(R_1^2 + R_2^2)}$$

$$\text{If } T_1 = T_2 = T, \text{ then } v_n^2(f) = 4(R_1 + R_2)kT, \quad i_n^2(f) = \frac{4kT}{R_1 + R_2}$$

A-2



$$i_n^2(f) = \frac{4k T_1}{R_1} + \frac{4k T_2}{R_2}$$

$$v_n^2(f) = |Z(f)|^2 i_n^2(f) = \left( \frac{R_1 R_2}{R_1 + R_2} \right)^2 \frac{4k R_2 T_1 + 4k R_1 T_2}{R_1 R_2} = \frac{4k R_1 R_2}{(R_1 + R_2)^2} (R_2 T_1 + R_1 T_2)$$

$$\text{If } T_1 = T_2 = T, \text{ then } i_n^2(f) = 4 \frac{R_1 + R_2}{R_1 R_2} kT \text{ and } v_n^2(f) = 4 \frac{R_1 R_2}{R_1 + R_2} kT$$

A-3

$$Z(f) = \frac{9(1 + jf)}{10 + jf} \Rightarrow R(f) = 9 \frac{10 + f^2}{100 + f^2} \Rightarrow v_n^2(f) = 36kT \frac{10 + f^2}{100 + f^2}$$

A-4

$$Z(f) = \frac{R(R - j/\omega C)}{2R - j/\omega C} \Rightarrow R(f) = R \frac{2R^2 + (1/\omega C)^2}{4R^2 + (1/\omega C)^2} \Rightarrow v_n^2(f) = 4RkT \frac{1 + 2(2\pi RCf)^2}{1 + 4(2\pi RCf)^2}$$



A-5

If  $qV/kT \gg 1$ , then  $I \gg I_S$  so  $i_n^2(f) \approx 2qI$  and  $r \approx kT/qI$ . Thus,  $i_n^2(f) \approx 2rkT = 4(r/2)kT$ , which looks like thermal noise from resistance  $R = r/2$ .

A-6

With  $R_S = r_i = r_o = 50 \Omega$ , Eq. (8) yields  $g_a(f) = \left( \frac{|H(f)|50}{100} \right)^2 \frac{50}{50} = \frac{1}{4} |H(f)|^2$ . Then,

since  $\Pi^2(\cdot) = \Pi(\cdot)$ , Eq. (10) becomes  $gB_N = \int_0^\infty g_a(f) df = \frac{200^2}{4} \int_0^\infty \Pi\left(\frac{f-f_c}{B}\right) df = 10^4 B = 10^{10}$ .

Then, from Eq. (11),  $kT_e = \frac{1}{gB_N} \int_0^\infty \eta_{\text{int}}(f) df = \frac{2 \times 10^{-16} B}{10^{10}} = 2 \times 10^{-20}$ . Finally, with  $T_s = T_0$ ,

Eq. (12) becomes

$$N_o = gB_N (kT_s + kT_e) \approx 10^{10} (4 \times 10^{-21} + 2 \times 10^{-20}) = 2.4 \times 10^{-10} = 240 \text{ pW}.$$

A-7

$$T_s = T_0: N_o = 10^5 k(T_0 + T_e) \times 20 \times 10^3 = 8 \times 10^{-12} \frac{T_0 + T_e}{T_0} = 80 \times 10^{-12} \text{ so } (T_0 + T_e)/T_0 = 10 \text{ and}$$

$$T_e = 9T_0, F = 1 + 9 = 10$$

$$T_s = 2T_0: N_o = 10^5 \times 4 \times 10^{-21} \frac{2T_0 + 9T_0}{T_0} \times 20 \times 10^3 = 88 \text{ pW}$$

A-8

$$T_s = T_0: N_o = gk(T_0 + T_e)B_N,$$

$$T_s = 2T_0: N_o = gk(2T_0 + T_e)B_N = \frac{4}{3} gk(T_0 + T_e)B_N \text{ so}$$

$$2T_0 + T_e = \frac{4}{3}(T_0 + T_e). \text{ Thus, } T_e = 2T_0, F = 1 + 2 = 3.$$

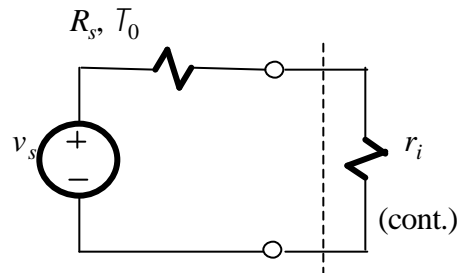
A-9

Assume matched impedances ( $R_s = r_i$ ), so

$$N_1 = CN_o = Cgk(T_0 + T_e)B_N$$

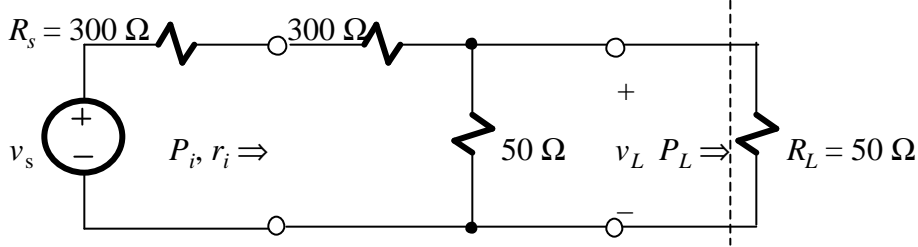
$$N_2 = N_1 + CgS = 2N_1 \Rightarrow S = k(T_0 + T_e)B_N$$

$$\text{Thus, } T_e = \frac{S}{kB_N} - T_0 \Rightarrow F = \frac{S}{kT_0 B_N}$$



To determine  $F$  by this method, we need an *absolute* power meter to measure  $S$ , and we must also know  $B_N$ . Both requirements are disadvantages compared to the noise-generator method.

A-10



$$r_i = 300 + \frac{50 \times 50}{50 + 50} = 325 \, \Omega \quad P_i = \left( \frac{325}{300 + 325} v_s \right)^2 / r_i$$

$$v_L = \frac{50 \parallel 50}{300 + 300 + 50 \parallel 50} v_s \quad P_L = \left( \frac{25}{625} v_s \right)^2 / R_L$$

$$g = \frac{P_L}{P_i} = \left( \frac{25 v_s}{625} \right)^2 \frac{1}{50} \times \left( \frac{625}{325 v_s} \right)^2 \times 325 = \frac{1}{26}. \text{ Thus, } F = L = 1/g = 26.$$

A-11

$$T_e = T_{e1} + T_{e2}/g \text{ where } F_2 = 13.2 \text{ dB} = 21 \Rightarrow T_{e2} = (21 - 1)T_0, \text{ so}$$

$$T_e = 3T_0 + 20T_0/10 = 5T_0 \text{ and}$$

$$\left( \frac{S}{N} \right)_o = \frac{S_s}{k(T_s + T_e)B_N} = \frac{S_s}{15kT_0B_N} = 10^3 \Rightarrow S_s = 6 \times 10^{-12} = 6 \text{ pW}$$

A-12

$$\text{Since } T_s = T_0, \left( \frac{S}{N} \right)_o = \frac{1}{F} \left( \frac{S}{N} \right)_s \geq 0.05 \left( \frac{S}{N} \right)_s \Rightarrow F \leq 20$$

$$\text{Since } F_1 = 1/g_1 = L \text{ and } F_2 = 7 \text{ dB} \approx 5, F = F_1 + \frac{F_2 - 1}{g_1} = L + L(5 - 1) = 5L$$

$$\text{Thus, we want } 5L \leq 20 \Rightarrow L \leq 4 = 6 \text{ dB} \Rightarrow \underline{L} \leq 6 \text{ dB} / 2 \text{ dB/km} = 3 \text{ km}$$

A-13

$$\text{Preamp: } F = 2 \text{ and } g = 100; \text{ Cable: } F = 4 \text{ and } g = 1/4; \text{ Receiver: } F = 20$$

(cont.)

$$(a) F = 2 + \frac{4-1}{100} + \frac{20-1}{100 \times 1/4} = 2.79 = 4.5 \text{ dB}$$

$$(b) F = 4 + \frac{2-1}{1/4} + \frac{20-1}{1/4 \times 100} = 8.76 = 9.4 \text{ dB}$$

A-14

$$L_1 = 10^{0.15} = 1.413, g_2 = 100, F_3 = 10$$

$$T_e = 0.413T + 1.413 \times 50\text{K} + \frac{1.413}{100}(10 - 1)290\text{K} = 0.413T + 107.5\text{K}$$

$$\text{so } T_e \odot 150 \text{ K} \Rightarrow T \odot 103 \text{ K}$$

A-15

With unit #1 first, cascade has  $F_{12} = F_1 + (F_2 - 1)/g_1$ . Similarly, interchange the subscripts for unit #2

$$\text{first. Thus, } F_{12} - F_{21} = \left( F_1 + \frac{F_2 - 1}{g_1} \right) - \left( F_2 + \frac{F_1 - 1}{g_2} \right) = \left( F_1 - \frac{F_1 - 1}{g_2} \right) - \left( F_2 - \frac{F_2 - 1}{g_1} \right)$$

If unit #1 first yields  $F_{12} < F_{21}$ , then  $F_{12} - F_{21} < 0$  and

$$F_1 - \frac{F_1 - 1}{g_2} = (F_1 - 1) \left( 1 - \frac{1}{g_2} \right) + 1 < F_2 - \frac{F_2 - 1}{g_1} = (F_2 - 1) \left( 1 - \frac{1}{g_1} \right) + 1 \text{ so}$$

$$\frac{F_1 - 1}{1 - 1/g_1} < \frac{F_2 - 1}{1 - 1/g_2} \Rightarrow M_1 < M_2$$